Cosmic Ray Transport in MHD Turbulence

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Recent advances in understanding of magnetohydrodynamic (MHD) turbulence call for revisions in the picture of cosmic ray transport. In this paper we use recently obtained scaling laws for MHD modes to obtain the scattering frequency for cosmic rays. We account for the turbulence cutoff arising from both collisional and collisionless damping. We obtain the scattering rate and show that fast modes provide the dominant contribution to cosmic ray scattering for the typical interstellar conditions in spite of the fact that fast modes are subjected to damping. We determine how the efficiency of the scattering depends on the characteristics of ionized media, e.g. plasma β. We show that streaming instability is suppressed by the ambient MHD turbulence.

1. Introduction

The propagation of cosmic rays (CRs) is affected by their interaction with magnetic field. This field is turbulent and therefore, the resonant interaction of cosmic rays with MHD turbulence has been discussed by many authors as the principal mechanism to scatter and isotropize cosmic rays. Although cosmic ray diffusion can happen while cosmic rays follow wandering magnetic fields ([14]), the acceleration of cosmic rays requires efficient scattering. For instance, scattering of cosmic rays back into the shock is a vital component of the first order Fermi acceleration.

While most investigations are restricted to Alfvén modes propagating along an external magnetic field (the so-called slab model of Alfvénic turbulence), obliquely propagating MHD modes have been included in [17] and later studies [25]. A more complex models were obtained by combining the results of the Reduced MHD with parallel slab-like modes have been also considered [3]. Here we attempt to use models that are motivated by the recent studies of MHD turbulence ([12], see [8] for a review and references therein).

The efficiency of scattering depends on turbulence anisotropy [20]. Therefore the calculations of CR scattering must be done using a realistic MHD turbulence model. An important attempt in this direction was carried out in [6]. However, only incompressible motions were considered. On the contrary, ISM is highly compressible. Compressible MHD turbulence has been studied recently (see review by [10] and references therein). [28] addressed the scattering by fast modes. But they did not consider the damping, which is essential for fast modes. In this paper we include various damping processes which can affect the fast modes.

2. MHD turbulence and its damping

2.1. Model of MHD turbulence

Analogous to MHD perturbations that can be decomposed into Alfvénic, slow and fast waves with well-defined dispersion relations, MHD perturbations that characterize turbulence can be separated into distinct modes. The separation into Alfvén and pseudo-Alfvén modes, which are the incompressible limit of slow modes, is an essential element of the GS95 [12] model. Even in a compressible medium, MHD turbulence is not an inseparable mess in spite of the fact that MHD turbulence is a highly non-linear phenomenon [9, 21]. The actual decomposition of MHD turbulence into Alfvén, slow and fast modes was addressed in [9, 10], who used a statistical procedure of decomposition in the Fourier space, where the basis of the Alfvén, slow and fast perturbations was defined.

Unlike hydrodynamic turbulence, Alfvénic one is anisotropic, with eddies elongated along the magnetic field. On the intuitive level it can be explained as the result of the following fact: it is easier to mix the magnetic field lines perpendicular to the direction of the magnetic field rather than to bend them. However, one cannot do mixing in the perpendicular direction to very small scales without affecting the parallel scales. This is probably the major difference between the adopted model of Alfvénic perturbations and the Reduced MHD [4]. In the GS95 model as well as in its generalizations for compressible medium mixing motions induce the reductions of the scales of the parallel perturbations.

The corresponding scaling can be easily obtained. For instance, calculations in [7] prove that motions perpendicular to magnetic field lines are essentially hydrodynamic. As the result, energy transfer rate due to those motions is constant $\dot{E}_k \sim v_k^2/\tau_k$, where τ_k is the energy eddy turnover time $\sim (v_k k_\perp)^{-1}$, where $k_\perp$ is the perpendicular component of the wave vector $\mathbf{k}$. The mixing motions couple to the wave-like motions parallel to magnetic field giving a critical balance condition, i.e., $k_\perp v_k \sim k_\parallel V_A$, where $k_\parallel$ is the parallel component of the wave vector $\mathbf{k}$, V_A is the Alfvén speed[1]. From these arguments, the scale dependent anisotropy $k_\parallel \propto k_\perp^{2/3}$and a Kolmogorov-like spectrum for the perpendicular motions $v_k \propto k^{-1/3}$ can be obtained [19].

It was conjectured in [21] that GS95 scaling should be approximately true for Alfvén and slow modes in moderately compressible plasma. For magnetically dominated, the so-called low β plasma, [9] showed that the coupling of Alfvénic and compressible modes is weak and that the Alfvénic and slow modes follow the GS95 spectrum. This is consistent with the analysis of HI velocity statistics [18, 29] as well as with the electron density statistics [2]. Calculations in [10] demonstrated that fast modes are marginally affected by Alfvén modes and follow acoustic cascade in both high and low β medium. In what follows, we consider both Alfvén modes and compressible modes and use the description of those modes obtained in [9, 10] to study CR scattering by MHD turbulence.

2.2. Damping of Turbulence

In many earlier papers Alfvénic turbulence was considered by many authors as the default model of interstellar magnetic turbulence. This was partially motivated by the fact that unlike compressible modes, the Alfvén ones are essentially free of damping in fully ionized medium[2]. However, it was shown that compressible fast modes are particularly important for cosmic ray scattering [31, 33]. For them damping is essential.

At small scales turbulence spectrum is altered by damping. Various processes can damp the MHD motions (see [33] for details). In partially ionized plasma, the ion-neutral collisions are the dominant damping process. In fully ionized plasma, there are basically two kinds of damping: collisional or collisionless damping. Their relative importance depends on the mean free path in the medium. If the wavelength is larger than the mean free path, viscous damping dominates. If, on the other hand, the wavelength is smaller than mean free path, then the plasma is in the collisionless regime and collisionless damping is dominant.

To obtain the truncation scale, the damping time Γ_d^{-1} should be compared to the cascading time τ_k. As we mentioned earlier, the Alfvénic turbulence cascades over one eddy turn over time $(k_\perp v_k)^{-1} \sim (k_\parallel V_A)^{-1}$. The cascade of fast modes takes a bit longer:

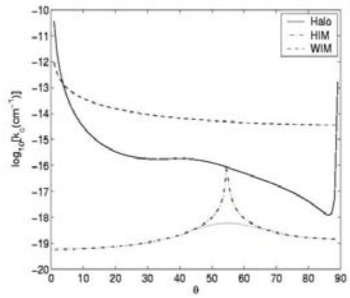

Figure 1: Damping scale vs. the angle θ between $\mathbf{k}$ and $\mathbf{B}$ in halo, HIM and WIM. The peak on the curve of HIM (dashdot line) is smeared out by randomization of both $\mathbf{k}$ and $\mathbf{B}$ [33].

$$\tau_k = \omega/k^2 v_k^2 = (k/L)^{-1/2} \times V_{ph}/V^2,$$

where V is the turbulence velocity at the injection scale, V_{ph} is is the phase speed of fast modes and equal to Alfvén and sound velocity for high and low β plasma, respectively [9]. If the damping is faster than the turbulence cascade, the turbulence gets truncated. Otherwise, for the sake of simplicity, we ignore the damping and assume that the turbulence cascade is unaffected. As the transfer of energy between Alfvén, slow and fast modes of MHD turbulence is suppressed at the scales less than the injection scale, we consider different components of MHD cascade independently.

We get the cutoff scale k_c by equating the damping rate and cascading rate $\tau_k \Gamma_d \simeq 1$. Then we check whether it is self-consistent by comparing the k_c with the relevant scales, e.g., injection scale, mean free path and the ion gyro-scale.

Damping is, in general, anisotropic, i.e., the linear damping (see [33] Appendix A) depends on the angle θ between the wave vector $\mathbf{k}$ and local direction of magnetic field $\mathbf{B}$. Unless randomization of θ is comparable to the cascading rate the damping scale gets angle-dependent[3]. The angle θ varies because of both the randomization of wave vector $\mathbf{k}$ and the wandering of magnetic field lines (see[33] for detail). With this input at hand, one can determine the turbulence damping scales given a medium.

[1] note that the linear dispersion relation is used for Alfvén modes.

[2] This picture contradicts to an erroneous assumption of strong coupling of compressible and incompressible MHD modes that still percolates the literature on turbulent star formation (see discussion in [15]). However, little cross talk between the different astrophysical communities allowed these two different pictures to coexist peacefully.

[3] The fast increase of collisionless damping with θ was applied to rotating stars, where [30] showed that the collisionless damping can be a dominant heating source for stellar wind.

3. Interactions between turbulence and particles

Basically there are two types of resonant interactions: gyroresonance acceleration and transit acceleration (henceforth TTD). The resonant condition is $\omega - k_\parallel v\mu = n\Omega$ ($n = 0, \pm 1, 2...$), where ω is the wave frequency, $\Omega = \Omega_0/\gamma$ is the gyrofrequency of relativistic particle, $\mu = \cos\xi$, where ξ is the pitch angle of particles. TTD formally corresponds to $n = 0$ and it requires compressible perturbations.

We employ quasi-linear theory (QLT) to obtain our estimates. If mean magnetic field is larger than the fluctuations at the injection scale, we may say that the QLT treatment we employ defines parallel diffusion. Taking into account only the dominant interaction at $n = \pm 1$, we obtain the Fokker-Planck coefficients (see also [27, 33]),

$$\left(\begin{array}{c} D_{\mu\mu} \\ D_{pp} \end{array}\right) = \frac{\pi\Omega^2(1-\mu^2)}{2}\int_{\mathbf{k_{min}}}^{\mathbf{k_{max}}} dk^3 \delta(k_\parallel v_\parallel - \omega \pm \Omega) \left[\begin{array}{c} \left(1+\frac{\mu V_{ph}}{v\zeta}\right)^2 \\ m^2V_A^2 \end{array}\right] \Big\{ (J_2^2(\frac{k_\perp v_\perp}{\Omega}) + J_0^2(\frac{k_\perp v_\perp}{\Omega})) \left[\begin{array}{c} M_{\mathcal{RR}}(\mathbf{k}) + M_{\mathcal{LL}}(\mathbf{k}) \\ K_{\mathcal{RR}}(\mathbf{k}) + K_{\mathcal{LL}}(\mathbf{k}) \end{array}\right] - 2J_2(\frac{k_\perp v_\perp}{\Omega})J_0(\frac{k_\perp v_\perp}{\Omega}) \left[e^{i2\phi}\left[\begin{array}{c} M_{\mathcal{RL}}(\mathbf{k}) \\ K_{\mathcal{RL}}(\mathbf{k}) \end{array}\right] + e^{-i2\phi}\left[\begin{array}{c} M_{\mathcal{LR}}(\mathbf{k}) \\ K_{\mathcal{LR}}(\mathbf{k}) \end{array}\right]\right]\Big\}, \quad (1)$$

where $\zeta = 1$ for Alfvén modes and $\zeta = k_\parallel/k$ for fast modes, $k_{min} = L^{-1}$, $k_{max} = \Omega_0/v_{th}$ corresponds to the dissipation scale, $m = \gamma m_H$ is the relativistic mass of the proton, $v_\perp$ is the particle's velocity component perpendicular to $\mathbf{B}_0$, $\phi = \arctan(k_y/k_x)$, $\mathcal{L}, \mathcal{R} = (x \pm iy)/\sqrt{2}$ represent left and right hand polarization.

The delta function $\delta(k_\parallel v_\parallel - \omega + n\Omega)$ approximation to real interaction is true when magnetic perturbations can be considered static[4] . For cosmic rays, $k_\parallel v_\parallel \gg \omega$ so that the resonant condition is essentially $k_\parallel v\mu - n\Omega = 0$. From this resonance condition, we know that the most important interaction occurs at $k_\parallel = k_{\parallel,res} = \Omega/v_\parallel$. This is generally true except for small μ (or scattering near 90^o).

4. Scattering of cosmic rays

4.1. Scattering by Alfvénic turbulence

As we discussed above, Alfvén modes are anisotropic, eddies are elongated along the magnetic field, i.e., $k_\perp > k_\parallel$. The scattering of CRs by Alfvén modes is suppressed first because most turbulent energy goes to $k_\perp$ due to the anisotropy of the Alfvénic turbulence so that there is much less energy left in the resonance point $k_{\parallel,res} = \Omega/v_\parallel \sim r_L^{-1}$. Furthermore, $k_\perp \gg k_\parallel$ means $k_\perp \gg r_L^{-1}$ so that cosmic ray particles have to be interacting with lots of eddies in one gyro period. This random walk substantially decreases the scattering efficiency. The scattering by realistic Alfvénic turbulence was studied in [31]. In case that the pitch angle ξ not close to 0, the analytical result is

$$\left[\begin{array}{c} D_{\mu\mu} \\ D_{pp} \end{array}\right] = \frac{v^{2.5}\mu^{5.5}}{\Omega^{1.5}L^{2.5}(1-\mu^2)^{0.5}}\Gamma[6.5, k_{max}^{-\frac{2}{3}}k_{\parallel,res}L^{\frac{1}{3}}] \left[\begin{array}{c} 1 \\ m^2V_A^2 \end{array}\right], \quad (2)$$

where $\Gamma[a, z]$ is the incomplete gamma function. The presence of this gamma function in our solution makes our results orders of magnitude larger than those[5] in [6] for the most of energies considered. However, the scattering frequency,

$$\nu = 2D_{\mu\mu}/(1-\mu^2), \quad (3)$$

are still much smaller than the estimates for isotropic and slab model [31]. As the anisotropy of the Alfvén modes is increasing with the decrease of scales, the interaction with Alfvén modes becomes more efficient for higher energy cosmic rays. When the Larmor radius of the particle becomes comparable to the injection scale, which is likely to be true in the shock region as well as for very high energy cosmic rays in diffuse ISM, Alfvén modes get important.

It's worthwhile to mention the imbalanced cascade of Alfvén modes [7]. Our basic assumption above was that Alfvén modes were balanced, meaning that the energy of modes propagating one way was equal to that in opposite direction. In reality, many turbulence sources are localized so that the modes leaving the sources are more energetic than those coming toward the sources. The energy transfer in the imbalanced cascade occurs at a slower rate, and the Alfvén modes behave more like waves. The scattering by these imbalanced Alfvén modes could be more efficient. However, as the actual degree of anisotropy of imbalanced cascade is currently uncertain, and the process will be discussed elsewhere[6].

[4] Cosmic rays have such high velocities that the slow variation of the magnetic field with time can be neglected.

[5] We compared our result with the resonant term as the non-resonant term is spurious as noted by [6].

[6] Preliminary results by one of us show that the inertial range over which the degree of anisotropy of imbalanced turbulence is small is very limited.

4.2. Scattering by fast modes

The contribution from slow modes is no more than that by Alfvén modes since the slow modes have the similar anisotropies and scalings. More promising are fast modes, which are isotropic. With fast modes there can be both gyroresonance and transit-time damping (TTD).

TTD happens due to the resonant interaction with parallel magnetic mirror force. The advantage of TTD is that it doesn't have a distinct resonant scale associated with it. TTD is thus an alternative to scattering of low energy CRs which Larmor radii are below the damping scale of the fast modes. Moreover, we shall show later that TTD can contribute substantially to cosmic ray acceleration (also known as the second order Fermi acceleration). This can be crucial in some circumstances, e.g., for γ ray burst ([16]), and acceleration of charged particles [32]. Different from gyroresonance, the resonance function of TTD is broadened even for CRs with small pitch angles. The formal resonance peak $k_\parallel/k = V_{ph}/v_\parallel$ favors quasi-perpendicular modes. However, these quasi-perpendicular modes cannot form an effective mirror to confine CRs because the gradient of magnetic perturbations along the mean field direction $\nabla_\parallel \mathbf{B}$ is small. Thus the resonance peak is weighted out and the Breit-Wigner-type [27] resonance function should be adopted.

Here we apply our analysis to the various phases of ISM. In low β medium, both collisionless and collisional damping increases with θ unless θ is close to $\pi/2$. This means that those quasi-parallel fast modes are least damped. For these modes the argument for the Bessel function in Eq.(1) is $k_\perp \tan\xi/k_{\parallel,res} < 1$ unless ξ is close to 90^o. So we can take advantage of the anisotropy of the damped fast modes and use the asymptotic form of Bessel function for small argument $J_n(x) \simeq (x/2)^n/n!$ to obtain the corresponding analytical result for this case:

$$\begin{bmatrix} D_{\mu\mu} \\ D_{pp} \end{bmatrix} = \frac{\pi(\Omega v\mu)^{0.5}(1-\mu^2)}{2L^{0.5}} \begin{bmatrix} (1-((\frac{k_{\perp,c}}{k_{\parallel,res}})^2+1)^{-\frac{7}{4}})/7 \\ (1-((\frac{k_{\perp,c}}{k_{\parallel,res}})^2+1)^{-\frac{3}{4}})m^2V_A^2/3 \end{bmatrix} \quad (4)$$

In high β medium and partially ionized medium, fast modes are subjected to severe damping and truncated at scale larger than the resonant scale of CRs, $\sim \Omega/c$. The only available scattering mechanism is TTD, which can still provide much more efficient scattering for CRs comparing with Alfvén modes (see [33] for detail).

A special case is that the cosmic rays propagate nearly perpendicular to the magnetic field, so called the 90^o scattering problem. It should be noted that with resonance broadening associated with the turbulence, the requirement $v_\parallel = V_{ph}/\cos\theta > V_{ph}$ is relieved. The contribution from TTD thus becomes dominant for sufficiently small μ. However, since quasi-linear approximation is not accurate in this regime, proper calculation should be carried out with non-linear effects taken into account [13, 23].

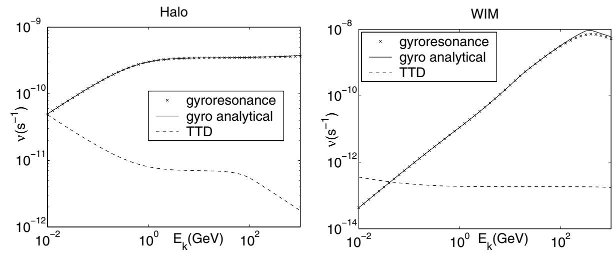

Figure 2: Scattering frequency ν given by Eq.(3) vs. the kinetic energy E_k of cosmic rays (a) in halo, (b) in WIM. The 'x' lines refer to scattering by gyroresonance and solid lines are the corresponding analytical results given by Eq.(4). The dashed line are the results for TTD (from [33]).

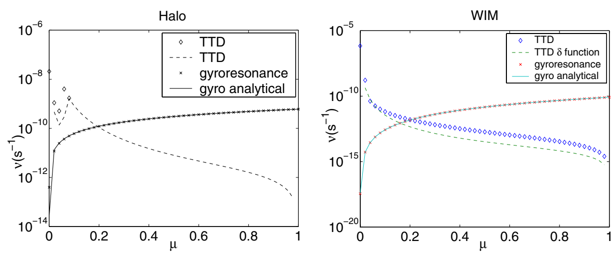

Figure 3: The scattering frequency ν vs. cosine of pitch angle μ of 1GeV CR (a) in halo, (b) in WIM. The 'x' lines refer to scattering by gyroresonance and solid lines are the corresponding analytical results given by Eq.(4). The dashed and diamond lines are the results for TTD adopting δ function and Breit-Wigner function respectively (from [33]).

5. Cosmic ray self confinement by streaming instability

When cosmic rays stream at a velocity much larger than Alfvén velocity, they can excite by gyroresonance MHD modes which in turn scatter cosmic rays back, thus increasing the amplitude of the resonant mode. This runaway process is known as streaming instability. It was claimed that the instability could provide confinement for cosmic rays with energy less than $\sim 10^2$GeV [5]. However, this was calculated in an ideal regime, namely, there was no background MHD turbulence. In other words, it was thought that the self-excited modes would not be appreciably damped in fully ionized gas.

This is not true for turbulent medium, however. [31] pointed out that the streaming instability is partially suppressed in the presence of background turbulence[7]. More recently, detailed calculations of the streaming instability in the presence of background Alfvénic turbulence were presented in [11].

For interaction with fast modes, it happens at the rate $\tau_k \sim (k/L)^{-1/2} V_{ph}/V^2$ (see Eq.(2.2)). By equating the growth rate [22],

$$\Gamma(k) = \Omega_0 \frac{N(\geq E)}{n_p}(-1 + \frac{v_{stream}}{V_A}) \tag{5}$$

and the damping rate Eq.(2.2), we can find that the streaming instability is only applicable for particles with energy less than

$$\gamma_{max} \simeq 1.5 \times 10^{-9} [n_p^{-1}(V_{ph}/V)(Lv\Omega_0/V^2)^{0.5}]^{1/1.1}, \tag{6}$$

which for HIM, provides $\sim$ 20GeV if taking the injection speed to be $V \simeq 25$km/s.

One of the most vital cases for streaming instability is that of cosmic ray acceleration in strong MHD shocks. Such shocks produced by supernovae explosions scatter cosmic rays by the postshock turbulence and by preshock magnetic perturbations created by cosmic ray streaming. The perturbations of the magnetic field may be substantially larger than the regular magnetic field. The corresponding nonlinear growth rate is ([26]):

$$\Gamma(k) \simeq \frac{ak\epsilon U^3}{cV_A(\gamma_{max}^a - (1+a)^{-1})(kr_{g0})^a(1+A_{tot}^2)^{(1-a)/2}}, \tag{7}$$

where ϵ is the ratio of the pressure of CRs at the shock and the upstream momentum flux entering the shock front, U is the shock front speed, $a-4$ is the spectrum index of CRs at the shock front, $r_{g0} = c/\Omega_0$, A_{tot} is the dimensionless amplitude of the random field $A_{tot}^2 = \delta B^2/B_0^2$.

Magnetic field itself is likely to be amplified through an inverse cascade of magnetic energy at which perturbations created at a particular k diffuse in k space to smaller k thus inducing inverse cascade. As the result the magnetic perturbations at smaller k get larger than the regular field. As the result, even if the instability is suppressed for the growth rate given by eq. (5) it gets efficient due to the increase of perturbations of magnetic field stemming from the inverse cascade.

Whether or not the streaming instability is efficient in scattering accelerated cosmic rays back depends on whether the growth rate of the streaming instability is larger or smaller than the damping rate. The precise picture of the process depends on yet not completely clear details of the inverse cascade of magnetic field. If, however, we assume that the small scale driving provides at the scales of interest isotropic turbulence the nonlinear damping happens on the scale of one eddy turnover time. Assuming the shock front speed U is low, we attain the maximum energy of particles accelerated in the shock by equating the growth rate of the instability Eq.(7) to the damping rate due to turbulence cascade Eq.(2.2):

$$\gamma_{max} \simeq (\frac{a\epsilon(LeB_0)^{0.5}U^3}{m^{0.5}V^2c^2})^{1/(0.5+a)}. \tag{8}$$

From this we can estimate $\gamma_{max} \simeq 2 \times 10^7 (t/kyr)^{-9/4}$ in HIM.

As shown in the Eq.(5) and (7), the growth rate depends on the CRs' density. In those regions where high energy particles are produced, e.g., shock fronts in ISM, γ ray burst, SN, the streaming instability is more important.

6. Summary

In the paper above we characterized interaction of cosmic rays with balanced interstellar turbulence driven at a large scale. Our results can be summarized as follows:

1. fast modes provide the dominant contribution to scattering of cosmic rays in different phases of interstellar medium provided that the turbulent energy is injected at large scales. This happens in spite of the fact that the fast modes are more subjected to damping compared to Alfvén modes.

2. As damping of fast modes depends on the angle between the magnetic field and the wave direction of propagation, we find that field wondering determined by Alfvén modes affects the damping of fast modes. At small scales the anisotropy of fast mode damping makes gyroresonant scattering within slab approximation applicable. At larger scales where damping is negligible the isotropic gyroresonant scattering approximation is applicable.

3. Transient time damping (TTD) provides an important means of cosmic ray transport. Use of δ function resonance entails errors, and therefore, resonance broadening is essential for TTD. Our study shows that it is vital for low energy and large pitch angle scattering. And it dominates scattering of cosmic rays in HIM and the partially ionized interstellar gas where fast modes are severely damped.

4. Streaming instability is subjected to non-linear damping due to the interaction of the emerging magnetic perturbations with the surrounding turbulence. The energy at which the streaming instability is suppressed depends on whether on the inverse cascade of magnetic energy as the instability gets more easily excited for low energy particles.

[7] The fast cascade induces fast non-linear damping of MHD turbulence. Essentially the damping of Alfvénic turbulence happens in one eddy turnover time for large eddies. This effect was invoked in [24] to explain the small transversal size of X-ray filaments observed ([1])

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