Warm Stellar Matter with and without Trapped Neutrinos

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In this work we calculate the properties of proton-neutron and neutron stars obtained from various equations of state (EOS) in relativistic models. We consider neutrino free and trapped neutrino equations of state either at zero temperature or fixed entropies. Possible mixed phases with hadrons and quarks in the interior of the stars are investigated. More sophisticated models which take into account delta meson interactions are also included.

1. INTRODUCTION AND FORMALISM

Landau predicted the possible existence of a neutron star after the neutrons were discovered by Chadwick in 1932. In 1934, it was suggested that neutron stars were formed after a supernova explosion, which happens when the core of a very massive star undergoes gravitational collapse. The first supernova explosion was registered in 1054 by the Chinese. Once the gravitational collapse of a massive star takes place, a proto-neutron star can be formed. Several different stages may happen during the evolution process. The proto-neutron stars cool down to form a neutron star. The structure of compact stars is characterized by its mass and radius, which in turn are obtained from appropriate equations of state (EOS) at densities about one order of magnitude higher than those observed in ordinary nuclei [1, 2].

In this work we investigate the equation of state (EOS) of cold and warm, β-equilibrium hadron/quark matter and apply it to determine the properties of mixed stars consisting of hadron matter with hyperons and quark matter. We build the mixed phase with hadron and quark matter and search for the possibility of a pure quark matter core inside compact stars. The calculations are performed for zero temperature and finite entropies in order to describe neutron and proto-neutron stars. We also verify the importance of including trapped neutrinos and consider entropies from zero to 2 Boltzmann units.

For the low density phase we consider two hadron models, the non-linear Walecka model (NLWM) [3] and the quark-meson-coupling model (QMC) [4], both with hyperons included. In the QMC model, the nucleon in nuclear medium is assumed to be a static spherical MIT bag in which quarks interact with the scalar and vector fields, σ, ω and ρ and these fields are treated as classical fields in the mean field approximation. We also study the effect of including the δ-meson in the NLWM. Its presence introduces in the isovector channel the same structure of relativistic interactions.

For the high density phase we consider three models, namely, the MIT bag model [5] to which we also refer as unpaired quark model (UQM), the Nambu-Jona-Lasinio model (NJL) [6] and the color flavor locked quark phase (CFL) [7]. The NJL model is a chiral symmetric model which allows us to study the effect of dynamical quark masses on the EOS.

At intermediate energies we build a mixed phase of hadrons and quarks by enforcing Gibbs coexistence conditions. All equations used for the above mentioned models in mean field theory are given in the literature. The details of the calculation are discussed in [9], [10], [11] where the complete formalism is shown. Once the EOSs are obtained we have expressions for the energy density $\mathcal{E}$, pressure P and baryonic density ρ_B, from which the corresponding star properties are computed.

β equilibrium and charge neutrality are basic conditions for the construction of the star EOS. They are enforced in hadronic matter respectively by

$$\mu_{B_i} = Q_i^B \mu_n - Q_i^e \mu_e \qquad (1)$$

and

$$\sum_B Q_B^e \rho_B + \sum_l q_l \rho_l = 0,$$

where Q_i^e is the electric charge of baryon or quark i, Q_i^B is the baryonic charge of baryon or quark i, and q_l stands for the electric charges of the leptons.

The same conditions are necessary in quark matter and they read

$$\mu_s = \mu_d = \mu_u + \mu_e, \qquad \mu_e = \mu_\mu \qquad (2)$$

and

$$\rho_e + \rho_\mu = \frac{1}{3}(2\rho_u - \rho_d - \rho_s).$$

In the mixed phase charge neutrality is imposed globally:

$$\chi\, \rho_c^{QP} + (1-\chi)\rho_c^{HP} + \rho_c^l = 0, \qquad (3)$$

where ρ_c^{iP} is the charge density of the phase i, χ is the volume fraction occupied by the quark phase and ρ_c^l is electric charge density of the leptons.

The Gibbs conditions for phase coexistence are given by

$$\mu_{HP} = \mu_{QP}, \quad T_{HP} = T_{QP},$$

$$P_{HP}(\mu_{HP}, T) = P_{QP}(\mu_{QP}, T), \qquad (4)$$

where QH (HP) refers to quark (hadron) phase. The chemical equlibrium conditions (1) and (2) in the mixed phase become

$$\mu_u = (\mu_n - 2\mu_e)/3, \; \mu_d = \mu_s = (\mu_n + \mu_e)/3 \qquad (5)$$

and the energy and baryonic densities read:

$$< \mathcal{E} >= \chi\, \mathcal{E}^{QP} + (1-\chi)\mathcal{E}^{HP} + \mathcal{E}^l, \qquad (6)$$

where $\mathcal{E}^l$ is the energy density of the leptons and

$$< \rho >= \chi\, \rho^{QP} + (1-\chi)\rho^{HP}. \qquad (7)$$

If neutrino trapping is imposed to the system, the beta equilibrium condition is altered from (1) to

$$\mu_{B_i} = Q_i^B \mu_n - Q_i^e(\mu_e - \mu_{\nu_e}).$$

According to [1], the total leptonic number Y_L is constant throughout the star. We take it to be

$$Y_L = Y_e + Y_{\nu_e} = 0.4.$$

In the sequel, we identify the neutrino free EOS with $Y_{\nu_e} = 0$ and the EOS with trapped neutrinos with $Y_L = 0.4$.

Properties of compact stars whose matter obeys the EOS calculated with the above models are obtained from the the equations for the structure of a relativistic spherical and static star composed of a perfect fluid, which were derived from Einstein's equations by Tolmann, Oppenheimer and Volkoff [12]:

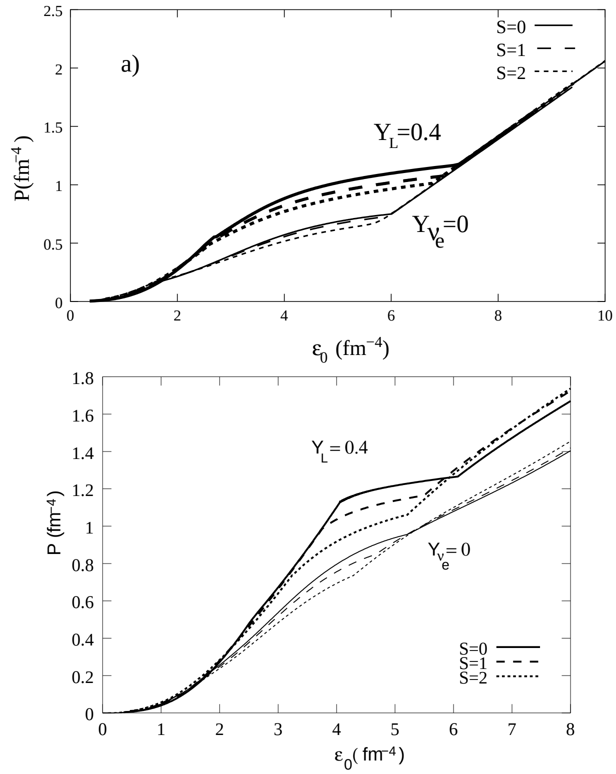
$$\frac{dP}{dr} = -\frac{G}{r}\frac{[\varepsilon + P]\left[M + 4\pi r^3 P\right]}{(r - 2GM)}, \qquad (8)$$

$$\frac{dM}{dr} = 4\pi r^2 \varepsilon,$$

where G is the gravitational constant and $M(r)$ is the enclosed gravitational mass. For all the EOS considered in the present work, the value of r $(= R)$, where the pressure vanishes defines the surface of the star.

2. RESULTS AND DISCUSSIONS

All the models used are parameter dependent. The parameters of the models are fixed by reproducing properties of nuclear matter or nucleon properties.



a)

b)

Figure 1: EOS obtained with the NLWM and a) the UQM b) the NJL model [9].

For the NLWM, we use the coupling constants defined in [2] for which the compression modulus is 300 MeV and the effective mass is $0.7M$. When δ mesons are included we take the parameter set of ref. [8], for which $E_B = -16$ MeV at the saturation density $\rho_0 = 0.16$ fm^{-1}, the symmetry coefficient is 32 MeV, the compression modulus is 240 MeV and the effective mass is $0.75M$. For the QMC the parameters are shown in [10] and they reproduce $E_B = -15.7$ MeV at $\rho_0 =$ 0.15 fm^{-3}, $a_{sym} = 32.5$ MeV, $K = 257$ MeV and $M^* = 0.774M$. For the NJL model the chosen parameters are given in [9] and they are fixed in order to fit the values in vacuum for the pion mass, the pion decay constant, the kaon mass, and the quark condensates. The value of the bag constant is mentioned throughout the text. For the CFL we use a gap parameter equal to $\Delta = 100$ MeV. Based on quark counting arguments we take the meson hyperon couplings $g_{\sigma B} = \sqrt{2/3}\, g_{\sigma N}$, $g_{\sigma B} = \sqrt{2/3}\, g_{\sigma N}$, $g_{\omega B} = \sqrt{2/3}\, g_{\omega N}$ and $g_{\rho B} = \sqrt{2/3}\, g_{\rho N}$.

We start by showing the results obtained with the NLWM for the hadron phase and the UQM and NJL models for the quark phase. In what follows the bag contant is $B^{1/4} = 190$ MeV.

From Fig. 1 one can see that the EOSs are harder and the mixed phase appears at higher densities if neutrino trapping is required independently of the model used.

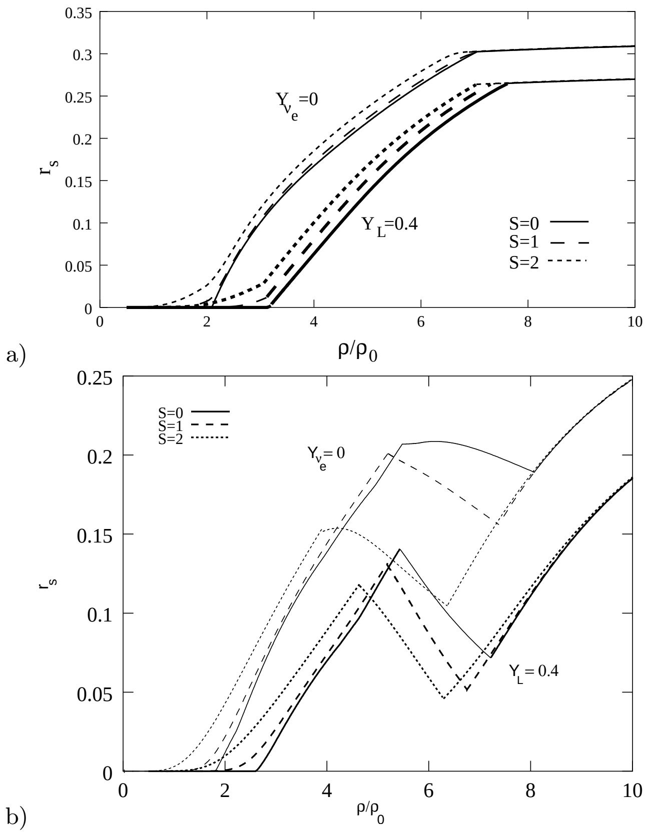


Figure 2: Strangeness fraction for the EOS with a) the UQM b) the NJL model for the quark phase.

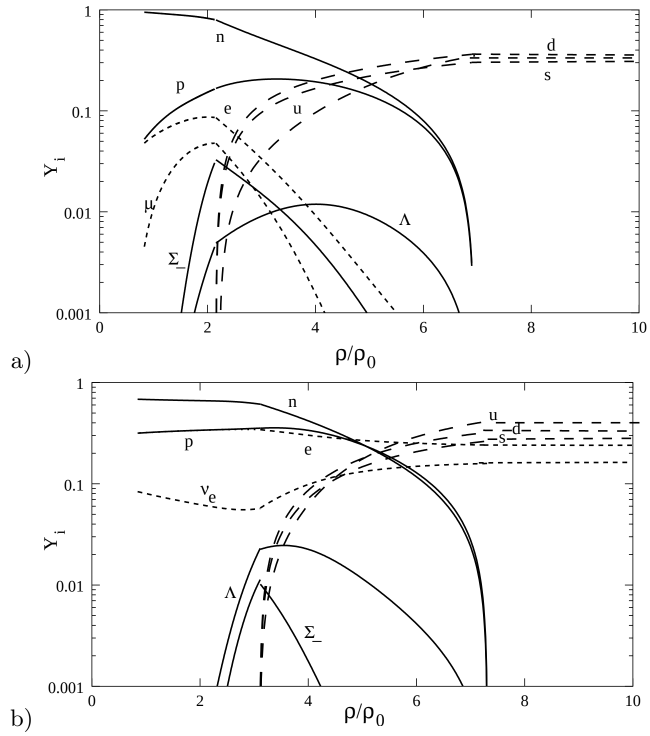


Figure 3: Particle fractions $Y_i = \rho_i/\rho$, for $i =$ baryons, leptons and quarks, obtained with the NLWM plus the UQM for $S = 1$ and a)$Y_{\nu_e} = 0$ b)$Y_L = 0.4$.

Finite temperature also makes the EOS softer because the number of states available increases with T. In the quark phase with neutrino trapping the NJL model shows a harder EOS than the neutrino free one due to a higher s-quark mass. Neutrino trapping shifts chiral symmetry restoration to higher densities.

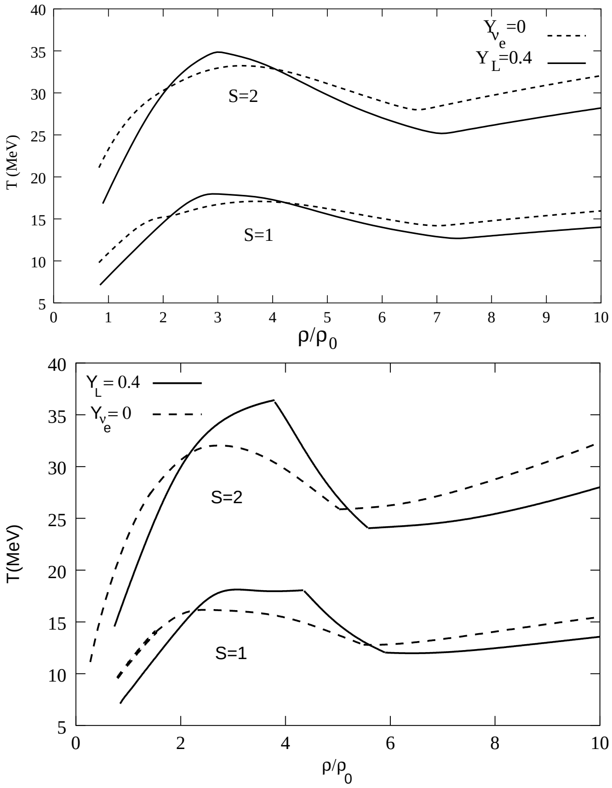
From Fig. 2 one can observe that when the quark phase is described by the Bag model the strangeness fraction rises steadly and, at the onset of the pure quark phase it has almost reached 1/3 of the baryonic matter if no trapping is imposed. It reaches a lower value once neutrino trapping is enforced. A constant lepton fraction implies higher fractions of electrons at high densities and, therefore, smaller s-quark fractions. The NJL model predicts a different behavior. In the mixed phase the strangeness fraction decreases with density. This behavior is due to fact that for the densities at which the mixed phase occurs the mass of the strange quark is still very high. The net effect of the entropy is to increase the strangeness fraction.

Notice, from Fig. 3 that the distribution of particles is altered if neutrino trapping is imposed. Trapping pushes the onset of hyperons, the mixed phase and the pure quark phase to higher energies. The pure quark phase is only slightly affected but the mixed phase can occur at a density that is $\rho_0 \sim 2\rho_0$ higher. The imposition of trapped neutrinos influences the threshold of hyperons and quarks through the conditions of charge conservation and chemical equilibrium.

It is important to determine the range of temperatures involved in the cooling process of a proto-neutron stars. This is plotted in Fig. 4, from where we can see that the maximum temperatures reach about 35 MeV if $S = 2$ and 17 MeV if $S = 1$. Moreover, it is shown that neutrino trapping makes the temperature vary more and reach a higher value in the mixed phase and a lower value in the quark phase. The reduction of T during the mixed phase is due to the opening of new degrees of freedom due to deconfinement.

For most EOSs studied and displayed in I and II, the central energy density ε_0 falls inside the mixed phase. The maximum masses of the stars decrease with increasing entropy and are systematically larger if neutrino trapping is enforced. For the present EOSs, the maximum mass of stable stars are larger if neutrinos are trapped. This difference is much smaller when the quark phase is described within NJL due, possibly, to the smaller s-quark content.

Next we show the results obtained with the QMC model for the hadron phase and the UQM and CFL models for the quark phase. The EOS are shown in Fig. 5. The EOSs are harder if neutrino trapping is



a)

b)

Figure 4: Temperature range obtained with the NLWM plus a) Bag model b) NJL model. In both figures the solid lines stand for the case with neutrino trapping and the dashed line without neutrino trapping.

Table I Hybrid star properties for the EOS obtained with the NLWM and the NJL model. Energy densities are given in fm^{-4}.

EOS	S	$M\mathrm{max}/M_\odot$	ε_0	ε_{min}	ε_{max}
$Y_{\nu_e}=0$	0	1.84	6.29	4.60	7.25
$Y_{\nu_e}=0$	1	1.84	5.99	4.35	6.62
$Y_{\nu_e}=0$	2	1.83	5.38	3.18	5.66
$Y_L=0.4$	0	2.05	6.14	4.92	6.94
$Y_L=0.4$	1	2.02	6.20	4.72	6.46
$Y_L=0.4$	2	1.96	5.72	4.50	6.00

Table II Hybrid star properties for the EOS obtained with the NLWM and the UQM. Energy densities are given in fm^{-4}.

EOS	S	$M\mathrm{max}/M_\odot$	ε_0	ε_{min}	ε_{max}
$Y_{\nu_e}=0$	0	1.64	4.58	1.81	6.06
$Y_{\nu_e}=0$	1	1.51	5.07	1.87	6.00
$Y_{\nu_e}=0$	2	1.45	4.95	1.98	5.81
$Y_L=0.4$	0	1.98	5.31	3.47	7.38
$Y_L=0.4$	1	1.89	5.20	3.03	6.99
$Y_L=0.4$	2	1.81	5.08	2.98	6.82

Table III Hybrid star properties - EOS with $Y_{\nu_e}=0$

model	$B^{1/4}$	$M\mathrm{max}/M_\odot$	ε_0	ε_{min}	ε_{max}
QMC+UQM	190	1.58	5.52	1.63	7.02
QMC+UQM	200	1.73	4.85	2.05	8.74
QMC+CFL	190	1.32	12.56	1.35	4.56
QMC+CFL	200	1.49	3.31	1.92	6.25
NLWM+UQM	190	1.64	4.58	1.81	6.06

Table IV Hybrid star properties - EOS with $Y_L=0.4$

model	$B^{1/4}$	m_s	$\frac{M\mathrm{max}}{M_\odot}$	$\frac{M_b}{M_\odot}$	R	ε_0
QMC+UQM	190	150	1.94	2.15	12.09	5.47
QMC+UQM	200	150	1.99	2.22	11.97	5.63
QMC+UQM ($Y_{\nu_e}=0$)	200	150	1.73	1.93	12.44	4.89
NLWM+UQM ($Y_{\nu_e}=0$)	190	150	1.64	1.83	12.84	4.5
NLWM+UQM ($Y_L=0.4$)	190	150	2.00	2.22	12.59	5.06
QMC+CFL	190	150	1.80	1.99	10.86	7.32
QMC+CFL	200	150	1.83	2.02	11.36	6.57
QMC+CFL ($Y_{\nu_e}=0$)	200	150	1.49	1.65	13.4	3.32

imposed, independently of the model used. A larger s-quark mass and a larger B parameter make the quark EOSs harder in the mixed phase, a fact that manifests itself on the maximum mass stellar configuration. If compared with the NLWM, the QMC provides softer EOS at low densities and harder at intermediate densities. The transition to a pure quark phase occurs at lower densities in the NWLM. This behavior has consequences on the properties of the corresponding families of stars.

In tables III and IV we display the hybrid star properties with and without trapped neutrinos respectively. The strange quark mass is $m_s = 150$ MeV, the energies are given in fm^{-4}, $B^{1/4}$ is given in MeV and the star radius R in Km. Notice that the maximum baryonic masses obtained within the NLWM are larger and the difference of maximum masses for trapped and untrapped matter is smaller for the QMC than for the NLWM. This means that the number of stars that would decay into a blackhole is much smaller in the QMC model and is probably due to the fact that no hyperons are formed in the interior of stars obtained with QMC for $m_s = 150$ MeV and $B^{1/4} = 190$ MeV contrary to the NLWM case. If the quark phase is a CFL state the baryonic mass difference between the neutrino rich stars and neutrino poor is greater than in the UQM. This is understood because in the quark phase there are no electrons and therefore the 0.4 fraction of leptons is all due to neutrinos. The greater flux of neutrinos carries out more energy. For

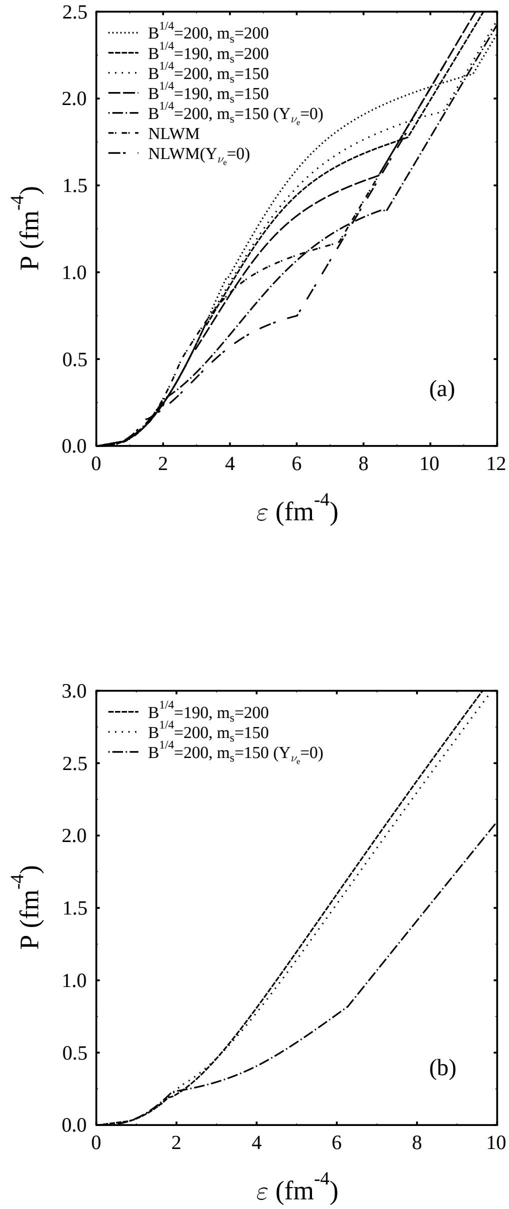


Figure 5: Equation of state obtained with the QMC model plus (a) UQM (b) CFL. In the figures above, $Y_L = 0.4$ unless stated otherwise.

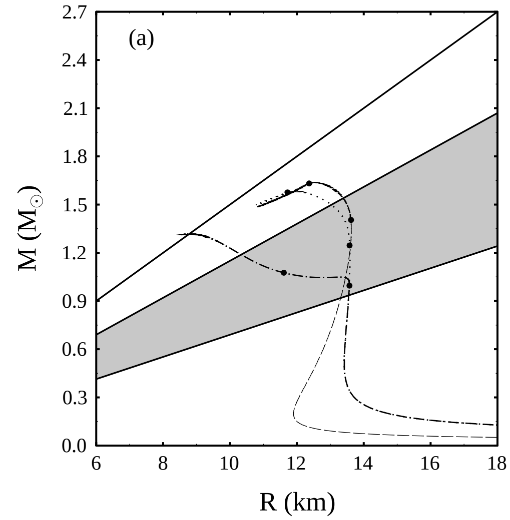


Figure 6: QMC plus UQM (solid line), QMC plus CFL (dotted line), NLWM plus UQM (long-dashed line), $B^{1/4}$=190 MeV

a finite temperature calculation we expect a smaller effect.

A straightforward method of determining neutron star properties is by measuring the gravitational redshift of spectral lines produced in neutron star photosphere which provides a direct constraint on the mass-to-radius ratio. Two recent measurements impose the constraints $M_\odot/R(km) = 0.15$ [13] and $M_\odot/R(km) = 0.12 - 0.23$ [14] respectively. From Fig. 6 we conclude that only the EOS for QMC plus CFL with $B^{1/4}$=190 MeV barely satisfies the first constraint. In fact, this constraint excludes all the EOS with hyperons, quarks or obtained within a relativistic mean-field approach. However, all the curves presented in Fig. 6 are consistent with the second constraint.

Until now, we have considered the exchange of the mesons σ, ω and ρ at the hadronic level. However, a formally consistent relativistic effective field model should include on the same footing isoscalar and isovector meson fields. Scalar isovector virtual $\delta(a_0(980))$ mesons are important in hadronic effective field theories when asymmetric nuclear matter is studied [11]. We include delta meson interactions in the NLWM for the hadronic phase and check its consequences. Only in this calculation the coupling contants with the hyperons are taken as 0.7 $g_{\sigma N}$ for the the σ and δ and are taken as 0.783 $g_{\omega N}$ for the ω and ρ.

For nucleonic stars the inclusion of the δ meson makes the EOS harder, as can be seen form Fig. 7. If hyperons (and quarks) are taken into account the EOS with δ meson suffers a transition to proton-neutron-hyperon phase at lower energies and becomes softer. From tables V and VI some conclusions can be drawn. In these tables NA means non-applicable and it is used when no mixed phases are constructed. The inclusion of δ in a star with hyperons results in opposite conse-

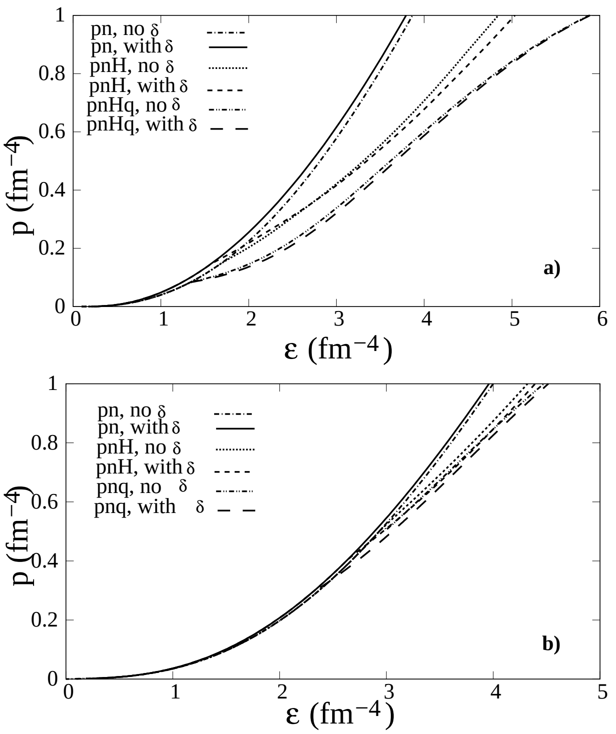


Figure 7: a) EOS with $Y_{\nu_e} = 0$; b) EOS with $Y_L = 0.4$

Table V Hadronic and hybrid star properties for the EOSs obtained with the NLWM and the UQM, $Y_{\nu_e} = 0$, $S = 0$, Bag$^{1/4}$=180 MeV, R is in Km and energies in fm^{-4}

	hadrons	$\frac{M_{\max}}{M_\odot}$	$\frac{M_b}{M_\odot}$	R	ε_0	ε_{min}	ε_{max}
no δ	pn	2.09	2.45	10.88	6.99	NA	NA
with δ	pn	2.15	2.56	11.35	6.52	NA	NA
no δ	pnH	1.72	1.95	10.76	7.41	NA	NA
with δ	pnH	1.71	1.94	11.31	6.71	NA	NA
no δ	pnHq	1.47	1.64	10.58	7.43	1.36	6.22
with δ	pnHq	1.45	1.61	10.49	7.87	1.26	6.31

Table VI Hadronic and hybrid star properties for the EOSs obtained with the NLWM and the UQM, $Y_L = 0.4$, Bag$^{1/4}$=180 MeV, R is in Km and energies in fm^{-4}.

	S	hadrons	$\frac{M_{\max}}{M_\odot}$	$\frac{M_b}{M_\odot}$	R	ε_0	ε_{min}	ε_{max}
no δ	0	pn	2.02	2.29	10.66	7.55	NA	NA
with δ	0	pn	2.04	2.31	10.73	7.33	NA	NA
no δ	0	pnq	1.82	2.02	11.62	6.31	2.98	7.68
with δ	0	pnq	1.80	1.99	11.85	6.25	2.33	7.55
no δ	0	pnH	1.89	2.10	10.91	7.06	NA	NA
with δ	0	pnH	1.88	2.09	10.51	7.03	NA	NA
with δ	0	pnHq	1.81	2.00	11.53	6.34	2.30	7.73
with δ	1	pn	2.04	2.28	-	7.35	NA	NA
with δ	1	pnq	1.78	1.95	-	6.36	2.25	7.82
with δ	1	pnH	1.86	2.04	-	6.80	NA	NA
with δ	1	pnHq	1.78	1.94	-	6.34	2.27	7.76

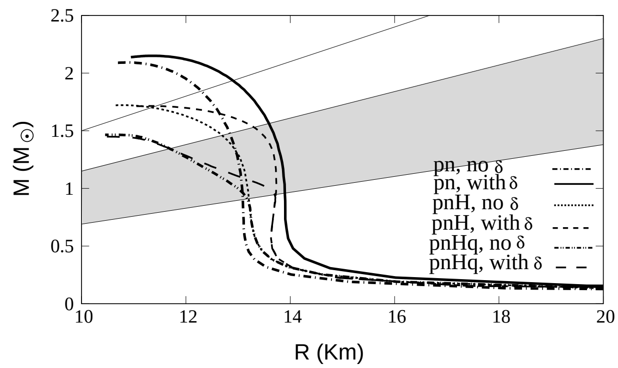


Figure 8: Compact stars: Mass versus radius

quences in the star properties if compared with a star only with protons and neutrons. In a nucleonic star the maximum gravitational and baryonic masses increase with the inclusion of the δ mesons whereas the central energy density decreases. In a pnH star, the masses decrease with the inclusion of the δ and the central energy density increases. So, while a harder EOS like the nucleonic with δ EOS supports a larger mass giving rise to a larger value for the maximum mass of a stable star, a softer EOS like the pnH or pnHq δ EOS supports a smaller mass and the maximum mass of a stable star is smaller. If we consider stars with $M > 0.5 M_\odot$, pn stars with δ-meson have always larger radius for a given mass than the corresponding stars without the δ-meson. This is not anymore true for hyperon or hybrid stars, due to the larger softening undertaken by the EOSs with the δ meson when the onset of hyperons and/or quarks occurs.

It is clear that the inclusion of the δ-meson makes the EOS for nuclear matter harder. However the onset of hyperons and/or quarks in these stars gives rises to a larger softening. In particular maximum stable stars have in these cases lower masses and smaller radius. All EOS with and without deltas satisfy both constraints imposed by spectral measurements, as can be seen from Fig. 8.

3. CONCLUSIONS

We have studied different EOSs in order to describe protoneutron and neutron stars. For this purpose, we have assumed that at low density the star is constituted of hadrons (nucleons and possibly hyperons) described within two different models NLWM and QMC, at intermediate energies it is constituted of a mixed phase of hadrons and quarks and at very high densities a possible quark phase was investigated. For the quark phase we have considered the MIT bag model with and without a CFL phase, and the NJL model. Moreover, we have studied EOS for stars after deleptonization (neutrino free) and with trapped neutrinos. Independently of the models used for the hadron and quark phases we have verified that the maximum stellar masses are larger once trapped neutrinos are considered, if hyperons and/or quarks are included. Depending on the amount of neutrinos present in the protoneutron star, it may decay into a low mass black hole or into a neutron star. The amount of neutrinos, on the other hand, depend on the EOS used [9–11]. Larger fractions of hyperons and/or s-quark correspond to larger fractions of neutrinos. Another point of interest is that generally the stellar central energy density corresponding to the most massive stable stars lies inside the mixed phase. Stars with quark core are not probable within the present models. It is important to stress that astrophysical measurements can impose constraints on the stellar mass-to-radius ratio, for instance. More precise measurements are necessary in order to rule out some of the EOSs used in the literature.

Ackowledgments

This work was partially supported by CNPq (Brazil), CAPES (Brazil)/GRICES (Portugal) under project 100/03 and FEDER / FCT (Portugal) under the projects POCTI/ FP/ FNU/ 50326/ 2003, POCTI/ FIS/ 451/ 94 and POCTI/ FIS/ 35304/ 2000.

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