# The Dynamics of Charged Particle of Ultra High Energy in CMB

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The dynamics of charged particle of ultra high-energy moving in the Cosmic Microwave Background (CMB) is considered. Since the CMB parameters, 1) the intensity and 2) the ratio of quantum's energy to the energy of charged particle are small, we handle the problem within the frameworks of Classical Electrodynamics. Solution of relativistic equation of motion results, after averaging over random phases of the waves, in a small additional acceleration, hence the well-known 'GZK cut-off' for charged Ultra High Energy Cosmic Rays (UHECR) becomes controversial. Owing to the relativistic invariance of the wave's phase, the huge increase of frequency both in the rest and in the center of mass reference frames of the ultra high-energy charged particle results in the corresponding decrease of the length of interaction. Therefore, the obtained energy gain is independent of chosen reference frames. Thus, there is no theoretical constrains for UHECR originated in far Universe during their long journey to Earth. Spectrum of UHECR is discussed and both the knee and the ankle are considered.

## 1 INTRODUCTION

The 'GZK cut-off' imposes limitation on the possible highest energy of protons of UHECR due to the photoproduction of pions on the photons of CMB radiation [1]. However, it will be shown below that the 'GZK cut-off" is caused only by the kinematical transformation into the reference frame of high-speed particle, where the frequency of quantum increases essentially and the energy of photon "becomes sufficient" for the photoproduction. However, this transformation should depend also on the relative directions of propagation of particles and, which is even more important, on the dynamical characteristics like duration or length of interaction.

From the point of view of the Random Processes Theory [2], the average density of electromagnetic energy, described by Plank's formula, represents by itself the spectral density of fluctuating electric $\vec{E}$ and magnetic $\vec{B}$ fields strengths of the thermal radiation field. In each of these plane waves $E = H$ and all the directions are equiprobable.

Thus, the dynamical and relativistic invariant turning on of charged UHECR particle interaction with the random electromagnetic fields of CMB [3] results in a simultaneous decrease of interaction time and photoproduction is impossible since an additional increase of energy of charged UHECR particle is negligible as compared with its initial energy even for the time comparable with the lifetime of the Universe.

The switching on of electromagnetic interaction with charged particles considered below from the Classical and Quantum points of view. It is shown that the procedure of imposition of initial conditions in Classical Electrodynamics corresponds to imposition of initial and free wave function in Quantum Electrodynamics.

## 2 QUANTUM APPOACH TO THE PROBLEM OF PROPAGATION OF CHARGED PARTICLE IN AN ELECTROMAGNETIC FIELDS

The solution of Dirac's equation for the charged particle moving in the field of plane electromagnetic wave (the so called Volkov's solution [4]), is well known and may be found in almost any book of Quantum Electrodynamics or Quantum Field Theory (see for example, [5, 6]). Due to its simplicity, especially for the case of plane electromagnetic wave of circular polarization, when squared term by four vector-potential becomes constant, this solution is widely used for calculations of various processes in external plane electromagnetic fields.

However, both the classical action, which enters the exponent of the wave function in Volkov's solution, and the wave function itself are not relativistic invariant. If an arbitrary constant is added to the four-dimensional vector potential, the action written for example in [6],

$$S = -px - \int_0^{\varphi} \left[ \frac{e}{(pk)} \cdot (pA) - \frac{e^2}{2(pk)} \cdot A^2 \right] \cdot d\varphi \qquad (1)$$

will be changed. Here $p_\mu$ is some not well-defined in literature four-dimensional vector, usually treated like initial, $A_\mu = A_\mu(\varphi)$ is the four-dimensional vector-potential, $\varphi = k_\mu \cdot x^\mu = \omega \cdot t - \vec{k} \cdot \vec{r}$ is the phase of electromagnetic wave at the point $x_\mu$ of 4-dimentional space, $k_\mu \equiv (\omega, \vec{k})$ is the four-dimensional wave vector, and in the vacuum $k_\mu^2 = k_\mu \cdot k^\mu = \omega^2 - \vec{k}^2 = 0$, $\omega$ is the cyclic frequency of the wave, $e$ is the charge of particle.

Besides, the procedure of switching on of the electromagnetic wave, as described by Eq. (1) is controversial: the field is supposed to be absent at the infinity $-\infty$, but the integration is carried from $0$ to $\varphi$. From the point of view of initial conditions, the case when particle is coming from infinity to the given point to interact with the photon, as is used in scattering matrix approach, contains two infinities: one infinite time is needed for particle to reach (with its finite velocity) a certain point, where the interaction takes place, and the same infinite time is needed to move to infinity again, where interaction is absent.

The situation is even worse in the medium, $k^2 \neq 0$, where solutions obtained without the due regard for initial conditions result by approaching to the Cherencov angle of incidence in an imaginary number of radiated and emitted photons [7], while they should result in a free motion of particle along the direction of propagation of electromagnetic wave, that is, there should be the limiting case of free particle, $q_\mu \to p_\mu$, if $pk \propto 1 - n \cdot \beta_0 \cdot \cos\vartheta_0 \to 0$, where $n$ is the refraction index, $\beta_0 = \frac{v_0}{c}$ is the ratio of velocity of particle to the velocity of light, $\vartheta_0$ is an incidence angle between the particle's propagation direction and the wave vector $\vec{k}$. The physical meaning of this limit is evident: transformations of fields into the reference frame of this Cherencov particle result in the vanishing of the electrical component of the electromagnetic field; at the same time the interaction of motionless particle with the static magnetic field of the wave is absent.

The cause of such historic error is that the experimental proof of all expressions for direct interaction for charged particles with electromagnetic waves was done only with the advent of lasers [8].

All these shortcomings are avoided, if we will seek for the solution of Dirac's or Klein-Gordon equations in Quantum Electrodynamics with imposition of initial free wave function, that is,

$$\psi \propto \exp(-i \cdot p_\mu x^\mu)$$

at the beginning, which results in the corresponding replacements $A_\mu(\varphi) \;\to\; A_\mu(\varphi) - A_\mu(\varphi_0)$ for vector potential and $0 \to \varphi_0$ for the lover limit of phase in the integral. However, it is better to use the Dirac or Klein-Gordon equations in the form, written in the **Appendix** of [3].

The exact modified solution for Dirac's equation may be written as follows:

$$\Psi = \frac{u}{\sqrt{2p_0}} \cdot \left[1 + \frac{e}{2(pk)} \cdot \hat{k} \cdot \left(\hat{A}(\varphi) - \hat{A}(\varphi_0)\right)\right] \cdot$$
$$\cdot \exp\Big[-i \cdot p_\mu x^\mu - i \cdot e \cdot A_\mu(\varphi_0) \cdot \left(x^\mu - x_0^\mu\right) -$$
$$-i \cdot \int_{\varphi_0}^{\varphi} \frac{2e\left(p\left(A(\varphi) - A(\varphi_0)\right)\right) - e^2\left(A(\varphi) - A(\varphi_0)\right)^2}{2(pk)} \cdot d\varphi\Big]$$

where $p_0$ is the zero component of four-vector, $\hat{k} \equiv k_\mu \cdot \gamma^\mu$, where $\gamma_\mu$ are Dirac's matrices, and $\frac{u}{\sqrt{2p_0}}$ is an arbitrary constant bi-spinor, coinciding with the bi-spinor amplitude of the free plane wave, normalized by equation $\bar{u} \cdot u = 2m$ where $m$ is the mass of particle.

## 3 CORRESPONDENCE BETWEEN QUANTUM AND CLASSICAL APPROACHES

In the solution of Dirac's equation for charged particle interacting with the electromagnetic wave in vacuum both the ratio of quantum's energy $\hbar\omega_{CMB} = E_\gamma \geq 6 \cdot 10^{-4}\,\mathrm{eV}$ to the energy of charged particle $\frac{\hbar\omega_{CMB}}{mc^2} << 1$ and the parameter of intensity for the wave $\xi = \frac{eE_{rms}}{mc\omega} << 1$ are small ($\xi \approx 10^{-9}$ for electrons and 1836 time lesser for protons), the problem may be considered within the frameworks of Classical Electrodynamics.

For the case of transversal wave of linear polarization in vacuum, that is, $A_\mu = a_\mu \cdot \cos\varphi$ $(k^2 = ka = 0)$ the most general solution for particles four-momentum may be written as

$$q_\mu = p_\mu - e \cdot \left(a_\mu - k_\mu \frac{(pa)}{(pk)}\right) \cdot \left(A(\varphi) - A(\varphi_0)\right) - \quad (3)$$
$$-k_\mu \cdot e^2 \cdot a^2 \cdot \frac{\left(A(\varphi) - A(\varphi_0)\right)^2}{2(pk)}$$

and the phase equation has the following form

$$\frac{d\varphi}{d\tau} = (p \cdot k)$$

where $\tau$ is the proper time. Since phase is relativistically invariant, the quantity $(p \cdot k) \cdot \tau$ is also invariant. Kinematical quantity $(p \cdot k)$ is proportional to the frequency of the wave and energy of the particle. The kinematical huge increase of frequency is considered as an ample ground of increasing of "photons" energy, which became now "sufficient" to produce pions. However, the proper time $\tau$ decreases as an inverse ratio of the energy $\tau \sim t / \left(\frac{E_p}{m_p \cdot c^2}\right)$, hence the dynamical invariant $(p \cdot k) \cdot \tau \equiv inv$ remains constant in any reference frame, as it should be according to the Principle of Relativity.

Moreover, additional, although small gain of energy is possible during the motion of charged UHECR particle in CMB. Thus for UHECR electrons and protons the pure gain approximately equal to

$\cong 10^7\,\mathrm{eV}$ and $\cong 10\mathrm{eV}$, respectively for the time comparable with the life time of the Universe. It is evident, that these values of energy are insufficient to produce even one pion.

In energy distribution peaks originate and shifted to the higher values

These curves are the results of interaction of initial Gauss-like distribution of charged particles with electromagnetic fields.

Worth noting that such an approach, based on imposition of initial conditions for charged particle motion in the field of electromagnetic wave in Classical Electrodynamics [9], has been used for particle acceleration in a slow electromagnetic wave, when $k^2 \neq 0$ [10] and to resolve the controversy with self-acceleration and runaway solutions in Lorentz-Dirac equation for radiation damping [11]. In Quantum Electrodynamics, to obtain the same physical phenomena, the solution of Dirac or Klein-Gordon equations with imposition of initial wave function in the form of free plane wave.

As related to knee and ankle in the spectrum of cosmic rays, thus situation is similar to the modification of energy spectrum in external electromagnetic waves, when expose to these fields results in origination and spreading of peaks [10].

## 4 RESOLVING THE CONTROVERSY WITH THE SELF-ACCELERATION IN THE RELATIVISTIC DIRAC'S EQUATION OF RADIATION DAMPING

As an example of imposition of initial conditions, the exact solution of the relativistic Lorentz-Dirac equation of radiation damping [11, 12] for free particle is obtained in this Section. The initial conditions are imposed on the four-dimensional coordinate and momentum of charged particle. It is shown that in the absence of initial four- acceleration there is no self-acceleration later, and the four-momentum of particle remains free and initial one.

The Lorentz-Dirac's equation of radiation damping may be written in the four-dimensional form [11]

$$\dot{q}_\mu = \frac{2e^2}{3}\cdot\left[\ddot{q}_\mu - q_\mu\left(q\ddot{q}\right)\right] \tag{4}$$

where $q_\mu$ and $\tau$ are the four-dimensional momentum and proper time and $m=c=1$ units are used. This equation should be solved with imposition of natural initial conditions:

$$x_\mu = x_{0\mu},\, q_\mu = q_{0\mu} \ \text{ at } \tau = 0$$

and, for generality, $\dot{q}_\mu \neq 0$; despite, for free particle there should be no four-acceleration at the beginning.

Four-dimensional force in the right hand side is orthogonal to the four-dimensional vector,

$$\ddot{q}_\mu - q_\mu\left(q\ddot{q}\right) = 0 \tag{5}$$

Which results in $q^2{}_\mu = 1$, and derivative of this expression by the proper time results in $\left(q\cdot\dot{q}\right)=0$.

The multiplying Eq. (4) by $\dot{q}_\mu$ results in the following equation

$$\frac{2e^2}{3}\cdot\frac{d}{d\tau}\left(\frac{\dot{q}^2}{2}\right) = \dot{q}^2, \tag{6}$$

which is integrated easily

$$\dot{q}_\mu{}^2 = \dot{q}_0^2\cdot e^{\frac{3}{2e^2}\cdot\tau} \tag{7}$$

And the initial Eq. (4) may be written in the form

$$\dot{q}_\mu = \frac{2e^2}{3}\left[\ddot{q}_\mu + q_\mu\cdot\dot{q}_0^2\cdot e^{\frac{3}{2e^2}\tau}\right] \tag{8}$$

The change of variable $\zeta = e^{\frac{2}{3e^2}\cdot\tau}$ reduces this equation to the well-known harmonic oscillation equation

$$q''_\mu(\zeta) + \frac{2e^2}{3}\cdot\dot{q}_0^2\cdot q_\mu(\zeta) = 0 \tag{9}$$

Solution of Eq. (6), which satisfies the natural initial conditions, may be written now as

$$q_\mu(\tau) = q_{o\mu}\cdot\cos\left[\sqrt{\dot{q}_0^2}\,\frac{\left(e^{\frac{3}{2e^2}\tau}-1\right)}{\left(\frac{3mc}{2e^2}\right)}\right] + \frac{\dot{q}_\mu}{\sqrt{\dot{q}_0^2}}\sin\left[\sqrt{\dot{q}_0^2}\,\frac{\left(e^{\frac{3}{2e^2}\tau}-1\right)}{\left(\frac{3mc}{2e^2}\right)}\right] \tag{10}$$

where the parameters of mass $m$ and velocity $c$ are restored.

If we now consider Eq. (10) in the limit of absence of initial four-acceleration, $\dot{q}_\mu \to 0$, we obtain that $q_\mu \to q_{0\mu}$, that is, there is no self-acceleration, and four-momentum of particle remains constant.

It is very interesting, that even in the *hypothetic case* of presence of some initial four-acceleration, three-dimensional velocity remains constant.

Since $\left(q\cdot\dot{q}\right)=0$, this four-acceleration vector is spacelike one, that is, $\dot{q}_0^2 \le 0$ ant temporal component of acceleration $\dot{q}_{00} = 0$. However, from Eq.(7) for three-dimensional velocity $\vec{v} = \frac{\vec{q}}{q_0}$ the trigonometric functions became the hyperbolic ones and in the limit $\tau \to \infty$ the initial three-dimensional velocity asymptotically tends to initial value, that is, the self-acceleration is absent even in this *hypothetic* case.

## 5. SUMMARY

1. Method of imposition of initial conditions both in Classical and Quantum Electrodynamics seemed to be fruitful and provides good correspondence between these approaches.
2. Owing to the relativistic invariance of the wave's phase, the huge increase of frequency (both in the rest and CM reference frames for UHECR charged particle) is accompanied by the corresponding decrease of the time of interaction.

3. After averaging over the random phases of EM plane waves of CMB, the solution of relativistic equation of motion, applied to UHECR charged particle, results in a small additional acceleration of particle, however insufficient to produce even one pion for the tame comparable to the life of the Universe.
4. Due to the dynamical relativistic invariance, these results are independent on the chosen reference frames.
5. There is no theoretical limitation for UHECR particles energy due to photoproduction of pions during charged particle propagation in the CMB.
6. Hence, the GZK cut-off for UHECR charged particles becomes controversial.

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