

Particle Acceleration Timescales in Relativistic Shear Flows

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We review the acceleration of energetic particles in relativistic astrophysical jets characterized by a significant velocity shear. The possible formation of power-law momentum spectra is discussed and typical acceleration timescales are determined for a variety of different conditions such as parallel and azimuthal shear flows. Special attention is given to the analysis of parallel shear flows with either a linear decreasing or a Gaussian-type velocity profile. It is shown that in the presence of a gradual shear flow and a particle mean free path scaling with the gyroradius, synchrotron radiation losses may no longer be able to stop the acceleration once it has started to work efficiently. Finally, the relevance of shear acceleration in small- and large-scale relativistic jets is addressed.

1. INTRODUCTION

Relativistic jet outflows are observed across a wide range of astrophysical scales, from galactic microquasars to GRBs and radio-loud AGNs [2, 16, 31]. The fact that velocity gradients are a generic feature of environments producing jets suggests that many (if not all) of these outflows may possess a significant internal velocity shear. Today, the phenomenological evidence for internal jet stratification is indeed mounting and comprises evidence for universal structured jets in GRBs [12, 24], internal jet rotation in AGNs [21, 22], and a multi-component jet structure consisting (at least) of a fast moving inner spine and a slower shear layer as indicated, for example, by the detailed analysis of the brightness and polarization systematics in kpc FR I jets [14, 15] or the intensity and polarization maps of pc-scale FR I + II jets [4, 20, 26]. It seems likely that in the presence of such a velocity shear efficient acceleration of particles to high energy may occur, provided particles are scattered across the shear by magnetic inhomogeneities carried within the flow [22, 23]. The theory of shear acceleration deals with those cases where the impact of the systematic velocity components of the scattering centers becomes dominant over their random motion components known as the source of second-order Fermi acceleration effects.

Substantial theoretical contributions in the field of shear acceleration have been given by several authors (cf. [22] for a review): Berezhko & Krymskii (1981), for example, showed that (non-relativistic gradual) shear acceleration can lead to power law particle momentum distributions resembling those of the classical Fermi theory, whereas Earl, Jokipii & Morfill (1988) derived the corresponding diffusive particle transport equation in the diffusion approximation including shear and inertial effects. The generalisation of the work by Earl et al. to the relativistic regime was achieved by Webb [28–30] using a mixed-frame approach and applied by Rieger & Mannheim (2002) to rotating and shearing jets in AGNs. Particle acceleration in non-gradual relativistic shear flows (i.e.,

in the presence of a relativistic velocity jump), on the other hand, has been analysed by Ostrowski [17–19] using Monte Carlo methods, showing that very flat momentum spectra may occur.

2. MICROSCOPIC PICTURE

Shear acceleration draws on a simple mechanism which seems to be applicable to a wide range of astrophysical flows such as accretion flows and structured jets in GRBs and AGNs (cf. also [10]). According to the underlying physical picture particles gain energy by scattering off (small-scale) magnetic field irregularities with different local velocities due to being systematically embedded in a collisionless velocity shear flow. The scattering process is assumed to occur in such a way that the particles are randomized in direction, with their energies being conserved in the local comoving fluid frame. In the presence of a velocity shear, the momentum of a particle travelling across the shear will change with respect to the local scattering frame so that for an isotropic particle distribution a net increase may occur [9]. For illustration consider a continuous, non-relativistic shear flow with velocity field given by

$$\vec{u} = u_z(x) \vec{e}_z. \quad (1)$$

Let $\vec{v} = (v_x, v_y, v_z)$ be the velocity vector, m the relativistic mass and \vec{p}_1 the initial momentum (relative to local flow frame) of the particle. Within one scattering time τ (initially assumed to be independent of momentum) the particle travels a distance $\delta x = v_x \tau$ across the shear, so that in the presence of a gradual shear the flow velocity will have changed by $\delta \vec{u} = \delta u \vec{e}_z$, with $\delta u = (\partial u_z / \partial x) \delta x$. Hence the particle's momentum relative to the flow becomes $\vec{p}_2 = \vec{p}_1 + m \delta \vec{u}$ (Galilean transformation), i.e.

$$p_2^2 = p_1^2 + 2 m \delta u p_{1,z} + m^2 (\delta u)^2. \quad (2)$$

As the next scattering event preserves the magnitude of the particle momentum (in the local scattering frame) the particle magnitude will have this value

in the local flow frame. Using spherical coordinates and averaging over solid angle assuming an almost isotropic particle distribution, the resultant average rate of momentum change and the average rate of momentum dispersion scale with the square of the flow velocity gradient [9, 23], i.e.,

$$\left\langle \frac{\Delta p}{\Delta t} \right\rangle \propto p \left(\frac{\partial u_z}{\partial x} \right)^2 \tau, \quad (3)$$

$$\left\langle \frac{\Delta p^2}{\Delta t} \right\rangle \propto p^2 \left(\frac{\partial u_z}{\partial x} \right)^2 \tau. \quad (4)$$

Generalizing these results to a momentum-dependent scattering time obeying a power law of the form $\tau \propto p^\alpha$, we may write down a simple Fokker-Planck transport equation for the phase space particle distribution function $f(p)$, assuming a mono-energetic injection of particles with momentum p_0 . Solving for the steady-state with $\alpha > 0$ one obtains (see [23])

$$f(p) \propto p^{-(3+\alpha)} H(p - p_0), \quad (5)$$

where $H(p)$ is the Heaviside step function. Hence for a mean scattering time scaling with the gyro-radius (Bohm case), i.e. $\tau \propto p^\alpha$, $\alpha = 1$, one obtains $f(p) \propto p^{-4}$, i.e., a power law particle number density $n(p) \propto p^{-2}$ which translates into a synchrotron emissivity $j_\nu \propto \nu^{-1/2}$. For a Kolmogorov-type ($\alpha = 1/3$) or Kraichnan-type ($\alpha = 1/2$) scaling, on the other hand, much flatter spectra may be obtained, i.e., the synchrotron emissivity becomes $j_\nu \propto \nu^{-1/6}$ and $j_\nu \propto \nu^{-1/4}$, respectively.

3. ACCELERATION TIMESCALES

The efficiency of shear acceleration can be studied by determining the acceleration timescales for typical shear flow velocity profiles. In the present context we will concentrate on well-collimated (cylindrical) relativistic AGN jets, leaving a detailed analysis of expanding relativistic jet flows to the near future (Rieger & Duffy, in preparation). In general, at least three different shear scenarios may be distinguished by means of observational and theoretical arguments [22]:

3.1. Gradual shear flow along the jet axis

As noted in the introduction, there is growing evidence today for an internal velocity structure parallel to the jet axis, with the simplest scenario consisting of a fast moving inner spine and a slower moving boundary layer. In addition, Laing et al. (1999) have shown that the intensity and polarization systematics in kpc-scale FR I jets are suggestive of a radially (continuously) decreasing velocity profile $v_z(r)$. In order to estimate the particle acceleration efficiency in a

longitudinal gradual shear flow, the following applications seem thus to be particularly interesting (see also Fig. 1): (i) a shear flow with velocity profile decreasing linearly from relativistic speed $u_{z,\max}$ to $u_{z,\min}$ over a characteristic length scale $\Delta r = (r_2 - r_1)$, and (ii) a Gaussian-type velocity profile with core radius r_c .

3.1.1. Linearly decreasing velocity profile

It can be shown (see Rieger & Duffy 2004) that for a linearly decreasing velocity profile the shear acceleration timescale becomes

$$t_{\text{acc}}(r) \simeq \frac{3}{\lambda} \frac{(\Delta r)^2}{\gamma_b(r)^2 [1 + \gamma_b(r)^2 u_z(r)^2/c^2]} \frac{c}{(\Delta u_z)^2} \quad (6)$$

where $\lambda(\gamma')$ is the mean free path for a particle with comoving Lorentz factor γ' , $\Delta u_z \equiv (u_{z,\max} - u_{z,\min})$ and $\gamma_b(r) = 1/(1 - u_z(r)^2/c^2)^{1/2}$ is the local bulk flow Lorentz factor. For nonrelativistic $u_{z,\min}$ and $\gamma_b(r_1)$ larger than a few, the minimum acceleration timescales is thus of order

$$t_{\text{acc}}(r_1) \simeq \frac{3(\Delta r)^2}{\gamma_b(r_1)^4 \lambda c}. \quad (7)$$

Note, however, that if $u_{z,\min}$ is still relativistic with $u_{z,\min} \sim 0.7 u_{z,\max}$ as suggested, for example, in the case of FR I jets [14], the minimum acceleration timescale might easily be an order of magnitude larger. The evolution of the acceleration timescale as a function of r is shown in Fig. 2. The acceleration timescale increases significantly (up to a factor $\sim 2 \cdot 10^3$ for the application in Fig. 2a) while going outwards with r due to the decrease in flow velocity, suggesting that the higher energy emission will be located closer to r_1 .

Any particle acceleration process will have to compete at least with radiative energy losses. If one requires efficient shear acceleration, this thus translates into an upper limit for the allowed width Δr of the transition layer. In the case of synchrotron radiation, for example, the cooling timescale is given by $t_{\text{cool}} \propto m^3 \gamma'^{-1} B^{-2}$, with m the rest mass of the particle. For a gyro-dependent particle mean free path, i.e., $\lambda \sim r_g(\gamma')$, the condition $t_{\text{acc}}(r_1) < t_{\text{cool}}$ thus implies an upper limit

$$\Delta r \lesssim (c^4/e^{5/2}) \gamma_b(r_1)^2 m^2 B^{-3/2}. \quad (8)$$

For typical pc-scale parameters, i.e., magnetic field strength of $B \sim 0.01$ Gauss and $\gamma_b(r_1) \sim 10$, one thus requires a width $\Delta r_e < 0.003$ pc and $\Delta r_p < (m_p/m_e)^2 \Delta r_e$ (improving the estimate in [22]) for efficient electron and proton acceleration, respectively. While efficient acceleration of protons is thus very likely, efficient acceleration of electrons on the pc-scale and below appears quite restricted, i.e., only possible if the transition layer is relatively thin. Note, however,

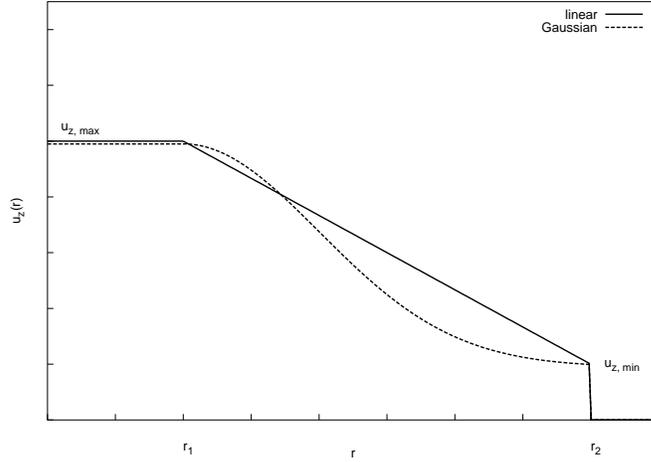


Figure 1: Schematic illustration of a linearly decreasing and a Gaussian-type velocity profile for a shear flow parallel to the jet axis, with $u_{z,\max}$ up to an inner radius r_1 and $u_{z,\min}$ at r_2 , and $r_c = r_1$ for the Gaussian profile.

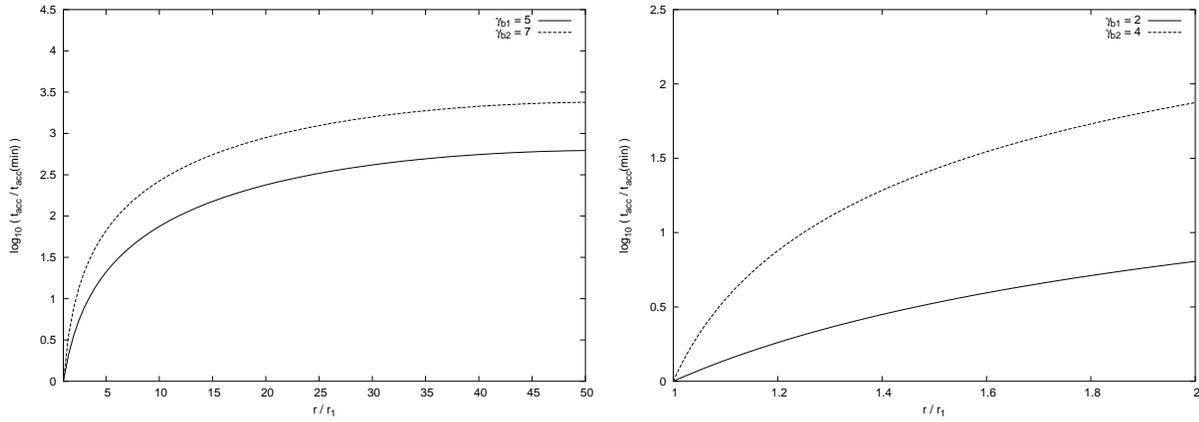


Figure 2: The acceleration timescales for a gradual, parallel shear flow with linear decreasing velocity profile as a function of the radial coordinate. *Left*: Shear flow with bulk Lorentz factors $\gamma_b(r_1) = 5$ and 7 , respectively, $r_2 = 50 r_1$ and $u_z(r_2) = 0.05 u_z(r_1)$. The minimum acceleration timescale is defined by $t_{\text{acc}}(\min) = t_{\text{acc}}(r_1)$ *Right*: Shear flow with conditions more appropriate for FR I jets, i.e., bulk Lorentz factors $\gamma_b(r_1) = 2$ and 4 , respectively, $r_2 = 2 r_1$ and $u_z(r_2) = 0.7 u_z(r_1)$.

that for powerful large-scale relativistic jets (such as the one in 3C 273) with typical parameters $\gamma_b(r_1) \sim 5$ and $B \sim 10^{-5}$ G, only $\Delta r_e < 30$ pc would be required, suggesting that efficient electron acceleration may be more likelier in large-scale relativistic jets. Note also, that for λ scaling with the gyro-radius, $t_{\text{acc}} \propto t_{\text{cool}} \propto 1/\gamma'$, so that radiation losses are no longer able to stop the acceleration process once it has started to operate efficiently.

3.1.2. Gaussian velocity profile

Consider for comparison a Gaussian-type velocity profile, which for $r \geq r_1$ decreases as

$$u_z(r) = u_{z,\min} + (\Delta u_z) \exp \left[-(r - r_1)^2 / (2 r_c^2) \right], \quad (9)$$

i.e., asymptotically approaches $u_{z,\min}$, where r_c is the core radius and Δu_z the velocity difference as defined

above. The acceleration timescale then becomes, cf. Rieger & Duffy (2004),

$$t_{\text{acc}}(r) = \frac{t_0 \exp[(r - r_1)^2 / r_c^2]}{(r/r_c - r_1/r_c)^2 \gamma_b(r)^2 [1 + \gamma_b(r)^2 u_z(r)^2 / c^2]} \quad (10)$$

with $t_0 = 3 c r_c^2 / [\lambda (\Delta u_z)^2]$ and $r > r_1$. The minimum of the acceleration timescale is then no longer located at r_1 , but still close-by. The evolution of the acceleration timescale as a function of the radial coordinate r is shown on the left hand side of Fig. 3. The acceleration timescale now first decreases up to a minimum before eventually increasing again significantly due to the diminishing velocity shear. An example, comparing the acceleration timescale for a Gaussian with the one for a linear-decreasing velocity profile, is shown on the right hand side of Fig. 3, illustrating the range

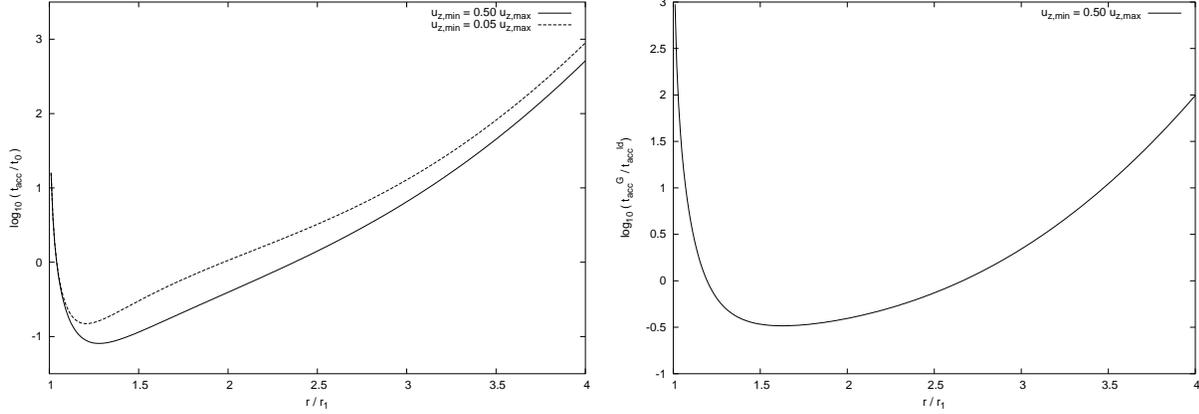


Figure 3: *Left:* The acceleration timescales for a longitudinal gradual shear flow with a Gaussian-type velocity profile as a function of the radial coordinate for $\gamma_b(r_1) = 5$, and $u_{z,\text{min}} = 0.5 u_{z,\text{max}}$ and $u_{z,\text{min}} = 0.05 u_{z,\text{max}}$, respectively, $r_c = r_1$ and $r \leq 4 r_1$. *Right:* The ratio of the acceleration timescale t_{acc}^G for a Gaussian profile versus the acceleration timescale $t_{\text{acc}}^{\text{id}}$ for a linear decreasing profile, shown using $\gamma_b(r_1) = 5$, $r_c = r_1$, $r_2 = 4 r_1$, and $u_{z,\text{min}} = 0.5 u_{z,\text{max}}$.

over which a Gaussian is slightly preferable. Note, however, that for the chosen parameters the minimum Gaussian acceleration timescale is still higher by a factor of a few than the minimum acceleration timescale for the linear decreasing profile.

3.2. Non-gradual shear flow along the jet axis

Suppose a particle becomes so energetic that its mean free path λ exceeds the width Δr of the velocity transition layer. The particle may then be regarded as essentially experiencing a non-gradual velocity shear, i.e., a discontinuous jump in the flow velocity. It has been pointed out by Ostrowski [17, 18] that the jet side boundary in powerful AGN jets may naturally represent a relativistic example where efficient non-gradual shear acceleration may occur. The minimum acceleration timescale (measured in the ambient medium rest frame) can be estimated using Monte-Carlo simulations [17, 18] and appears to be comparable to the minimum timescale of nonrelativistic shock acceleration in the Bohm limit ($\lambda \sim r_g$), i.e.,

$$t_{\text{acc}} \sim 10 \frac{r_g}{c} \quad \text{provided } r_g > \Delta r, \quad (11)$$

where r_g denotes the particle gyro-radius. Balancing the acceleration timescale with the synchrotron cooling timescale then translates into an upper limit for the maximum attainable particle Lorentz factor of $\gamma < 4 \cdot 10^7 (m/m_e) B^{-1/2}$. Efficient acceleration thus requires

$$\Delta r < 2 \cdot 10^{-8} \left(\frac{m}{m_e} \right)^2 B^{-3/2} \text{ pc}. \quad (12)$$

Hence, for a pc-scale magnetic field strength of $B \sim 0.01$ G, a width of $\Delta r_e < 2 \cdot 10^{-5}$ pc and $\Delta r_p <$

$(m_p/m_e)^2 \Delta r_e$ would be needed in order to allow for electron and proton acceleration, respectively. Efficient electron acceleration on the pc-scale thus appears unlikely, while efficient acceleration of protons could be well possible as long as the particle mean free path remains smaller than the width of the jet. Note that for the large-scale case with $B \sim 10^{-5}$ G one would still need $\Delta r_e \lesssim 0.6$ pc, suggesting that electrons are also not accelerated efficiently on kpc-scales.

3.3. Gradual transversal shear flow

While the formation of relativistic jets generally seems to be connected to the presence of an accretion disk, a central black hole is not necessarily required [5]. If jets indeed originate as collimated disk winds, however, a significant amount of rotational energy of the disk may be channeled into the jet leading to a flow velocity field which is characterized by an additional rotational component (cf. [21, 22] for more details). In order to evaluate the acceleration potential associated with such a shear flow, we have recently analyzed the transport of energetic particles in rotating relativistic jets [21]. Based on an analytical (mixed-frame) approach, and starting with the relativistic Boltzmann equation and a (BKG) relaxation scattering term, we have studied the acceleration of particles in a cylindrical jet model with relativistic outflow velocity u_z and different azimuthal rotation profiles. It turned out, for example, that in the case of a simple (non-relativistic) azimuthal Keplerian profile, where shear effects dominate over centrifugal effects, local power law distributions $f(p) \propto p^{-(3+\alpha)}$ are generated for $\tau \propto p^\alpha$, $\alpha > 0$, whereas for more complex rotational profiles (e.g., flat rotation, where centrifugal effects become relevant) steeper spectra may be produced.

In order to gain insights into the acceleration efficiency, we may consider a flow velocity field with constant relativistic u_z and azimuthal Keplerian rotation profile of the form $\Omega(r) = \Omega_k (r_{\text{in}}/r)^{3/2}$, where Ω_k is a constant. The acceleration timescale is then of order (see [22])

$$t_{\text{acc}}(r) \simeq \frac{c}{\lambda} \frac{1}{\gamma_b(r)^4 (1 - u_z^2/c^2) \Omega_k^2} \left(\frac{r}{r_{\text{in}}} \right)^3, \quad (13)$$

assuming the flow to be radially confined to a region $r_{\text{in}} \leq r \leq r_j$, where r_j is the jet radius and $\gamma_b(r) = 1/[1 - \Omega(r)^2 r^2/c^2 - u_z^2/c^2]^{1/2}$ is the local flow Lorentz factor. According to Eq. (13) efficient particle acceleration generally requires a region with significant rotation. As the acceleration timescale increases considerably with r , the higher energy emission will generally be concentrated closer to the axis (i.e., towards smaller radii). A comparison of acceleration and cooling timescales indicates that efficient electron acceleration on the pc-scale is only possible close to r_{in} , whereas proton acceleration is not subject to such a restriction. Note again that for $\lambda \propto \gamma$, the ratio of acceleration timescale to cooling timescale is independent of γ , suggesting that losses are no longer able to stop the acceleration process once it has started to operate efficiently.

4. CONCLUSIONS

There is growing evidence today that astrophysical jets may indeed be characterized by a significant shear within their flow velocity field. Internal jet rotation, for example, is likely to be present at least in the initial parts of the flow. On the other hand, a significant and continual velocity shear parallel to the jet axis, is expected for most powerful jet sources. Applying the results derived above thus suggests the following conclusions:

- Gradual shear acceleration of electron occurring in small-scale relativistic jets may naturally account for a steady second population of synchrotron-emitting particles, contributing to the observed emission in addition to shock-accelerated [11] ones. Depending on the level of turbulence (i.e., the momentum index α for the mean scattering time) shear-accelerated electrons may lead to very flat synchrotron emission spectra. On the other hand, if the Alfvén velocity is very high, the radio and optical emission properties of large-scale relativistic jets, such as the one in 3C 273 [7, 8], may be dominated by synchrotron emission from relativistic electrons accelerated via second-order Fermi processes (cf. [25]). The concrete details, however, will depend on the energy scale at which shear acceleration takes over [22].
- Gradual and non-gradual shear acceleration processes usually work very well for protons, which indicates that it may be possible to accelerate cosmic-rays to ultra-high energies along powerful relativistic jets, cf. also [18, 19].
- Shear acceleration may generally represent an efficient way of dissipating a considerable part of the kinetic energy of the jet, thus possibly accounting for the substantial deceleration of jets in lower-power radio sources (e.g., [27]).
- If a jet and its shear layer decelerates and expands significantly with distance as expected, for example, in FR I sources, the minimum shear acceleration timescale will increase as well, suggesting the existence of a distance scale above which shear acceleration of electrons may no longer overcome the energy losses.
- Compared with a simple homogeneous jet model, the emission properties of astrophysical shear flows may be much more complex, e.g., the inverse Compton emission can be strongly boosted as each jet part will see an enhanced radiation field from the other parts, cf. [6].

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