Pulse Emission from Relativistic Collapsing Objects

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We discuss observable form of the radiation emitted from a surface of a collapsing object using a simplified model in which a radiation of massless particles has a sharp in time profile and it happens at the surface at the same moment of comoving time. Its redshift and bending angle are affected by the strong gravitational field. We obtain a simple expression for the observed flux of the radiation as a function of time. To find an explicit expression for the flux we develop an analytical approximation for the bending angle and time delay for null rays emitted by a collapsing surface at $R > 2R_g$. We obtain an approximate analytical expression for the observed flux and study its properties.

1. Introduction

Light propagating in the vicinity of compact relativistic objects like neutron stars and black holes are is affected by a strong gravitational field. For the description of the photon propagation under these conditions the general relativity is required. it was demonstrated recently that general relativistic effects might be important for understanding the features of the radiation coming from these objects [2]-[6]. In particular, according to the general relativity, because of the gravitational bending of light rays, a distant observer can see a part of the opposite side of the neutron star which is invisible in a flat spacetime. The radiation emitted from this part gives an important contribution and has a visible impact on the form of the signal from the X-ray burst.

The effects of the general relativity also modify considerably light curves for continuous in time radiation from the surface of a collapsing star as seen by a distant observer[7] [8][9]. The main attention was focused on the details connected with light emitted near (unstable) circular photon orbits at $3r_g/2$, where r_g is a gravitational radius of a collapsing star. In such considerations there were usually adopted a number of simplifying assumptions, such as: (1) Spherical geometry; (2) Dust-like (pressure free) equation of state; (3) Radiation comes only from the (free-falling) surface of the star; and (4) It is continuous in time.

In this work we would like to discuss a slightly different set up, when the assumption (4) is changed. Namely we assume that the radiation emitted from the surface of a collapsing spherically symmetric stellar object has a profile of a sharp in time pulse. Such radiation may occur in different situations. For example, suppose a neutron star or a proto-neutron star looses its stability as a result of the accretion of matter onto it or due to the softening of equation of state [10] at the center which is supposed to be already several times higher than the nuclear density . During the collapse, the matter density of a compact object is growing and the whole system evolves into the much higher density than the normal nuclear density [11]–[12], beyond which new hadronic phase transitions might take place [13]. One might expect a possible sharp-in-time emission of massless particles (photons and neutrino) during such phase transition [14].

In this work we assume that a radiation of massless particles has a sharp in time pulse profile and it happens at the surface at the same instant of time (from a point of view of a comoving observer). The time required for the radiation to reach a distant observer depends on the position of a radiative region on the collapsing surface. For this reason the pulse emission results in a continuous flux received by the observer during some finite interval of time. During this interval the flux as well as the redshift factor changes. In principle, knowing the redshift and light curves allows one to obtain direct information about the collapsing body at the moment when the radiation occurs.

We consider a photon emitted from a collapsing spherical surface and propagating to the observer at infinity in the background of Schwarzschild metric

$$ds^2 = -f dt^2 + f^{-1} dr^2 + r^2 d\Omega\,, \tag{1}$$

where $f = f(r) = 1 - 2M/r$ and M is the mass of the collapsing object. We adopt the natural units, $c = G = \hbar = 1$. In [7] and [11] the motion of a spherical surface during the gravitational collapse was discussed under assumption that the dynamical role of the pressure can be neglected [15], while the surface follows a radial geodesic in the Schwarzchild geometry [16].

Denote by τ the proper time as measured by an observer comoving with the collapsing surface. We suppose that the collapse starts at $\tau = 0$ and the initial surface radius is R_0. For a freely falling surface the invariant radial velocity at the moment when its radius is R is given by

$$v_i = f^{-1}(R)\frac{dR}{dt} = -\sqrt{\frac{2M}{R}}\frac{\sqrt{1-R/R_0}}{\sqrt{1-2M/R_0}}\,. \qquad (2)$$

Consider a photon emitted from the surface. Its trajectory lies in the plane. Without loss of generality we assume that it coincides with a plane of the fixed coordinate ϕ, so that the vector of the 4-momentum of the photon is $p^\mu = (p^t, p^r, p^\theta, 0)$ Because of the symmetry of the Schwarzschild metric, $E = -p_t$ (the energy at infinity) and $L = p_\theta$ (the angular momentum) are constants of motion. Instead of the angular momentum L we shall use the impact parameter $l = L/E$. The radial momentum p^r is given by $p^r = \sigma E Z$, where $Z = Z(l,r) = \sqrt{1 - l^2 f(r)/r^2}$. Here and later σ denotes a sign function which takes the values $+$ and $-$ for a forward ($p^r > 0$) and backward ($p^r < 0$) emission, respectively.

There is an upper limit for the impact parameter, l_{max}, given by $l_{max} = \frac{R}{\sqrt{f(R)}}$. We consider only the light with the emission angle, $\beta \le \pi/2$, as measured by an observer comoving with the surface. For a tangentially emitted photon with respect to a comoving observer($\beta = \pi/2$) the corresponding impact parameter $l = l_T$ is determined by the condition $Z = -v_i$: $l_T = \frac{R}{\sqrt{1-2M/R_0}}$. To escape delicacies connected with more complicated behavior of the photon orbit we assume that $R > 3\sqrt{3}\sqrt{1-2M/R_0}M$. In this case for a photon which reaches the infinity the possible ranges of an impact parameter are $0 \le l \le l_{max}$ and $l_T \le l \le l_{max}$ for a forward and backward emission, respectively. A discussion of the allowed ranges of the impact parameter for the smaller radius up to $R \sim 2M$, can be found in [7] and [8].

For a given ray, the redshift factor Φ is defined as the ratio of the emitted frequency $\nu^{(e)}$ to the observed at infinity frequency $\nu^{(o)}$

$$\Phi = \frac{\nu^{(e)}}{\nu^{(o)}} = \frac{1 - \sigma v_i Z(l,R)}{\sqrt{f}\sqrt{1-v_i^2}} \qquad (3)$$

for a ray with the impact parameter l emitted from the surface of the radius R.

We use the freedom in the choice of spherical coordinates to put the angle θ in the direction to an observer at infinity to be equal to zero, $\theta^{(o)} = 0$. Consider a null ray emitted by the collapsing surface when its radius is R and which reaches the distant observer. Suppose its impact parameter is l. Then such a ray is emitted by the collapsing surface from the region at the angle $\theta^{(e)}$. For forward emission this bending angle is

$$\theta_+^{(e)} = \Theta(l,R) = l\int_R^\infty \frac{dr}{r^2 Z(l,r)}\,. \qquad (4)$$

For a backward-emission a photon before it reaches the infinity should pass through a turning point, $r_t < R$, which is determined by $Z(l,r_t) = 0$. One can see that, for $l = l_{max} = R^2/(1-2M/R)$, $r_t = R$ as expected. Then we get

$$\theta_-^{(e)} = 2\Theta(l,r_t) - \Theta(l,R)\,. \qquad (5)$$

Consider a null ray with the impact parameter l emitted from the collapsing surface at the moment τ when it has the radius $R(\tau)$. Denote by $t_\pm^{(o)}$ the time when it reaches a distant observer at radius $r^{(o)}$ for the forward/backward ray. It is evident that $t^{(o)} \to \infty$ when $r_0 \to \infty$. For this reason it is more convenient to consider a finite quantity, the time difference between arrival of two null rays emitted at two different moments of proper time, τ and τ_e, respectively. For the second ray, emitted at τ_e, we put $l = 0$. Such a ray goes radially. We denote this time difference by $\Delta t(l;\tau,\tau_e)$. In the limit when $r^{(o)} \to \infty$ this quantity remain finite. For the forward ray it is given by the following expression

$$\Delta t_+(l;\tau,\tau_e) = t^{(e)}(\tau) - t^{(e)}(\tau_e) + T(l, R(\tau))$$

$$+R(\tau_e) - R(\tau) + 2M\ln\frac{R(\tau_e) - 2M}{R(\tau) - 2M}, \qquad (6)$$

where

$$T(l,R) \equiv \int_R^\infty \frac{dr}{f(r)}\left[\frac{1}{Z(l,r)} - 1\right]. \qquad (7)$$

Similarly for the backward ray one has

$$\Delta t_-(l;\tau,\tau_e) = t^{(e)}(\tau) - t^{(e)}(\tau_e) + 2T(l,r_t) - T(l,R(\tau)) \qquad (8)$$

$$+R(\tau_e) + R(\tau) - 2r_t + 2M\ln\frac{(R(\tau)-2M)(R(\tau_e)-2M)}{(r_t-2M)^2}.$$

The integrals for Θ and T (see relations (4) and (7), respectively) can be expressed in terms of the elliptic functions. However, for practical calculations it is very convenient to have approximations for these quantities in terms of simple elementary functions. In the next section, we develop high accuracy analytic approximations, for the integrals $\Theta(l,R)$ and $T(l,R)$.

2. Analytic Approximation

It is convenient to use the dimensionless quantities

$$x = M/r\,, \quad q \equiv M/R\,, \quad \hat{l} = l/M\,. \qquad (9)$$

We also denote

$$\hat{Z} = \hat{Z}(\hat{l},x) = \sqrt{1 - \hat{l}^2 x^2(1-2x)}\,, \qquad (10)$$

so that

$$\Theta = \int_0^q dx \frac{\hat{l}}{\hat{Z}(\hat{l},x)} \,. \tag{11}$$

In a flat metric with $f = 1$, one can calculate the integral (11) analytically to get

$$\Theta_{flat} = \arccos(\sqrt{1-\hat{l}^2 q^2}). \tag{12}$$

Leahy [3] discovered that in a wide range of its arguments the exact integral for the bending angle can be approximated by a simple analytical expression. A simple elegant form of the approximative expression was proposed later by Beloborodov [4]: $\Theta_0 = \arccos\left[\frac{\hat{Z}-2q}{1-2q}\right]$. We shall refer to this relation as to Beloborodov–Leahy (or BL–) approximation.

The typical accuracy of the BL–approximation is of order of 1% for the light rays emitted from the surface $R = 6M$. For smaller R the accuracy of the BL–approximation is worse. For example it becomes of order of 10% for $R = 4M$. In order to use this approximation for our purposes we first slightly modify it to improve the accuracy.

Comparing expansions of Θ and Θ_0 at small values of $\hat{l}$, we propose an ansatz as follows

$$\hat{\Theta}(\hat{l},q) = \arccos\left[\frac{\hat{Z}(\hat{L},q)-2q}{1-2q}\right] + b_5 q^2 \mathcal{Z}^5 \,, \tag{13}$$

where

$$\mathcal{Z} \equiv \frac{1-\hat{Z}(\hat{l},q)}{1-2q}. \tag{14}$$

Numerical calculations show that the accuracy of the approximation is very good for $b_5 = 0.0884$. Thus the approximate expression for the forward-emitted rays is

$$\theta_f^{(e)} \approx \hat{\Theta}(l, q = M/R) \,. \tag{15}$$

The relative error $\Delta_\theta = (\theta_f^{(e)} - \hat{\Theta})/\theta^{(e)}$ of the approximate expression is very small. It is less than 0.5% for $R \geq 4.5M$ for all the allowed values of l. For $R = 4M$ the error Δ_θ is slightly larger. It is still less than 0.8 % every where excluding a narrow vicinity of $s = 1$ where it reaches 2%.

We shall use the formula (13) to approximate the bending angle for the forward emission. For the backward emission the approximate formula is

$$\theta_b^{(e)}(l,R) \approx 2\hat{\Theta}(\hat{l}, M/r_t) - \hat{\Theta}(\hat{l}, M/R) \,. \tag{16}$$

Now we consider the arrival time. Using the dimensionless version of T, $\mathcal{T} = T/M$, we can rewrite the expression (7) in the following form:

$$\mathcal{T} = \mathcal{T}(\hat{l},q) = \int_0^q dx \frac{\hat{l}^2}{\hat{Z}(\hat{l},x)(1+\hat{Z}(\hat{l},x))} \,. \tag{17}$$

We want to obtain an analytic approximation for T.

For f =const in the integral (17), this integral can be calculated exactly :$\mathcal{T}_0 = \frac{1-\sqrt{1-\hat{l}^2 q^2 f}}{f\,q}$. We restore the dependence f on q and use this expression with $f = 1-2q$ as a starting point for our approximation. The corresponding expression can be written as

$$\mathcal{T}_0 = \frac{\mathcal{Z}}{q} \,. \tag{18}$$

One can check that this approximation is very good for small q. Our ansatz for the improved approximation is

$$\hat{\mathcal{T}} = \mathcal{T}_0 + Q(\mathcal{Z}) \,, \tag{19}$$

where

$$Q(\mathcal{Z}) = \frac{1}{4}\mathcal{Z}^2 + a_3 \mathcal{Z}^3 + a_4 \mathcal{Z}^4 \,. \tag{20}$$

Numerical calculations show that the accuracy of the approximation is very good for the following choice of the parameters: $a_3 = 1/15 \,, \quad a_4 = 1/25$.

The relative error $\Delta_T = (T - \hat{T})/T$ of the approximate expression is is less than 0.5% for $R \geq 4.5M$ for all the allowed values of l. For $R = 4M$ the error Δ_T is slightly larger. It is still less than 1 % every where excluding a narrow vicinity of $s = 1$ where it reaches 3%. On the other hand the approximation for the derivative $\hat{\mathcal{T}}_{,\hat{l}}$ works slightly worse than the approximation for $\hat{\mathcal{T}}$. We denote

$$\Delta_{T,l} = (\mathcal{T}_{,\hat{l}} - \hat{\mathcal{T}}_{,\hat{l}})/\mathcal{T}_{,\hat{l}} \tag{21}$$

the relative error. It is found that the maximum value of the relative error (for $R = 4.5M$) reached 5% near $s = 1$.

3. Flux and intensity for a short flash

We shall use superscripts (e) and (o) for emitted and observed radiation, respectively.

Consider a light ray with the impact parameter l emitted from the collapsing surface at the moment of the proper time τ, and let t be the time when it reaches a distant observer at r_0. One can show (see e.g. Exercise 22.17 in [18]), that the quantity $I_\nu(\nu)/\nu^3$ remains constant along a photon's world line.

The specific flux as measured by a distant observer at time t is

$$\mathcal{F}^{(o)}_{\nu_0}(t) = \frac{2\pi}{r_0^2} \int l dl \Phi^{-3} \mathcal{I}^{(e)}_{\nu_e}(l, \nu_e, \tau(t,l)) \,. \tag{22}$$

Here $\Phi = \Phi(l,R)$ is given by eq.(3). The integral over l in eq.(22) can be rewritten as an integral over the proper time, τ:

$$\mathcal{F}^{(o)}_{\nu_0}(t) = \frac{2\pi}{r_0^2} \int d\tau W \Phi^{-3} \mathcal{I}^{(e)}_{\nu_e}(l, \nu_e, \tau) \,, \tag{23}$$

where $W \equiv l \left| \frac{dl}{d\tau} \right|$.

For a very short in time flash from the surface at the moment τ_e, the intensity can be approximated as

$$\mathcal{I}^{(e)}_{\nu_e}(l, \nu_e, \tau) = I^{(e)}_{\nu_e}(l, \nu_e)\delta(\tau - \tau_e) \tag{24}$$

and we get

$$F^{(o)}(t) = \frac{2\pi}{r_0^2} W_e \Phi_e^{-4} I^{(e)}(l_e)\,, \tag{25}$$

where the intensity $I^{(e)}(l)$ is

$$I^{(e)}(l) = \int d\nu_e I^{(e)}_{\nu_e}(l_e, \nu_e)\,. \tag{26}$$

This form radically simplifies the study of the light curves.

We denote the flux registered by the distant observer at the moment t as $F^{(o)}(t)$ and we denote by $F^{(o)}(0)$ the flux at the moment when the first ray arrives to the distant observer. It is convenient to normalize the observed time-dependent flux $F^{(o)}(t)$ to the value $F^{(o)}(0)$. We denote this ratio

$$\mathbf{F}(t) = \frac{F^{(o)}(t)}{F^{(o)}(0)} = \frac{W_e(t)}{W_0}\left(\frac{\Phi_e(t)}{\Phi_0}\right)^{-4}\mathbf{I}\,, \tag{27}$$

where $\mathbf{I} = \frac{I^{(e)}(l_e)}{I^{(e)}(0)}$.

The intensity of the radiation from the surface of star can be written [20] as

$$I^{(e)}(l) = a + b\cos(\beta(l)), \tag{28}$$

where a and b depend on the details of the emission process. In this work, we calculate two extreme cases: (A) $a \neq 0$, $b = 0$, and (B) $a = 0$, $b \neq 0$.

4. Flash from a collapsing surface

To illustrate the obtained results, we consider now special examples. As a first example, we consider a neutron star which looses its stability. In this case an initial radius R_0 is $R_{NS} = 12-20$ km and the mass is of order of $M \sim 1.5M_\odot$ [21], and hence $R_0/M = 5.4-9$. Another example is a proto neutron star $R_{PNS} \sim 20$ km and $M \sim 1.5M_\odot$ [1]. In this case $R_0/M = 9$. In this section, we discuss in detail two cases with initial radii: $R_0 = 5.4M$ and $R_0 = 9M$.

For a freely falling surface with $R_0 = 5.4M$, the turning point r_t on the trajectory of a backward ray lies within the valid range of the analytic approximation, $r_t > 4.5M$, provided $R_e \geq 4.8M$. In accordance with this we choose $R_e = 4.8M$ (case I).

For a freely falling surface with $R_0 = 9M$, the analytic approximation can be applied to the emission at $R_e \geq 5.5M$. In this case we calculate a bending angle, redshift and a fluxes registered by a distant observer for the following 3 values of $R_e/M = 5.5, 6.5, 7.5$ (cases IIa, IIb, and IIc, respectively).

The maximum arrival time difference is assigned for the backward ray emitted with an impact parameter l_T, and it is

$$\begin{aligned}\Delta t_{max} &= \Delta t_-(l_T; \tau_e, \tau_e) = 2T(l_T, r_t) - T(l_T, R_e), \\ &+2R_e - 2r_t + 4M \ln\frac{(R_e - 2M)}{(r_t - 2M)}.\end{aligned} \tag{29}$$

In the case I, for $R_0/M = 5.4$ and $R_e/M = 4.8$, the time delay is calculated is $\Delta t_{max}/M = 13.8$. In the case II, for $R_0/M = 9$ the time delay for different values of R_e is given in the Table.

Time delay for $R_0/M = 9$

Case	IIa	IIb	IIc
R_e/M	5.5	6.5	7.5
$\Delta t_{max}/M$	16.9	15.4	14.4

In what follows it is convenient to use a normalized arrival time difference defined as $\delta \equiv \Delta t/\Delta t_{max}$. We shall call this quantity the time parameter. The time parameter is always changes in the interval $[0, 1]$. The time parameter for forwardly emitted light increases as l increases from $l = 0$ to l_{max}. The backward emission starts with l_{max} and ends at l_T and the time parameter for a backward emission is increasing as l changes from l_{max} to l_T.

The bending angle as a function of the time parameter is a monotonously increasing function. The smaller is R_e/M, the faster is the radial motion of the radiating surface, and the larger is the observed region with the backward emission. As a result the range of bending angle for smaller values of R_e/M becomes larger for a given R_0.

The frequency observed at infinity is different from the frequency at emission because of two reasons: (1) Difference of the gravitational potential at the point of emission and observation, (gravitational redshift), and (2) The velocity of the emitting surface (Doppler shift). The photons emitted from the surface of R_e experience the same gravitational redshift independent of their angular positions (bending angle) of emission. However Doppler shift depends on the relative velocity of the surface of emission with respect to the distant observer, and hence it depends on the bending angle (or the impact parameter l). Since the arrival time depends on the impact parameter as well, the frequency shift then can be plotted as a function of the arrival time. The calculated ratio of emitted frequency to the observed one, Φ, for a short flash as a function of the time parameter δ is shown in Fig. 1. Three curves which meet one another at $\delta = 1$ correspond to the three cases IIa,b,c. The forth curve corresponds to the case I.

It is interesting that the redshift due to the gravity is substantially cancelled by the Doppler shift for the

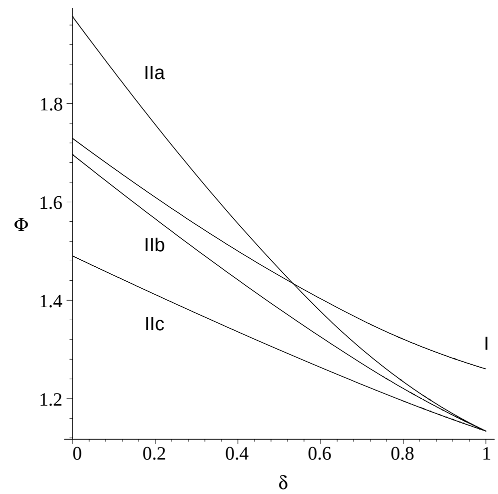

Figure 1: Redshift factor for a freely collapsing surface as a function of the time parameter δ for the cases I, and IIa,b,c [22].

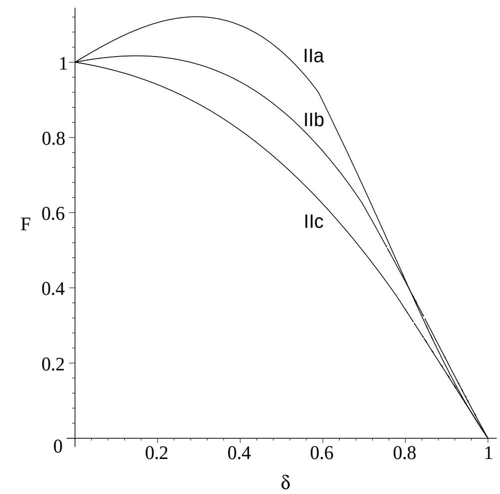

Figure 2: Light curves for $\mathbf{I} = 1$ for the cases IIa,b,c[22].

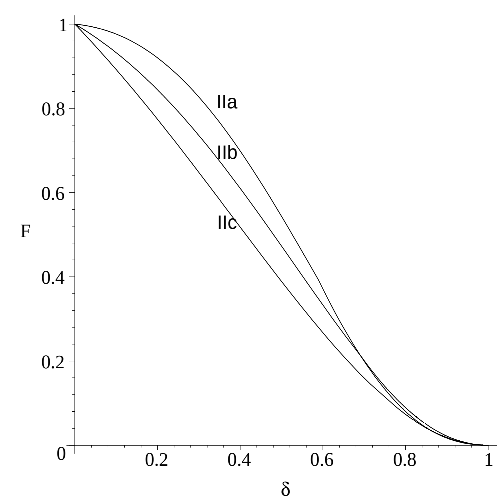

Figure 3: Light curves for $\mathbf{I} = \cos\beta(l)$ for the cases IIa,b,c[22].

tangentially emitted light. For a free fall from the radius R_0 one has $\Phi_T = \frac{1}{\sqrt{1-2M/R_0}}$. It means that for the "last rays" (that is for rays with $l = l_T$), the redshift depends only on the initial radius R_0 and does not depend on the radius of emission R_e. For this reason the three curves IIa,b,c in Fig. 1 merge at the same value $3/\sqrt{7} \approx 1.134$ at $\delta = 1$ (that is for $l = l_T$). Relation (4) also shows that for $R_0 = \infty$ the gravitational redshift is exactly cancelled by the Doppler shift [8].

Since for direct radial rays ($l = 0$) both effects "work" in the same direction, one can expect that for a given R_0 the redshift will be larger for smaller values of R_e. The Fig. 1 clearly demonstrates this.

5. Light curves

Let us discuss now normalized flux as a function of the time parameter δ. We call the corresponding graph a light curve.

For the case II ($R_0 = 9M$) the light curves for A and B type of the radiating surface are shown in Fig. 2 and Fig. 3, respectively. Each of the figures contains 3 curves corresponding to IIa,b,c cases.

Let us discuss now qualitative behavior of the light curves. The observed normalized flux $\mathbf{F}(t)$ is a product of 3 factors: (1) a kinematic term $W_e(t)/W_0$, (2) a redshift factor $(\Phi_e(t)/\Phi_0)^{-4}$, and (3) a normalized intensity of the emission $\mathbf{I}$. The third factor depends on the model of the radiating surface and it does not depend on the arrival time. The first two factors are time dependent. The arrival time dependence of W_e is essentially determined by the factor of $|Z - \sigma v_i|$, which is a decreasing function of δ and vanishes for $\delta = 1$. Hence every light curves should cross the zero-flux axis at $\delta = 1$. For a static surface $v_i = 0$ and Z (and hence $W_e(t)$) vanishes at $\delta = 1$, where $l = l_{max}$. For a collapsing surface $v_i < 0$ and the observable flux vanishes not for l_{max} (where $Z = 0$) but for the backward emission with $l = l_T$. Hence one can expect longer duration of observed flux for the emission from a collapsing surface compared to the emission from a static surface. The effect of motion of the collapsing surface becomes stronger for larger v_i. For example for a given $R_0 = 9M$, Δt_{max} is calculated to be larger for smaller R_e for which v_i is larger (see Table).

The redshift factor Φ depends basically on the relative receding velocity of the emitting region (determined by the bending angle) with respect to the distant observer. The relative receding velocity is decreasing as the bending angle is increasing. Since the arrival time difference becomes larger for a ray with larger bending angle, one can expect the enhancement of a factor, Φ^{-4}, for larger δ. The main effect of the frequency shift for the observed flux due to the collapsing surface is the enhancement of the flux for lately arriving rays. As a result the shape of the light curve for a collapsing surface is changing from that of a static surface in such way that the flux decreasing in δ is delayed and the sharp forward peak at $\delta = 0$ becomes a rather smooth peak. For sufficiently large collapsing velocity, for example for $R_0 = 9M$ and $R_e = 5.5M$, one can observe that position of the peak in the light curve also changes from $\delta = 0$ to a later arrival time $\delta \neq 0$ for the isotropic intensity profile(A) as shown at Fig. (2). The emission angle with respect to the normal to the surface, β, varies from 0 to $\pi/2$ as the impact parameter l varies from 0 to l_{max} and further to l_T. Hence the intensity profile of (B) with $I^{(e)} = b\cos(\beta(l))$ suppresses the enhancement due to the factor Φ^{-4} for lately arriving rays substantially as shown in Fig. 3.

Comparison to the light curves from the static surface is in order. Even for a static object the effects of the General Relativity allows one to "see" a part of its opposite side surface. For a collapsing object this effect is more profound. As a result, the duration of the flux is elongated. Another difference is that the sharp decrease in time for the static surface is delayed and "smoothed out" so that the peak becomes broader. For a sufficiently large collapsing velocity the peak position can even be shifted to $\delta > 0$. We demonstrated these features by considering two examples of collapses starting at $R_0 = 5.4M$ and $R_0 = 9M$.

Though in this paper we focused on a model of brief in time flash emission, some of its results (improved analytic approximation) might be of the interest for other astrophysically interesting problems.

Acknowledgment

This work was supported by the Natural Sciences and Engineering Research Council of Canada and by the Killam Trust. HLK was supported also in part by Korea Research Foundation Grant(KRF-2004-041-C00085). HKL would like to thank Valeri Frolov for the kind hospitality during his visit to University of Alberta and also Roger Blandford for his kind invitation to KIPAC and thanks Andrei Zelnikov for discussions.

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$$F^{(o)} = \frac{2\pi}{r_0^2}[\int_0^{l_{max}} ldl\, I_+^{(o)} + \int_{l_T}^{l_{max}} ldl\, I_-^{(o)}]\,. \qquad (30)$$

We shall use this convension for the other similar integrals.
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