MHD Simulations of Accretion Flows

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Recent collaborators:
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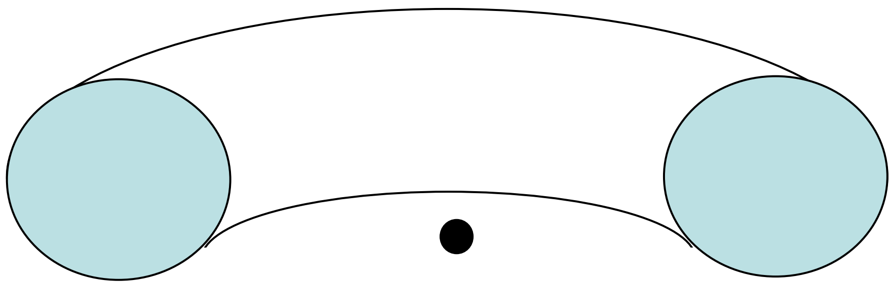
- Peter Teuben (UMd)
- Neal Turner (JPL)
- Julian Krolik, Shigenobu Hirose (JHU)

Outline of Talk

1. *Global* hydro and MHD simulations
2. MRI in *radiation dominated* disks
3. Local simulations of the MRI with *new Godunov scheme* for MHD

I. Global hydro and MHD simulations

- In last 5 years, numerical “experiments” have studied physics of global accretion flows
- Most begin evolution from rotationally supported torus (an exact equilibrium state in axisymmetry)



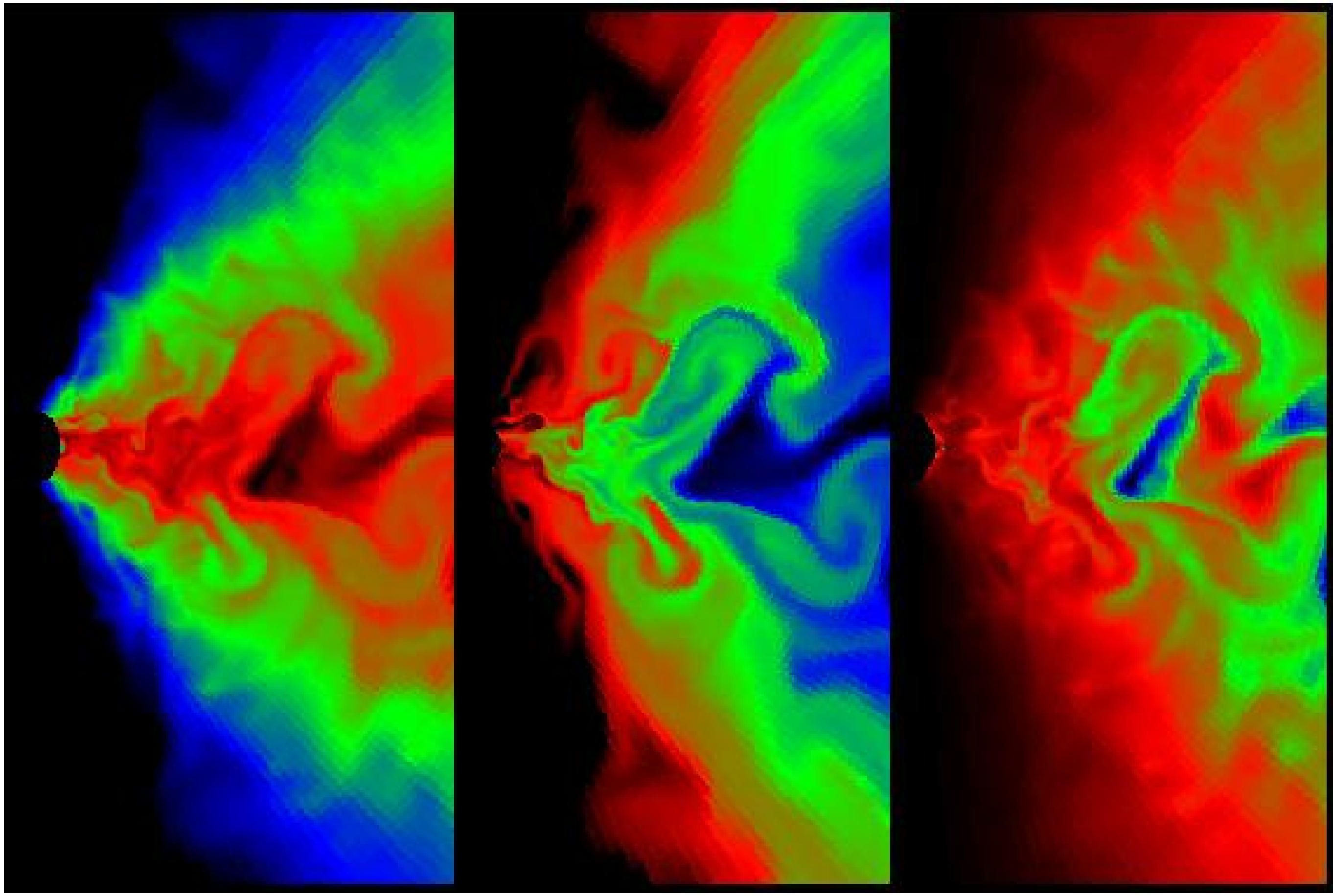
- Hydro: assume anomalous stress which follows the “ ” prescription
- MHD: stress provided by MRI
- Use spherical polar grid with factor ~10^2 range in radius
- Since $t_{orbital} \sim r^{3/2}$, must evolve for ~10^3 orbits in inner regions

Snapshot of inner 10% of *hydro* simulation after 3000 orbits

(Stone, Pringle, & Begelman 1999; Igumenshchev & Abramowicz 1999; 2000)

Animation of Log( )

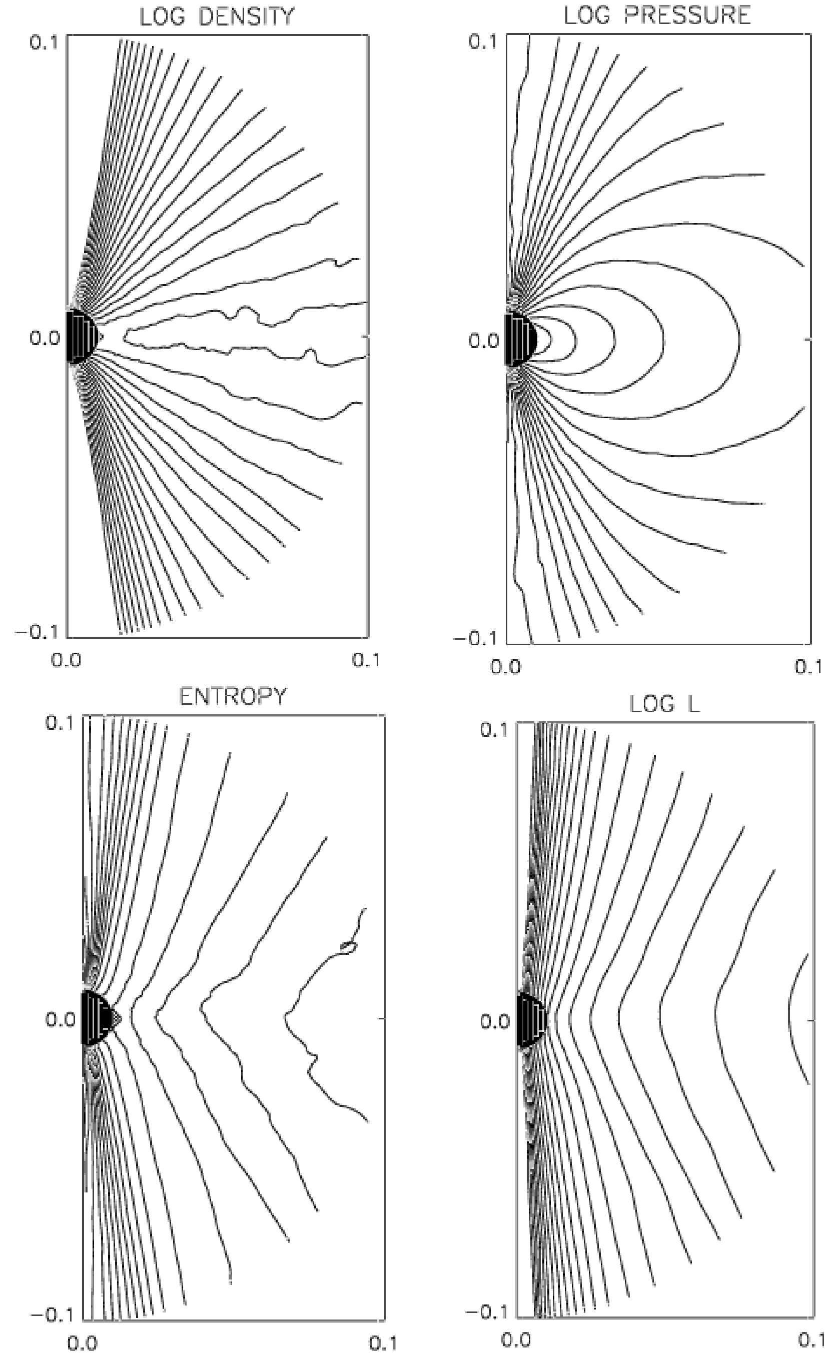
QuickTime™ and a
GIF decompressor
are needed to see this picture.



Flow dominated by convection.

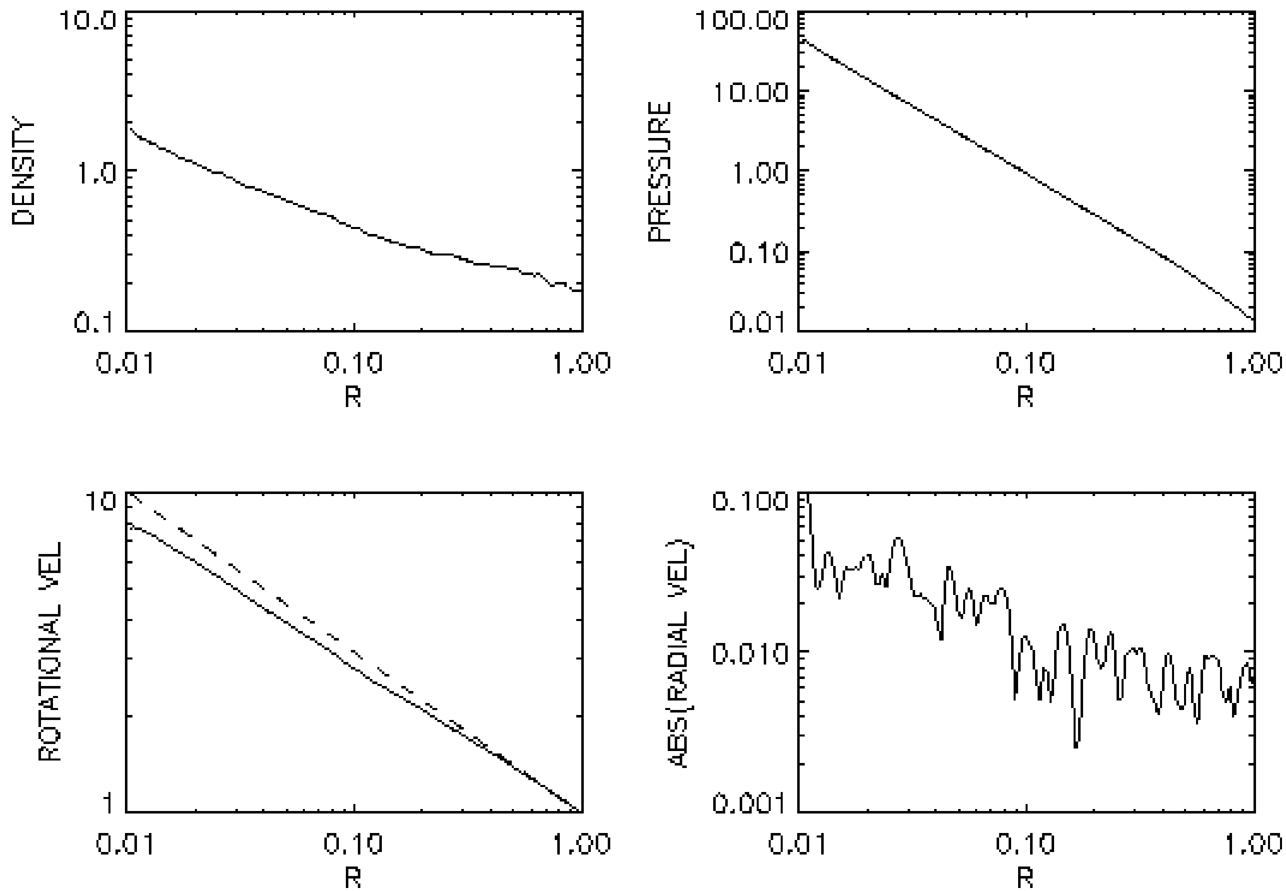
In hydro, time-averaged variables show that…

Contours of P and   very different.



Contours of S and L nearly parallel → marginal stability to one of Hoiland criterion

Time-averaged radial profiles are simple power laws



Simulations have $r^{-1/2}$, but an ADAF predicts $r^{-3/2}$

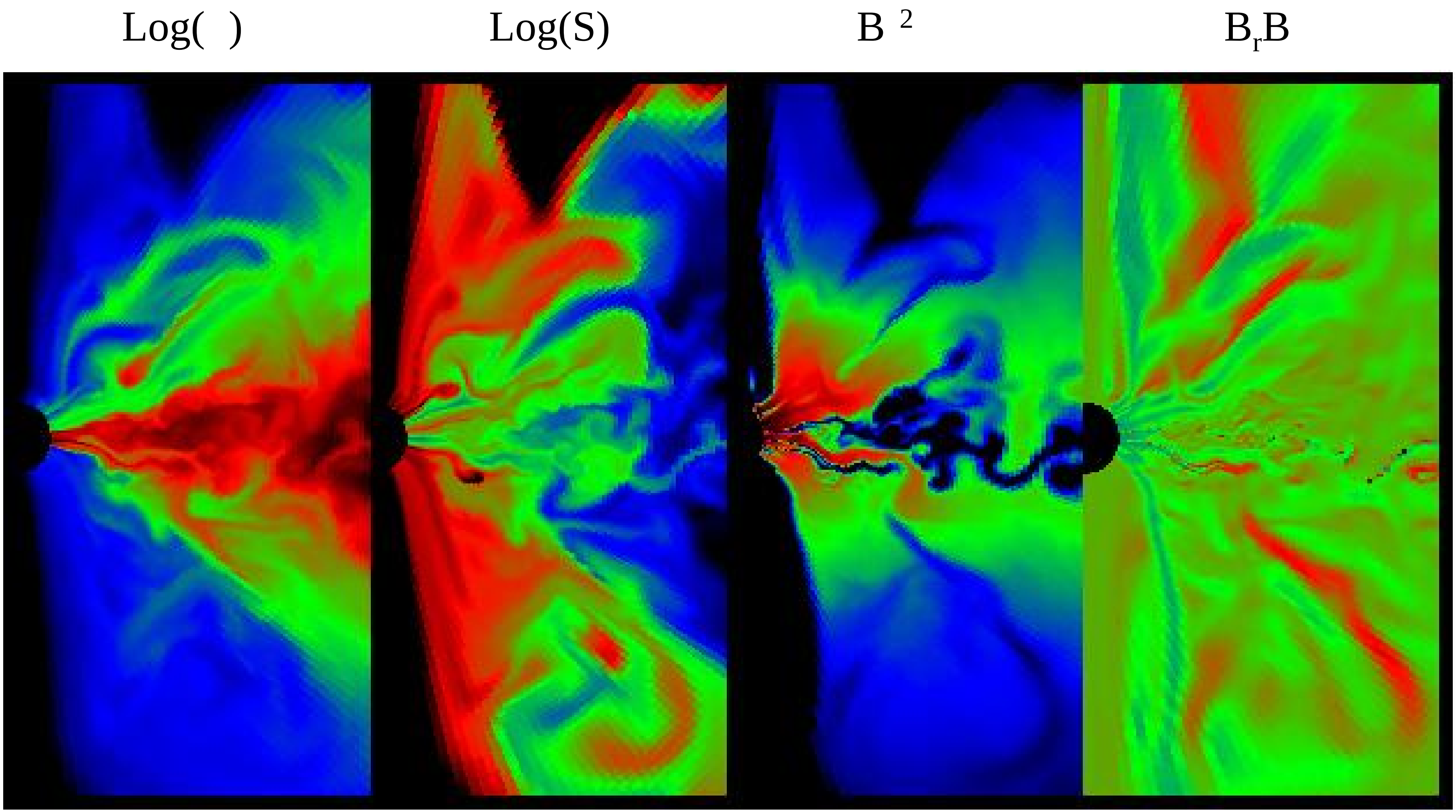
→ Much lower accretion rate in the center

Using condition that flow is marginally stable to convection, can derive new class of steady-state solutions: CDAFs (Narayan et al. 2000; Quataert & Gruzinov 2000)

In *MHD*; MRI produces turbulence and inward accretion

Snapshot of inner 10% of grid at t = 3250 orbits.

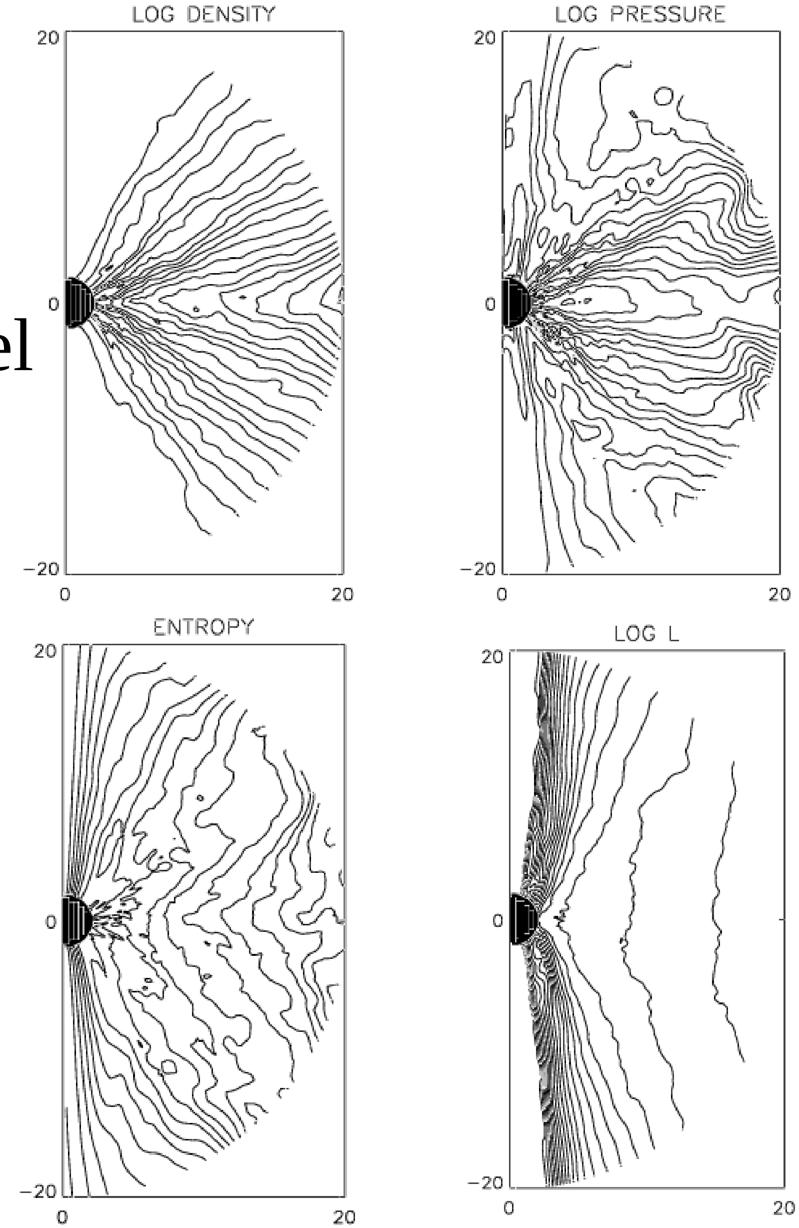
(Stone & Pringle 2001)



Time-averaged variables in MHD are different than hydro…

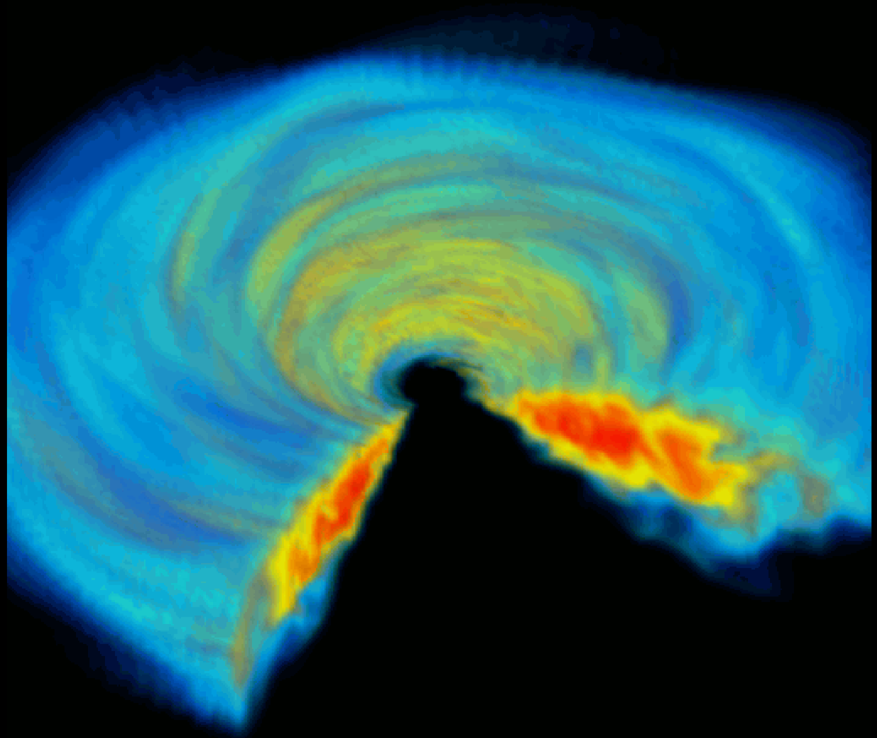
Contours of P and   nearly parallel → gas is barytropic

Contours of S and L no longer parallel → Hoiland criterion no longer applies



…not clear CDAF solutions are appropriate for MHD flows

Current state-of-the-art: Fully GR 3-D global models of geometrically thick accretion flows in Kerr metric.



See, e.g., J.Hawley's talk in afternoon session

II: radiation dominated disks

Studying this regime requires solving the equations of radiation MHD:

(Stone, Mihalas, & Norman 1992)

$$\frac{D\rho}{Dt} + \rho\nabla\cdot\mathbf{v} = 0,$$

$$\rho\frac{D\mathbf{v}}{Dt} = -\nabla p - \rho\nabla\Phi + \frac{1}{4\pi}(\nabla\times\mathbf{B})\times\mathbf{B} + \frac{1}{c}\chi_F\mathbf{F},$$

$$\rho\frac{D}{Dt}\left(\frac{E}{\rho}\right) = -\nabla\cdot\mathbf{F} - \nabla\mathbf{v}:\mathsf{P} + 4\pi\kappa_P B - c\kappa_E E,$$

$$\rho\frac{D}{Dt}\left(\frac{e+E}{\rho}\right) = -\nabla\mathbf{v}:\mathsf{P} - p\nabla\cdot\mathbf{v} - \nabla\cdot\mathbf{F},$$

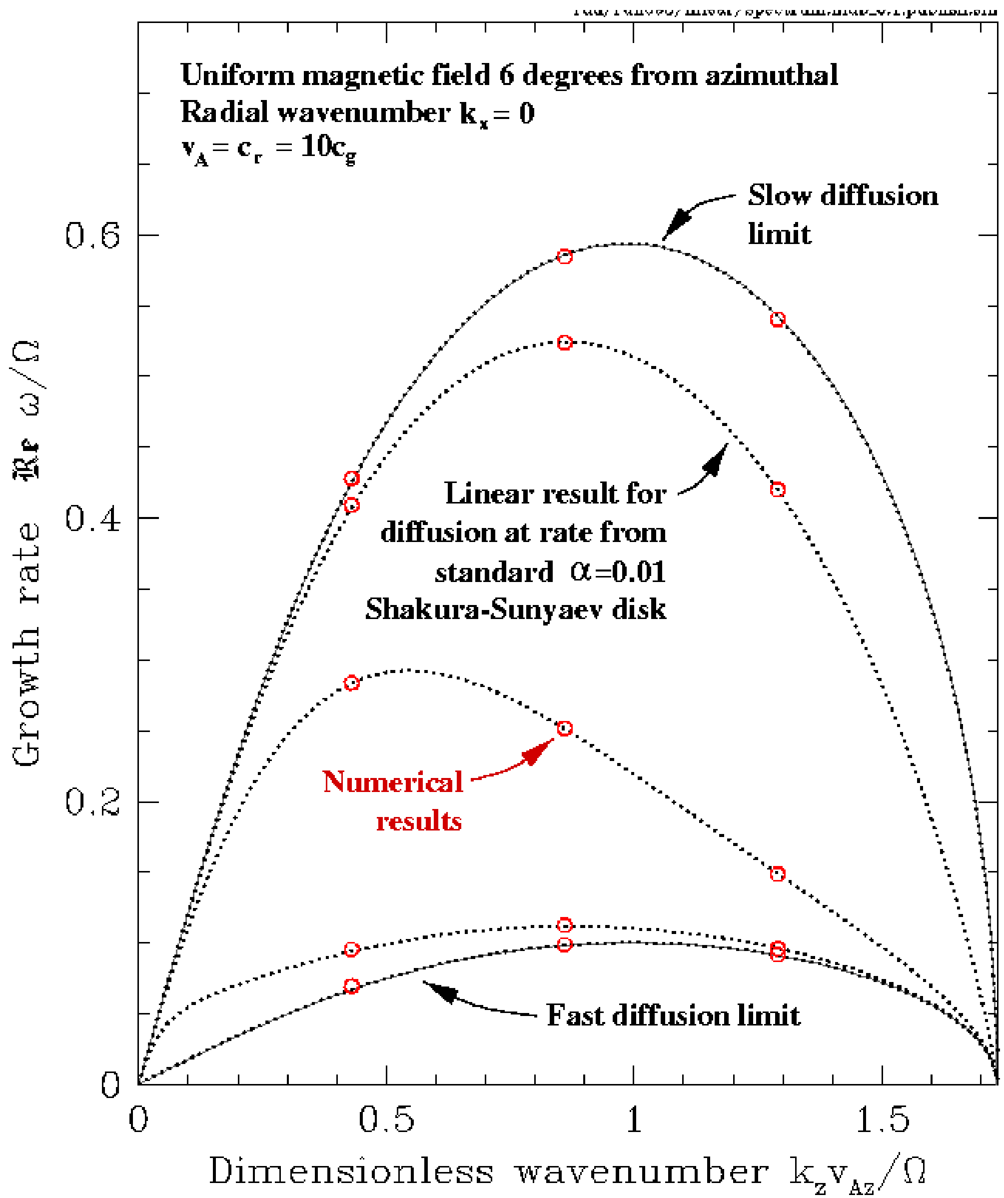
$$\frac{\rho}{c^2}\frac{D}{Dt}\left(\frac{\mathbf{F}}{\rho}\right) = -\nabla\cdot\mathsf{P} - \frac{1}{c}\chi_F\mathbf{F},$$

$$\frac{\partial\mathbf{B}}{\partial t} = \nabla\times(\mathbf{v}\times\mathbf{B}),$$

$$\Phi = -\frac{GM}{r}$$

Use ZEUS with flux-limited diffusion module (Turner & Stone 2001)

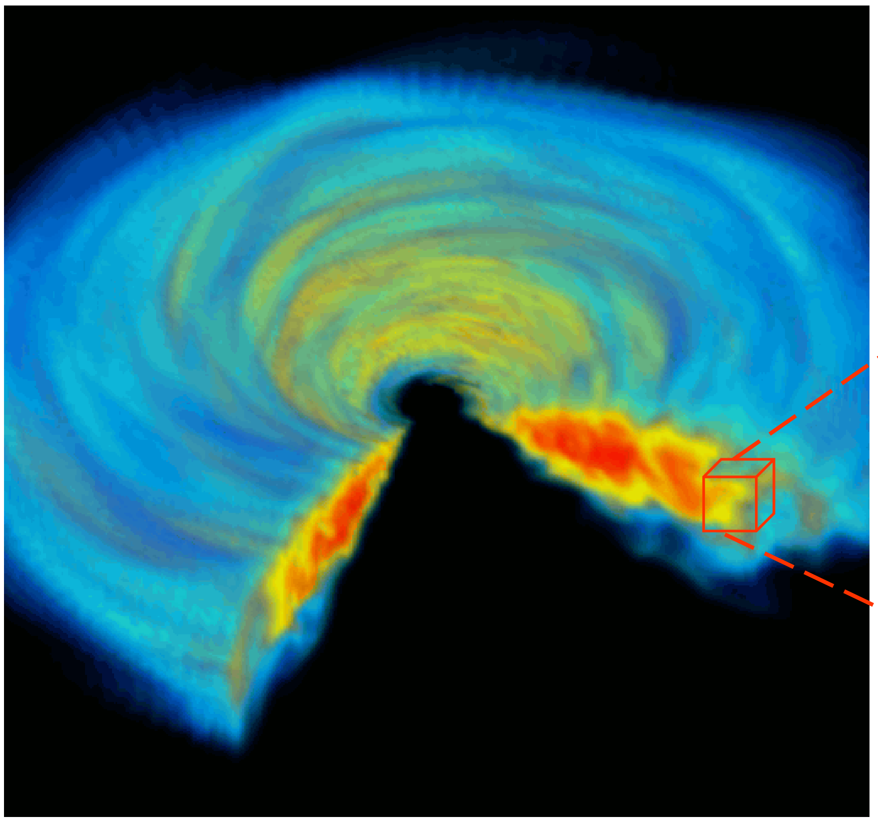
Linear growth rates of the MRI are changed by radiative diffusion (Blaes & Socrates 2001)



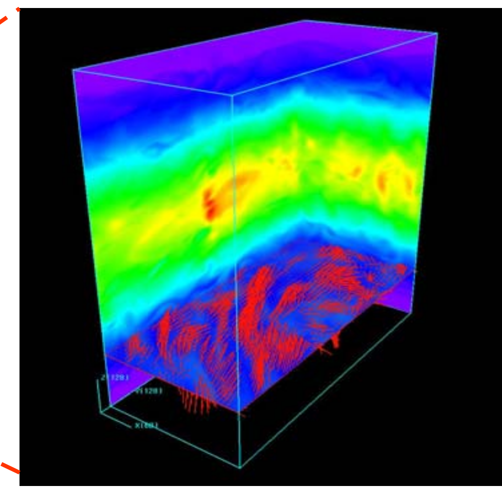
(Turner, Stone, & Sano 2002)

These simulations use a small, local patch of a disk termed the *shearing box*

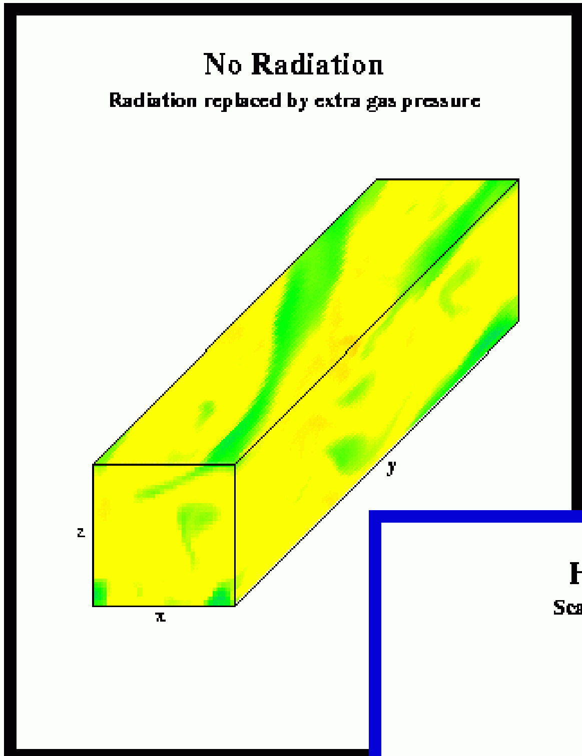
Hawley, Gammie, & Balbus 1995; 1996; Brandenburg et al. 1995; Stone et al. 1996; Matsumoto et al. 1996; Miller & Stone 1999



Global simulation

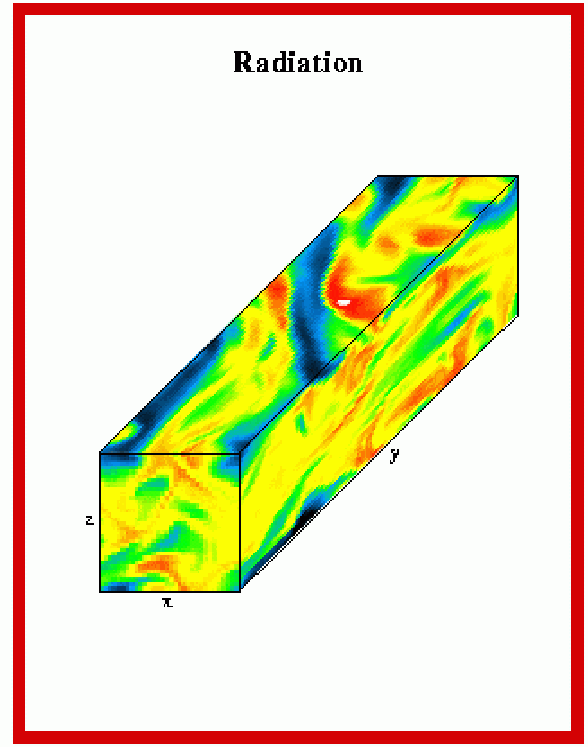


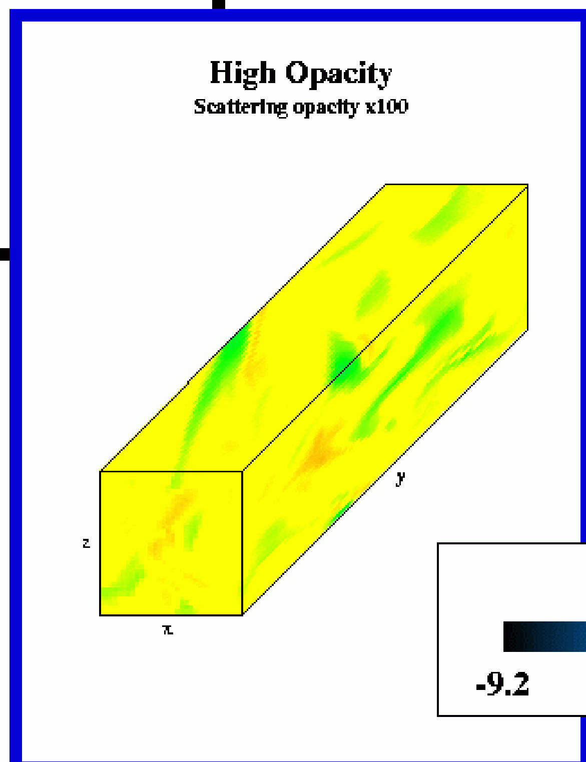
Local simulation



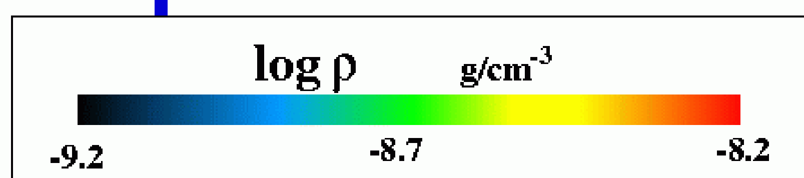
Uniform Vertical Initial Fields

at 20 orbits



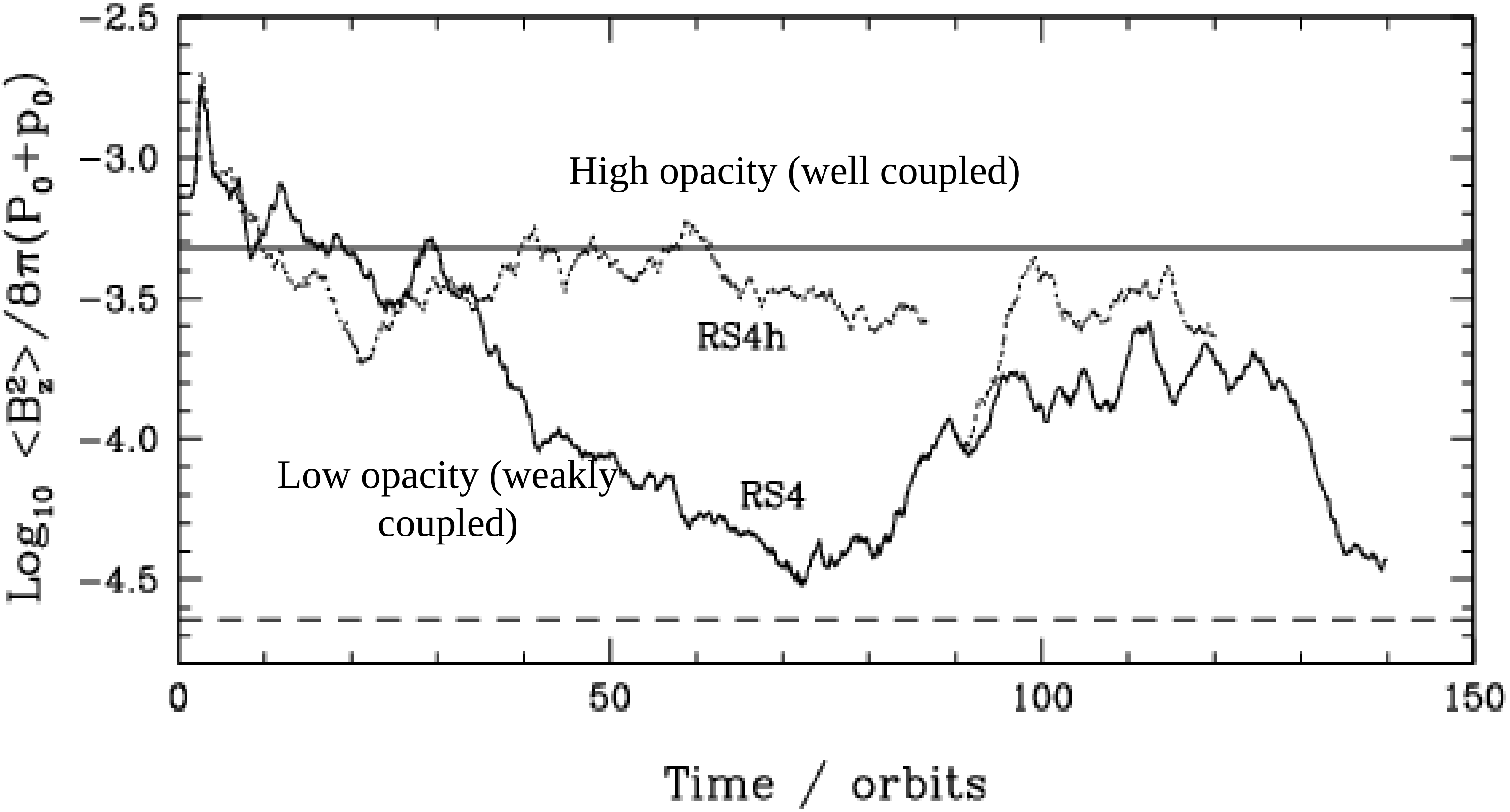


Density on faces of computational volume



Initial
$P_{rad}/P_{gas} = 100$

Saturation amplitude depends on total pressure if radiation and gas are well coupled, gas pressure if they are not.



Initial $P_{rad}/P_{gas} = 10$

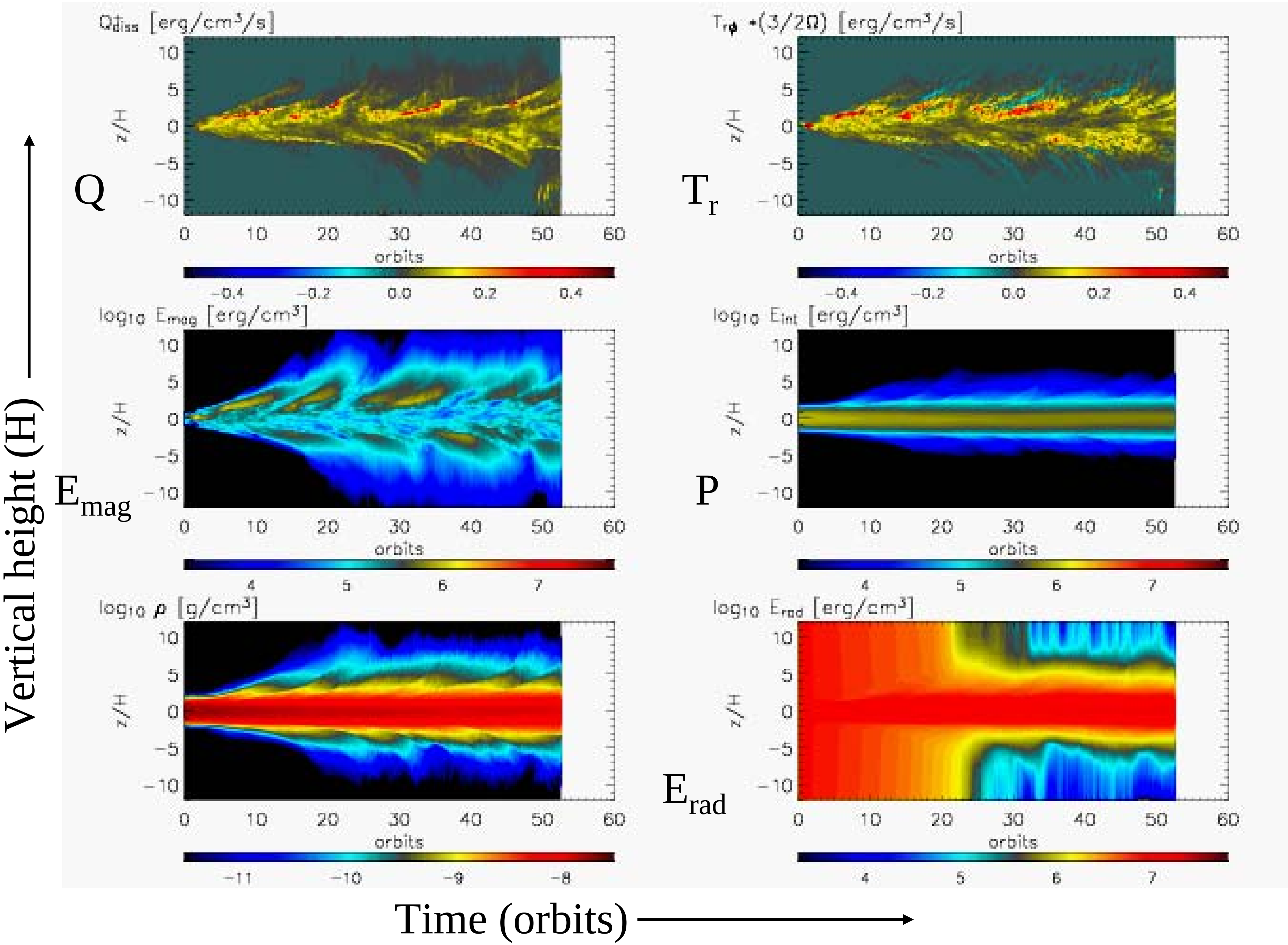
Vertically stratified radiation dominated disks

Hirose, Krolik, & Stone 2005

Motivation:

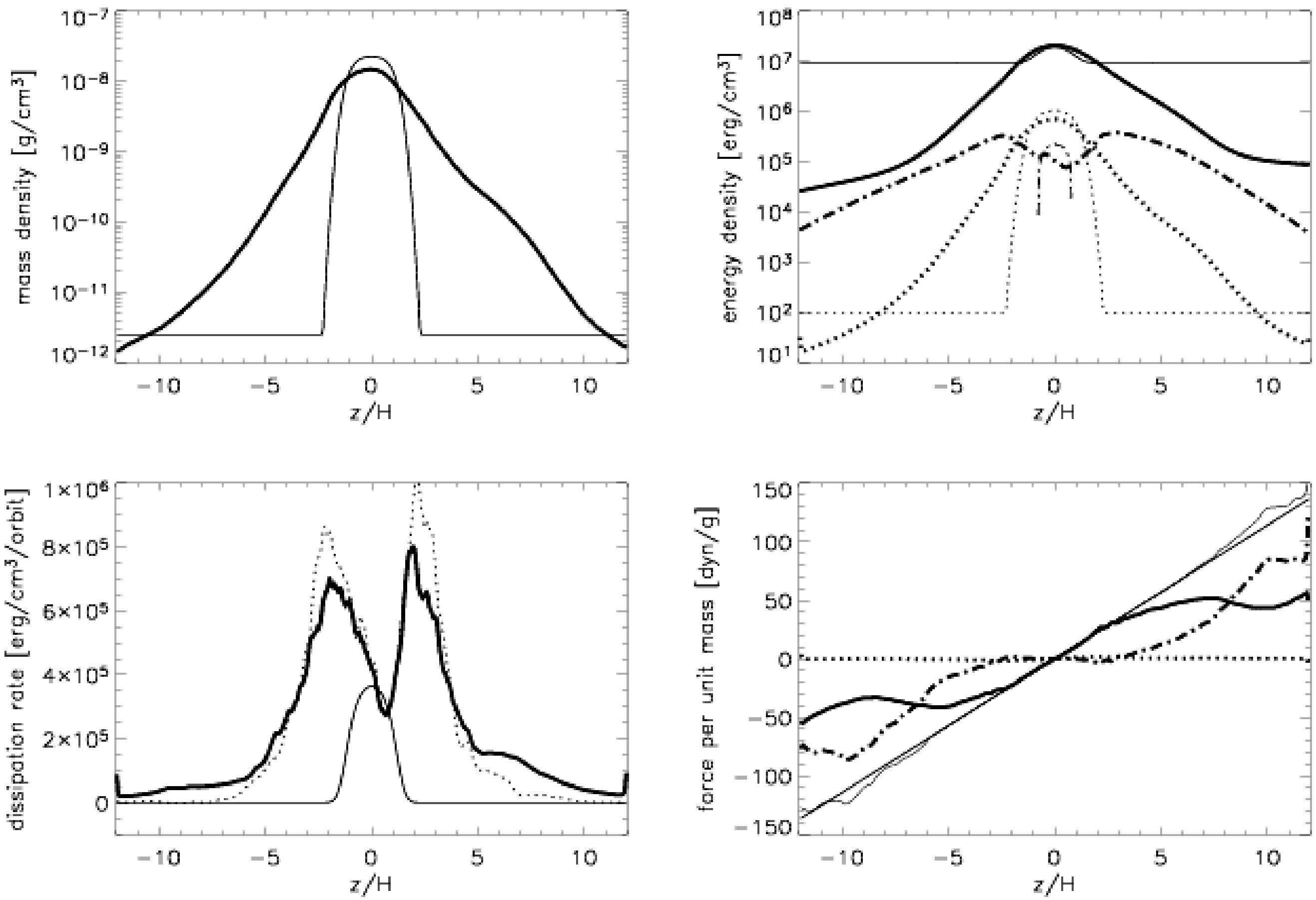
1. What is vertical structure of radiation dominated disk?
2. Need to include radiation to balance heating for truly steady-state disk models → spectra.

- Parameters same as Turner (2004) but lower density floor.
- Starts from SS model with R = 100 R_s
- Grid is 2*H* x 4*H* x 24*H* (32x64x384)
- P_{rad}/P_{gas} = 10, initial P_{rad}/P_{mag}= 25, zero-net-flux



Q
Tr
Emag
P
Erad
Vertical height (H)
Time (orbits)
Qdiss [erg/cm3/s]
Trφ *(3/2Ω) [erg/cm3/s]
log10 Emag [erg/cm3]
log10 Eint [erg/cm3]
log10 ρ [g/cm3]
log10 Erad [erg/cm3]
z/H
orbits

Vertical profiles (averaged over orbits 30-50)
Thick lines = initial distribution



E_{rad}

E_{mag}

P

Q

T_r

- Final vertical profiles much different than SS disk,
- Disk much thinner than in Turner (2004)
- $_{rad}$ = 0.02, saturation amplitude determined by P_{rad}

No evidence for photon bubble instability

Gammie (1998) and Blaes & Socrates (2001) have shown magnetosonic waves are linearly unstable in radiation dominated atmospheres

QuickTime™ and a
TIFF (LZW) decompressor
are needed to see this picture.

Turner et al. (2004) have shown they evolve into shocks in nonlinear regime:

Perhaps MRI destroys photon bubble modes?
Perhaps vertical profile emerging in disk is stable?

III. Local Simulations of MRI with a new MHD Code

Global model of *geometrically thin* (*H*/R << 1) disk covering 10*H* in R, 10*H* in Z, and 2 in azimuth with resolution of shearing box (128 grid points/*H*) will require nested grids.

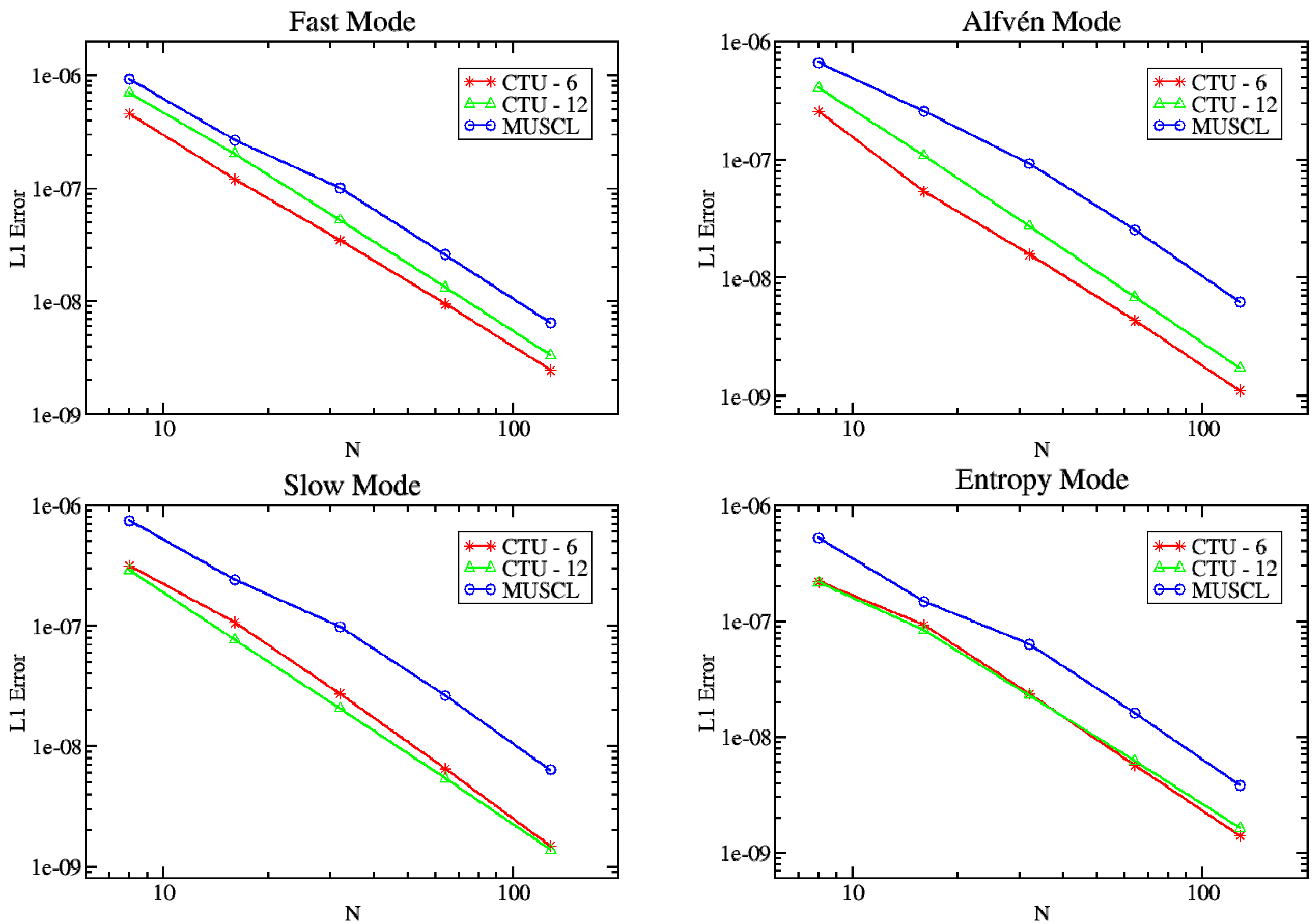
Nested (and adaptive) grids work best with single-step Eulerian methods based on the conservative form

Algorithms in ZEUS are 15+ years old - a new code could take advantage of developments in numerical MHD since then.

Athena – What is it?

- PPM Godunov Algorithm for MHD
- Evolves **B** using Constrained Transport (·**B** = 0)
- Unsplit Integration Algorithm (CTU; Colella 1991)
- 2D Algorithm Paper (Gardiner & Stone 2004, JCP)
- Fully conservative, 2nd order accurate method
- Ideal for nested grid (AMR) calculation
- 1D and 2D versions released in C & F95 with docs
- www.astro.princeton.edu/~jstone/athena.html

Linear Wave Convergence
(2N x N x N) Grid



2D MRI

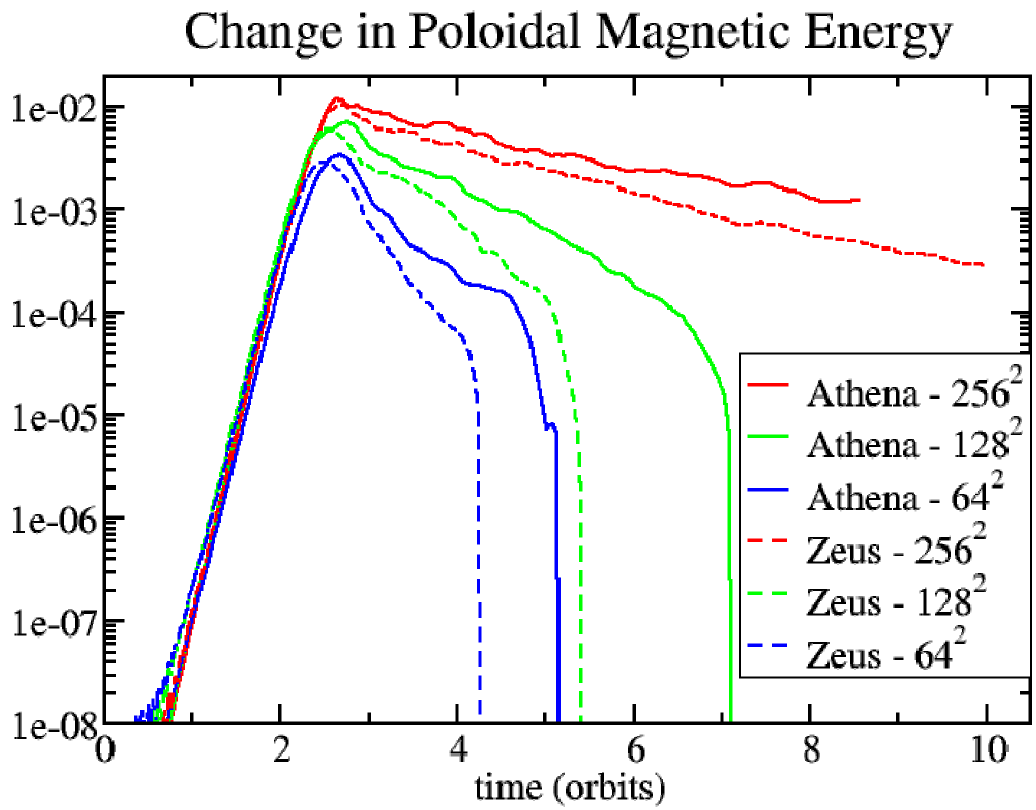
Animation of angular velocity fluctuations: ♎$V_y = V_y + 1.5$⌖$_0 x$ shows saturation of MRI and decay in 2D

QuickTime™ and a
GIF decompressor
are needed to see this picture.

CTU with 3rd order reconstruction, 256^2 Grid

♌$_{min}$=4000, orbits 2-10

Magnetic Energy Evolution
ZEUS vs. Athena



Numerical dissipation is ~1.5 times smaller with CTU & 3rd order reconstruction than ZEUS.

3D MRI

Animation of angular velocity fluctuations: $\delta V_y = V_y + 1.5\Omega_0 x$
Initial Field Geometry is Uniform B_y

QuickTime™ and a
GIF decompressor
are needed to see this picture.

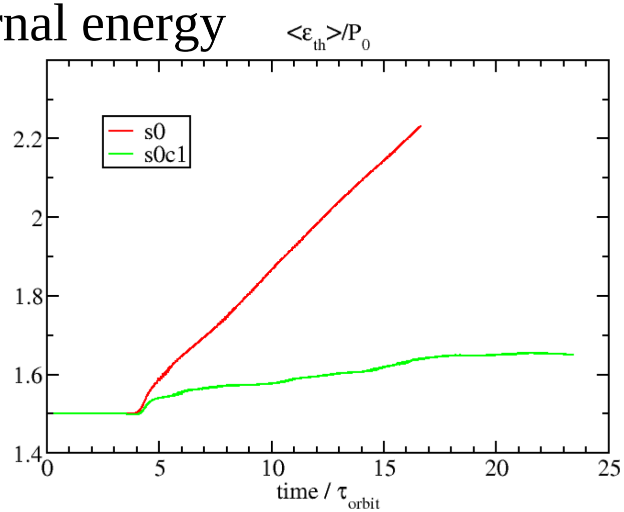
CTU with 3rd order reconstruction,
128 x 256 x 128 Grid
$\beta_{min} = 100$, orbits 4-20

Goal: Since Athena is strictly conservative, can measure spectrum of T fluctuations from dissipation of turbulence

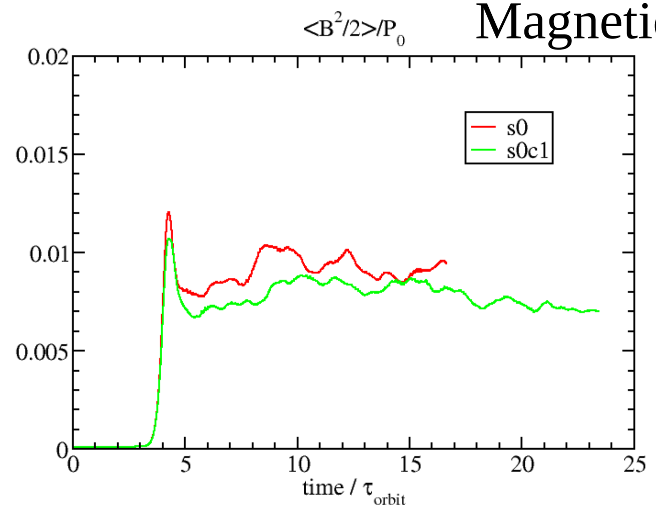
Dependence of saturated state on cooling

Red line: no cooling; Green line: $_{\text{cool}} = Q$

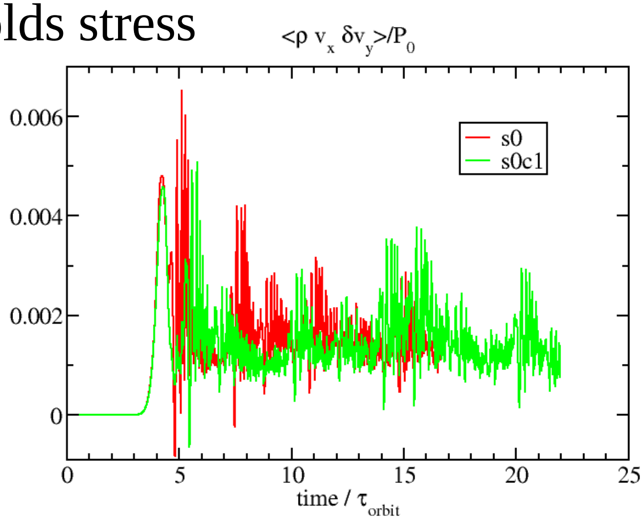
Internal energy



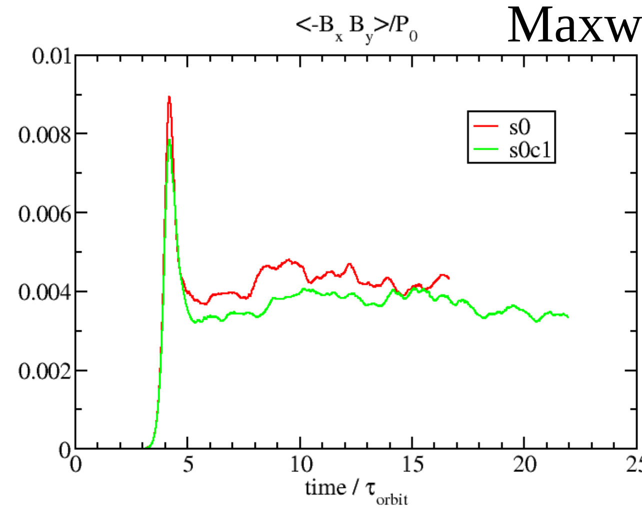
Magnetic energy



Reynolds stress

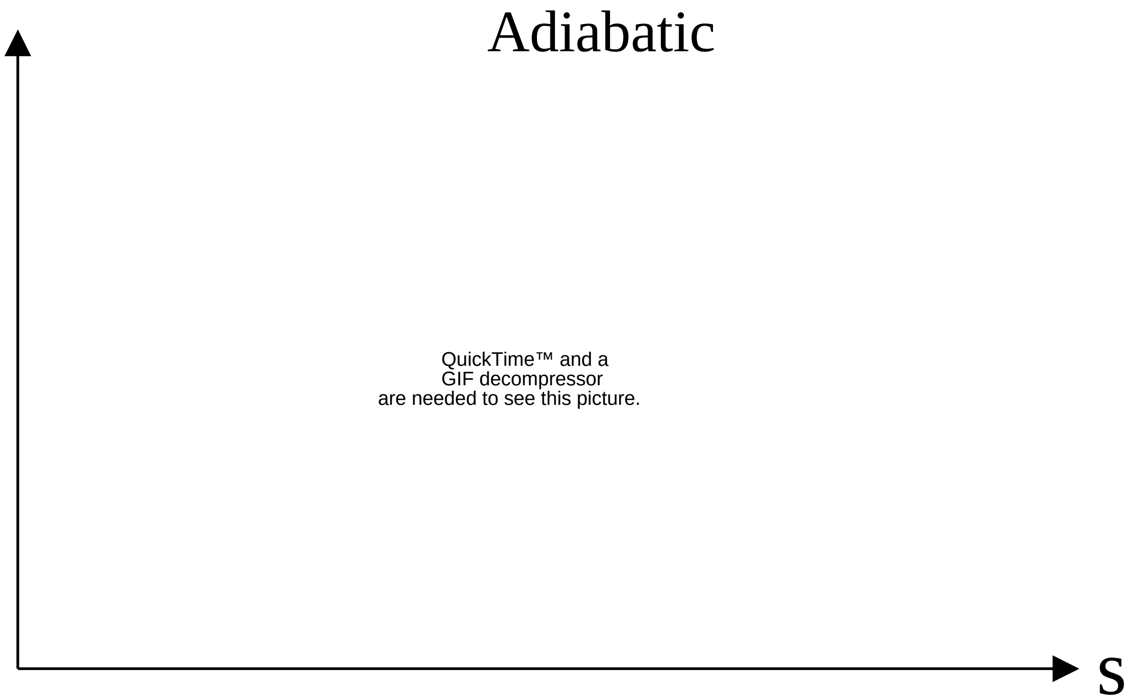


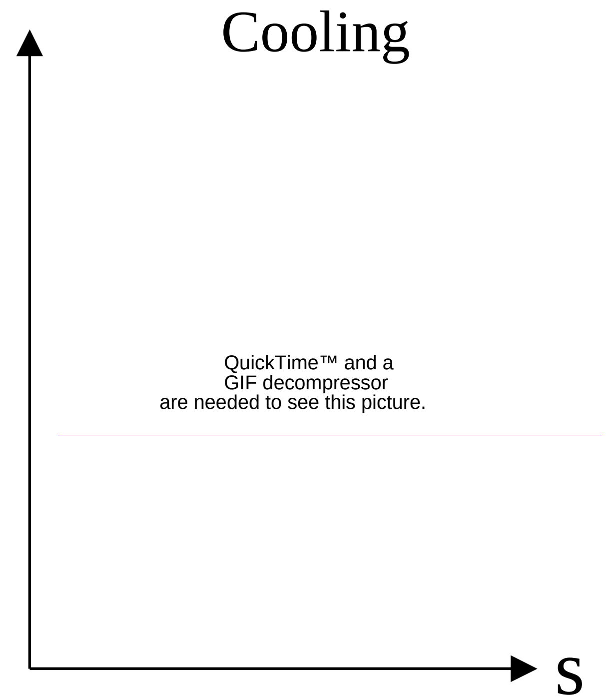
Maxwell stress



Cooling has almost no effect except on internal energy

Probability Distribution s)





- Dissipation / Cooling translates the distribution to the right / left
- Adiabatic waves redistribute the PDF vertically
- Temperature fluctuations dominated by compressive waves

Conclusions

1. 3D global simulations of geometrically thick disks are routine (see afternoon session). Thin disks are next.
2. Local simulations of radiation dominated disks allow first-principles disk models (structure, heating rate, spectra?)
3. A new fully conservative MHD code is allowing new studies of MRI: with nested grids will be ideal for global thin disk models.

Conclusions

- 3D global simulations of geometrically thick disks are routine (see afternoon session). Thin disks are next.
- Local simulations of radiation dominated disks reveal:
  - Saturation amplitude of the MRI depends on $P_{rad} + P_{gas}$ if radiation is strongly coupled to the gas, P_{gas} if it is not
  - Vertical profile of radiation dominated disk different than SS
- A new conservative algorithm is being used to study energy dissipation in MHD turbulence driven by MRI
  - saturation amplitudes are insensitive to cooling.
  - Temperature fluctuations dominated by compressive waves.