Relativistic Solutions to the Problem of Jets with Time–Dependent Injection Velocities

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We present a ballistic description of the propagation of the working surface of a relativistic jet. Using simple laws of conservation of mass and linear momentum at the working surface, we obtain a full description of the jet flow parametrised by the initial velocity and mass injection. This analysis will soon be applied to particular cases of time-dependent injection of mass and velocity into the jet.

1. Introduction

The apparent superluminal knots observed along the relativistic jets of quasars and microquasars are usually interpreted as shock waves moving through the jet. It is not perfectly understood what is the mechanism that can generate internal working surfaces that move along an astrophysical jet, but it is generally accepted that the formation of these shocks is produced by a variation on the ejection flow velocity of the jet material [see for example 1, and references within].

In this work we present a full relativistic generalisation of the non–relativistic one dimensional dynamical description of internal working surfaces made by Cantó et al. [2] that can easily be applied to the most energetic jets associated with quasars, microquasars and GRB's.

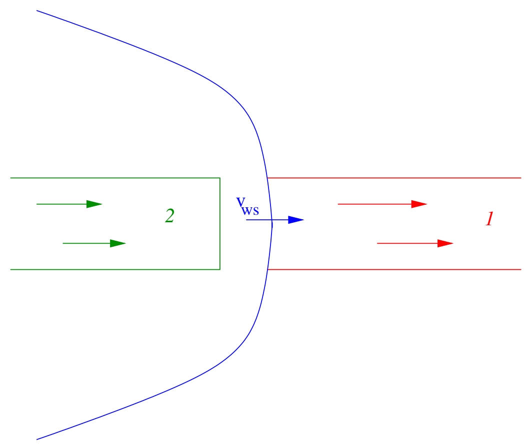

Figure 1: When a fast velocity flow 2 moves over a slow velocity flow 1, a working surface moving with velocity $v_{\rm ws}$ is generated as a result of the interaction.

2. Dynamics of relativistic working surfaces

To follow the evolution of the working surfaces, we consider a source ejecting material in a preferred direction x with a velocity $u(\tau)$ and a mass ejection rate $\dot{m}(\tau)$, both dependent on time τ.

Once the material is ejected from the source, we assume it will flow in a free-stream fashion [see e.g. [1]]. The formation of a working surface is studied as the intersection of two distinct parcels of material ejected at times $\tau_1 = 0$ and $\tau_2 = \Delta t$ labelled by their flow velocities $u_1 = u(\tau_1)$ and $u_2 = u(\tau_2) = u_1 + \alpha\,\Delta t$ respectively (see Figure 1). If $\alpha > 0$, the second parcel will eventually reach the first parcel. At the time τ_2, the distance between the parcels is $u_0 \Delta t$ and thus the time $t_{\rm m}$ (measured in the reference frame of the source) when both parcels merge is given by

$$\begin{aligned} t_{\rm m} &= \frac{1}{\alpha} u_1 \gamma^2(u_1) \left\{ 1 - \frac{u_1^2}{c^2} - \frac{\alpha u_1 \Delta t}{c^2} \right\}, \\ &= \frac{u_1}{\alpha} \left\{ 1 - \frac{\gamma^2 \alpha u_1 \Delta t}{c^2} \right\}, \end{aligned} \qquad (1)$$

where $\gamma^{-2}(u) := 1 - u^2/c^2$ represents the Lorentz factor of the flow with velocity u. The working surface is formed at a distance $d_f = u_1(t_m + \Delta t)$ from the source.

Following the non–relativistic formalism first proposed by Cantó et al. [2], we assume that the working surface is thin and that there are no mass losses within it (e.g. by sideways ejection of material [see [3, 4]). Using the free-streaming condition, we can then calculate the position $x_{\rm ws}$ of the working surface from the downstream flow

$$x_{\rm ws} = u_1(t - \tau_1), \qquad (2)$$

or from the upstream flow

$$x_{\rm ws} = u_2(t - \tau_2). \tag{3}$$

On the other hand, since the flow is free–streaming, the velocity of the working surface is given by the velocity $v_{\rm ws}$ of it's centre of mass, which is determined by [5]

$$v_{\rm ws} = \frac{1}{M_\gamma} \int_{\tau_1}^{\tau_2} \gamma\left(u(t)\right) \dot{m}(t) u(t) {\rm d}t, \tag{4}$$

where the weighted mass M_γ ejected between times τ_1 and τ_2 is

$$M_\gamma = \int_{\tau_1}^{\tau_2} \gamma\left(u(t)\right) \dot{m}(t) {\rm d}t. \tag{5}$$

With this velocity, the position of the working surface is given by

$$x_{\rm ws} = (t-\tau_2)v_{ws} + \frac{1}{M_\gamma} \int_{\tau_1}^{\tau_2} \gamma\left(u(t)\right)\, \dot{m}(t)\, u(t)\, (t-\tau_2)\, {\rm d}t. \tag{6}$$

For a given value of position $x_{\rm ws}$, expressions (2), (3) and (6) establish a relation between the times τ_1 and τ_2. The other is used to eliminate t. Taking τ_2 as a parameter, we can construct the position and velocity of the working surface as a function of τ_2 and calculate relevant quantities such as the energy available on the working surface.

To calculate the amount of energy radiated as the working surface moves, we take into account the energy E_0 the material had when it was ejected, i.e.

$$E_0 = \int_{\tau_1}^{\tau_2} \dot{m}(\tau)\, \gamma\left(u(\tau)\right)\, c^2 {\rm d}\tau, \tag{7}$$

and the energy $E_{\rm ws}$ of the material inside the working surface, which is given by

$$E_{ws} = m\gamma_{ws}c^2, \tag{8}$$

where the Lorentz factor $\gamma_{\rm ws}$ of the working surface material is given by $\gamma_{\rm ws}^{-2} = 1 - v_{\rm ws}^2/c^2$.

If we assume now that the energy loss along the jet, $E_r = E_0 - E_{\rm ws}$, is completely radiated away, then the luminosity $L = {\rm d}E_{\rm r}/{\rm d}t$ of the working surface given by

$$\begin{aligned} L = &\frac{\dot{m}(\tau_2)c^2}{{\rm d}t/{\rm d}\tau_2} \left\{ \gamma_{\rm ws} + \frac{m}{M_\gamma} \frac{\gamma_{\rm ws}^3 \gamma_2}{c^2} \left(v_{\rm ws} u(\tau_2) - v_{\rm ws}^2 \right) - \gamma_2 \right\} - \\ &- \frac{\dot{m}(\tau_1)c^2}{{\rm d}t/{\rm d}\tau_2} \frac{{\rm d}\tau_1}{{\rm d}\tau_2} \left\{ \gamma_{\rm ws} + \frac{m}{M_\gamma} \frac{\gamma_{\rm ws}^3 \gamma_1}{c^2} \left(v_{\rm ws} u(\tau_1) - v_{ws}^2 \right) - \gamma_1 \right\}, \end{aligned} \tag{9}$$

where the Lorentz factors $\gamma_{1,2}^{-2} := 1 - v^2(\tau_{1,2})/c^2$ and we keep τ_2 as a free parameter in this expression.

3. Conclusion

We have shown how a full relativistic solution can be constructed to the problem of a ballistic working surface travelling along an astrophysical jet. Our main goal is to find analytic and numerical solutions to equation (9) so that we can compare with actual observations of high–energy jets. This will be published elsewhere soon.

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