Search for Possible Exotic Contributions to Atmospheric Neutrino Oscillations

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Neutrino-induced upward-going muons in MACRO are analyzed in terms of relativity violating effects, keeping "standard" mass-induced oscillations as the dominant source of $\nu_{\mu} \rightarrow \nu_{\tau}$ oscillations. Stringent 90% C.L. limits are placed on the Lorentz invariance violation parameter $|\Delta v|$ as a function of the mixing angle θ^{v} or on the equivalence principle violation parameter $|\phi \Delta \gamma|$.

1. INTRODUCTION

Two flavor $\nu_{\mu} \rightarrow \nu_{\tau}$ mass-induced oscillations are a solid explanation of the experimental data concerning atmospheric neutrinos [1–4]. Other alternatives as the inclusion of sterile neutrinos [5, 6], $\nu_{\mu} \leftrightarrow \nu_{e}$ oscillations [4] or other exotics [7, 8], are strongly disfavored by the data, at least as "stand alone" interpretations.

In this paper we consider the mass-induced neutrino oscillations as the leading mechanism for flavor transitions, and estimate upper limits on possible contributions of relativity violating effects (violation of the Lorentz invariance (VLI), or of the equivalence principle (VEP)) as subdominant ones [9, 10], using a subset of the MACRO upward-going muon data [11]. Both "exotic" contributions are described within the same formalism; in the following, for simplicity, we will refer only to VLI.

We assume that neutrinos can be described in terms of three distinct bases: flavor eigenstates, mass eigenstates and velocity eigenstates, the latter being characterized by different maximum attainable velocities (MAVs), and consider that only two families contribute to the atmospheric neutrino oscillations. Thus, we may write

$$\begin{aligned} |\nu_{\mu}\rangle &= |\nu_{2}^{i}\rangle\cos\theta^{i} + |\nu_{3}^{i}\rangle\sin\theta^{i} \\ |\nu_{\tau}\rangle &= -|\nu_{2}^{i}\rangle\sin\theta^{i} + |\nu_{3}^{i}\rangle\cos\theta^{i} . \end{aligned}$$
 (1)

In Eq. 1 the upper index i = m or i = v for mass, and MAV neutrino eigenstates, respectively.

When both mass-induced and VLI-induced neutrino oscillations are considered simultaneously, the ν_{μ} survival probability can be expressed as [9, 12–14]

$$P_{\nu_{\mu} \to \nu_{\mu}} = 1 - \sin^2 2\Theta \sin^2 \Omega \tag{2}$$

where the global mixing angle Θ and the term Ω are given by:

$$2\Theta = \arctan(a_1/a_2)$$

$$\Omega = \sqrt{(a_1^2 + a_2^2)} .$$
(3)

The terms a_1 and a_2 in Eq. 3 contain the relevant



Figure 1: Energy dependence of the $\nu_{\mu} \rightarrow \nu_{\mu}$ survival probability for mass induced oscillations alone (continuous curve), and mass-induced + VLI oscillations for two different values of the Δv parameter. The neutrino path length was fixed as $L = 10^4$ km.

physical information

$$a_{1} = 1.27 |\Delta m^{2} \sin 2\theta^{m} L/E + 2 \cdot 10^{18} \Delta v \sin 2\theta^{v} LEe^{i\eta}|$$

$$a_{2} = 1.27 \left(\Delta m^{2} \cos 2\theta^{m} L/E + 2 \cdot 10^{18} \Delta v \cos 2\theta^{v} LE\right)$$

(4)

where the muon neutrino path length L is expressed in km, the neutrino energy E in GeV and the oscillation parameters $\Delta m^2 = m_{\nu_3}^2 - m_{\nu_2}^2$ and $\Delta v = v_{\nu_3}^v - v_{\nu_2}^v$ are in eV² and c units, respectively. The unconstrained phase η refers to the connection between the mass and velocity eigenstates; in the following we consider $\eta = 0$, for simplicity.

If the VEP scenario is preferred as source of an additional contribution to the atmospheric neutrino oscillations, in Eqs. 4 one should perform the substitution $\Delta v/2 \mapsto \phi \Delta \gamma$, where ϕ is the gravitational potential (adimensional in natural units) and $\Delta \gamma$ is the difference between the gravitational coupling constants of the two "gravitational" neutrino eigenstates involved in the oscillation.

As an example, Fig. 1 shows the $\nu_{\mu} \rightarrow \nu_{\mu}$ survival probabilities versus neutrino energy, assuming only mass-induced oscillations with the MACRO parameters $\Delta m^2 = 2.3 \cdot 10^{-3} \text{ eV}^2$ and $\sin^2 2\theta^m = 1$ (the solid curve), compared with the oscillation probabilities assuming additional VLI contributions: $\Delta v = 4 \cdot 10^{-26}$ and $2 \cdot 10^{-25}$, dashed and dotted curves, respectively. In both cases, we assumed $\sin^2 2\theta^v = 1$.

From Fig. 1 it is clear that VLI contributions significantly modify the oscillation probabilities for neutrino energies larger than few tens of GeV, while are completely negligible at lower energies. Since ν mass is expected to be below 1 eV, such energies correspond to very high values of the Lorentz factor γ .

2. EXPERIMENTAL DATA AND ANALYSIS

MACRO [15] was a multipurpose large area detector (~10000 m² sr acceptance for an isotropic flux) located in the Gran Sasso underground Laboratory, shielded by a minimum rock overburden of 3150 hg cm⁻². ν_{μ} 's were detected via charged current interactions $\nu_{\mu} + N \rightarrow \mu + X$; upgoing muons were identified with the streamer tube system (for tracking) and the liquid scintillator system (for time-of-flight measurements). Early results concerning atmospheric neutrinos were published in [1] for the up-throughgoing muon sample, and in [16] for the low energy semicontained and upgoing-stopping muon events. The global analysis of all neutrino data was presented in [2].

In order to analyze the data in terms of VLI, we used a subsample of 300 up-throughgoing muons whose energies were estimated via multiple Coulomb scattering in the 7 horizontal rock absorbers of the lower apparatus [11, 17]. The energy estimate was obtained using the streamer tubes in drift mode, which allowed to considerably improve the spatial resolution of the detector (~ 3 mm). The overall neutrino energy resolution was of the order of 100%, mainly dominated by muon energy losses in the rock below the detector (we remind that $\langle E_{\mu} \rangle \simeq 0.4 \langle E_{\nu} \rangle$). Upgoing muons in this sample have zenith angles larger than 120° and the median value of the neutrino path length is slightly larger than 10000 km.

We used two independent and complementary analyses: one based on the χ^2 criterion and the Feldman and Cousins prescription [18], and a second one based on the maximum likelihood technique.

2.1. χ^2 Analysis

Following the analysis in Ref. [11] we selected a low and a high energy samples, requiring that the reconstructed neutrino energy should be less than 30



Figure 2: 90% CL upper limits on the LVI parameters. The dashed limit is obtained applying the same energy cuts as in [11]; the solid line corresponds to the improved cut discussed in this paper. See Section 2.1 for details.

GeV and larger than 130 GeV, respectively. The numbers of events surviving these cuts are $N_{low} = 49$ and $N_{high} = 58$; the corresponding median energies, estimated via Monte Carlo, are 13 GeV and 204 GeV (assuming mass-induced oscillations). We fixed the neutrino oscillation parameters to the values of Ref. [2] ($\Delta m^2 = 2.3 \cdot 10^{-3} \text{ eV}^2$ and $\sin^2 2\theta^m = 1$), and we scanned the plane of the two remaining free parameters (Δv , $\sin^2 2\theta^v$) using the function

$$\chi^2 = \sum_{i=low}^{high} \frac{\left(N_i - \alpha N_i^{MC}(\Delta v, \theta^v; \Delta m^2, \theta^m)\right)^2}{(\sigma_i^{stat})^2 + (\sigma_i^{syst})^2} \quad (5)$$

where αN_i^{MC} is the number of events predicted by the Monte Carlo (MC) normalized to the number of observed events N_i , and σ_i^{stat} and σ_i^{syst} are the statistical and systematic uncertainties.

The used MC is described in [11] and we considered different neutrino fluxes in input, in order to estimate the contribution of the simulated ν fluxes to the systematic uncertainties of our analysis. The largest relative difference of the extreme values of the MC expected ratio N_{low}/N_{high} is 13%; in this work we use a conservative 16% theoretical systematic error on this ratio. The experimental systematic error was estimated to be 6%. In the following, we show the results obtained with the neutrino flux computation of Ref. [19].

The inclusion of the VLI effect does not improve the value of χ^2 in any point of the $(\Delta v, \sin^2 2\theta^v)$ plane, compared to stand-alone mass-induced oscillations, and upper limits on the VLI parameters were obtained. The 90% confidence level (CL) limit computed with the Feldman and Cousins prescription is shown as the dashed line in Fig. 2.

The energy cuts considered above (the same as used in Ref. [11]) were optimized for mass-induced neutrino oscillations. We maximized the sensitivity of the procedure for VLI induced oscillations by performing a blind analysis, based only on MC events, and found that the best performances are obtained by requiring that the reconstructed neutrino energy should be less than 28 GeV and larger than 142 GeV, respectively. The corresponding number of events in the real data are $N'_{low} = 44$ and $N'_{high} = 35$. The limit obtained with this selection is shown in Fig. 2 as the continuous curve; as expected, this limit is more stringent than the previous one.

We should stress that in the analysis presented above we considered $\sin^2 2\theta^v$ as a free parameter: this limits the range of sensitivity to the "velocity" mixing angle to $0 \le \theta^v \le \pi/4$; since in Eq. 4 we considerred $\eta = 0$, the limits in Fig. 2 assume $\Delta m^2 > 0$ and $\Delta v > 0$.

2.2. Maximum Likelihood Analysis

A different and complementary analysis of VLI contributions to the atmospheric neutrino oscillations was made on the MACRO muon data corresponding to parent neutrino energies in the range 25 GeV $\leq E \leq$ 75 GeV. This energy region is characterized by the best energy reconstruction, and the number of muons satisfying this selection is 106. These events are outside the energy ranges used in the analysis discussed in Section 2.1, and thus the expected sensitivity to VLI (or VEP) contributions to the atmospheric neutrino oscillations should be lower; on the other hand, the maximum likelihood technique (MLT) has the advantage to exploit the information event-by-event (is a bin-free approach).

Given a specific hypothesis, MLT allows to determine the set of parameters **a** of the problem (in our case $\mathbf{a} = (\Delta m^2, \theta^m, \Delta v, \theta^v)$) that maximizes the probability of the realization of the actual measurements \mathbf{x} (here $\mathbf{x} = (E, L)$), by minimizing the negative loglikelihood function:

$$\mathcal{L} = -2\sum_{i=1}^{n} \ln f(\mathbf{x}_i; \mathbf{a}) , \qquad (6)$$

where the sum is over the number of observed events, and $f(\mathbf{x}_i; \mathbf{a})$ is (at least proportional to) the probability of realization of a given event, which in our case is:

$$f(\mathbf{x}; \mathbf{a}) = \mathcal{K}(\mathbf{a}) \cdot \mathcal{P}_{MC}(\mathbf{x}) \cdot P_{\nu_{\mu} \to \nu_{\mu}}(\mathbf{x}; \mathbf{a}) .$$
(7)

In Eq. 7, $\mathcal{K}(\mathbf{a})$ is a normalization factor meant to ensure that the integral of $f(\mathbf{x}; \mathbf{a})$ over the observables \mathbf{x} space is independent on the parameters \mathbf{a} (otherwise



Figure 3: 90% CL upper limits on the $\Delta v/2$ parameter, versus the Δm^2 parameter varying inside the 90% CL MACRO global result [2].

 \mathcal{L} would not converge), $\mathcal{P}_{MC}(\mathbf{x})$ is the MC probability to observe the event \mathbf{x} in the no-oscillation hypothesis and $P_{\nu_{\mu} \to \nu_{\mu}}(\mathbf{x}; \mathbf{a})$ is the ν_{μ} survival probability given by Eq. 2.

MLT has a drawback: \mathcal{L} is not a true goodness of fit estimator, as the more popular χ^2 we used in the analysis in Section 2.1; at the same time, it has the power to be very effective when the experimental statistics is limited ¹.

We have chosen different fixed values of the Δm^2 and $\sin^2 2\theta^m$ mass-oscillation parameters on the 90% CL border reported in Ref. [2] and found the relative Δv and $\sin^2 2\theta^v$ that minimize Eq. 6. Fig. 3 shows the 90% CL upper limit of the VLI parameter $\Delta v/2$ versus the assumed Δm^2 values. The limit shown in Fig. 3 was obtained as a convolution of the 90% CL upper limits of $\Delta v/2$ corresponding to each chosen Δm^2 value.

3. CONCLUSIONS

We have searched for "exotic" contributions to "standard" mass-induced atmospheric neutrino oscillations arising from a possible violation of Lorentz invariance. We used a sample of the MACRO upthroughgoing muon events for which an energy mea-

¹We tested the MLT on the 106 events sub-sample of the data in the hypothesis of only mass-induced neutrino oscillations, obtaining results perfectly compatible with those reported in [2], based on the full MACRO statistics.

surement was made via multiple Coulomb scattering. Two different and complementary analyses were performed on the data, both of them yielding compatible upper limits for the VLI contribution.

The first approach, described in Section 2.1, uses two sub-sets of events referred to as the low energy and the high energy samples. The mass neutrino oscillation parameters have been fixed to the values determined in Ref. [2], and we mapped the evolution of the χ^2 estimator in the plane of the VLI parameters, Δv and $\sin^2 2\theta^v$. No χ^2 improvement was found, so we applied the Feldman Cousins method to determine 90% CL upper limits on the VLI parameters. The obtained limits, using the same energy cuts as in [11], and a set of cuts optimized in order to increase the sensitivity to VLI effects are shown in Fig. 2. The best limit ranges from $\Delta v < 6 \cdot 10^{-24}$ for $\sin^2 2\theta^v = 0$, to $\Delta v < 2.5 \cdot 10^{-26}$ for $\sin^2 2\theta^v = 1$.

The second approach is less conventional and is described in Section 2.2. It exploits the information contained in a data sub-set characterized by intermediate muon energies. It is based on the maximum likelihood technique, and considers the mass neutrino oscillation parameters inside the 90% border of the global result [2]. The obtained 90% CL upper limit on the Δv VLI parameter (shown in Fig. 3 versus the assumed Δm^2 values) is also around 10^{-25} .

The two analyses yielded compatible results.

The limits reported in this paper are comparable to those estimated using Super-Kamiokande and K2K data [9, 10].

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