Interaction Between Stars and an Inactive Accretion Disc in a Galactic Core

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We discuss the structure of a central stellar cluster whose dynamics is influenced by gravitation of a supermassive black hole and by the dissipative interaction of orbiting stars with an accretion disc. We also take the effect of disc self-gravity into account. We show that the cluster properties are determined predominantly by the radial profile of the disc surface density. To this aim we develop a simple steady-state model of the central cluster and we estimate the rate at which stars migrate to the centre.

This model is relevant for central regions of an active galactic nucleus (AGN) containing a rather dense accretion medium. In passing we also briefly mention a possibility that a fossil (inactive) disc could exist in the centre of our own Galaxy. Such a hypothetical disc could perturb the motion of stars and set them on highly elliptic trajectories with small pericentre distances. The required mass of the disc is less than one percent of the central black hole mass, i.e. below the upper limit permitted by present accuracy of the orbital parameters of S-stars in the Galactic Centre.

1. INTRODUCTION

We examined the long-term orbital evolution of stars forming a dense stellar cluster surrounding a central black hole with an embedded accretion disc. This configuration is relevant for central regions of active galactic nuclei [10, 13] and it may be applied also to the center of our own Galaxy, assuming that rapid accretion took place and a gaseous disc was formed at some stage of its history [8]. Indeed, the presence of a dissipative gaseous environment can provide a mechanism driving stars towards the black hole, while the gravitational influence of the disc may pump eccentricity of the orbits to large values at some moments of the evolution [11, 12]. This would help us to understand the puzzling nature of young stars in the close vicinity of the black hole in the centre of our Galaxy. An attractive feature of this calculation is that it provides a well-defined model allowing to estimate the expected time-scales of the orbital migration as well as the distribution of eccentricities. It may turn out to be more likely that a different (non-standard) kind of a gaseous disc or a dusty torus plays the role, but this would not change the essence of the model.

2. RESULTS

We idealise a galactic core as a system consisting of a central black hole, an accretion disc and a dense stellar cluster. The three components interact with each other. Naturally, various approximations need to be imposed in order to keep our model sufficiently simple and tractable. The aim is to examine the structure of a stellar system in the region of black hole gravitational dominance $R_{\rm h}$, including the effects due to a gaseous disc. Two regions of the cluster can be distinguished according to the characteristic time-scale of processes dominating the stellar motion. The outer cluster is assumed to reach a gravitationally relaxed form [1], acting as a reservoir of fresh satellites that are being continuously injected inwards. The inner cluster is defined as a region where star-disc collisions take over.

Individual orbits

Stars loose their orbital energy and momentum by means of successive dissipative passages through an accretion disc. If the orbit is inclined with respect to the disc plane, each encounter with the disc slab slightly modifies the orbital parameters. The overall trend is to circularize orbits and to decline them into the plane of the disc [13]. Characteristic time of aligning stellar orbits with the disc is $t_{\rm d}(a) \approx t_0\, M_8\, (\Sigma_*/\Sigma_\odot)^{-1}\, (a/R_{\rm g})^q$ yr, where a is semi-major axis, t_0 and q are constants (order of unity for the standard disc model) and $M_8 \equiv M_\bullet/(10^8 M_\odot)$ is the fiducial value of the central mass in AGN [4].

Different modes of radial migration apply to stars in the disc plane, depending whether a star succeeds to open a gap in the disc medium, or if it remains embedded entirely and proceeds via density waves excitation. We switch between relevant modes in the numerical integration.

A gap is cleared in the disc if the embedded star is sufficiently massive and its Roche radius exceeds the characteristic vertical thickness of the disc slab at a corresponding radius, $r_{\rm L} \approx (M_*/M)^{1/3} r \gtrsim h(R)$. In this case the motion of the star is coupled with the disc inflow,

$$\dot{a}_{\rm gap} = f(a, \mu_{\rm E}, M_\bullet; b, q_i) \qquad (1)$$

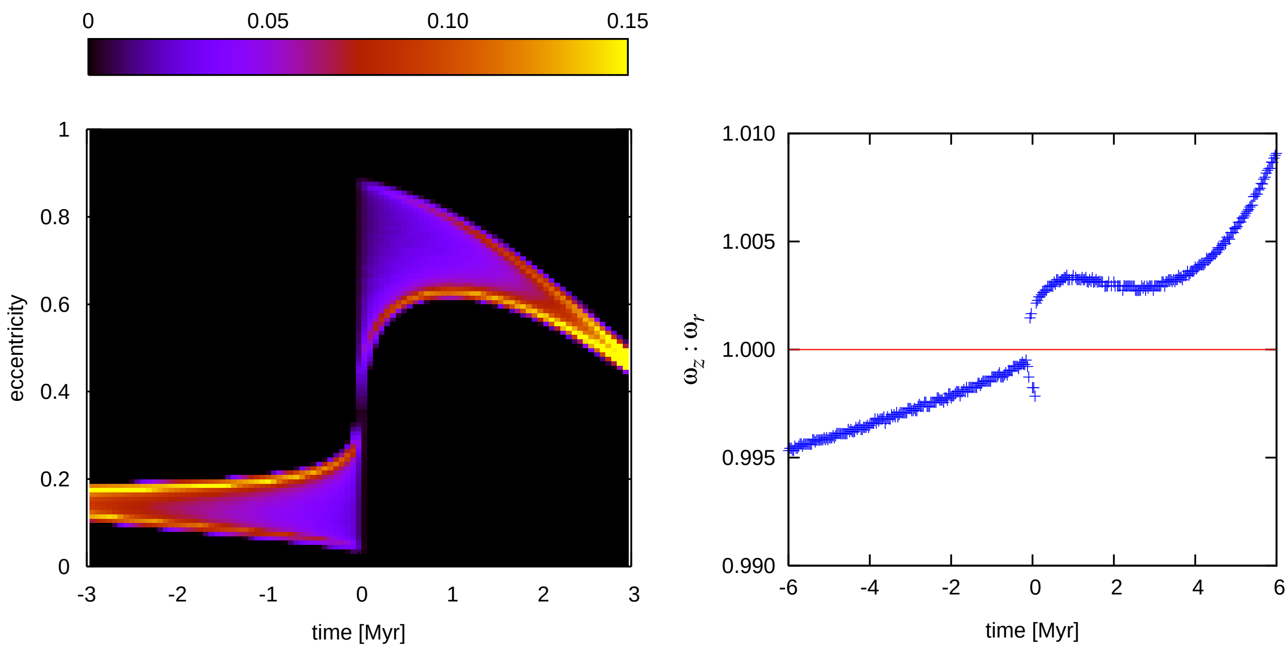

Figure 1: An example of the episode during which the stellar orbit becomes highly elliptical. Left: occasional jumps toward high eccentricity occur due to the Kozai-type mechanism in the field of the central black hole and a gravitating disc. Prevailing circularisation is expected because of continuing energy losses and the resulting orbital decay via star-disc collisions, however, fraction of highly eccentric orbits always persists in the cluster, as indicated by the colour scale. These trajectories bring stars close to the black hole where they can be preferentially captured or destroyed. Right: the ratio of mean latitudinal frequency ω_z to the mean radial epicyclic frequency ω_r (for the same orbit as on the left). It turns out that the jump of orbital parameters occurs at the moment when the ratio $\omega_z/\omega_r = 1$. The indicated time-scale corresponds to the case of $M_\bullet = 4 \times 10^6 M_\odot$ and a hypothetical disc with the mass $M_\mathrm{d} = 0.01 M_\bullet$.

with

$$f(a, \mu_\mathrm{E}, M_\bullet; b, q_i) \equiv -bM_8^{q_1}\mu_\mathrm{E}^{q_2}\alpha^{q_3}\left(\frac{a}{R_\mathrm{g}}\right)^{-q_4}$$

with μ_E being the accretion rate in units of the Eddington accretion rate and α viscosity parameter. The numerical factor b and power-law indices $q_{1\ldots4}$ are determined by details of the particular model adopted for the disc [4].

On the other hand, if the star is too tiny to create the gap, than density-waves are the dominating migration process. The resulting orbital decay can be written in the form

$$\dot{a}_\mathrm{dw} = \frac{M_*}{M_\odot}\, f(a, \mu_\mathrm{E}, M_\bullet; b', q'_i). \qquad (2)$$

Differences in the rate of stellar migration are thus introduced in the model already within this very simplified picture where the process of satellite sinking is driven by the gas medium. The dependence on the orbital parameters, the stellar masses and sizes of stars causes the gradual segregation of different stellar types in the nuclear cluster.

The gravitational field of the disc provides a perturbation capable of exciting large fluctuations of eccentricity. Assuming that these fluctuations occur on the time-scale substantially longer than the orbital period, one can apply an averaging techique and evolve equation for mean orbital parameters. See e.g. Kozai [5] and Lidov & Ziglin [6] for a general introduction to the formalism; see Vokrouhlický & Karas [14] and Šubr & Karas [12] for its modification to the present situation. We computed the gravitational field of the disc and took its effect into account for the long-term orbital decay of stellar trajectories. The effect of eccentricity oscillations is shown in Figure 1. Here, we have set the current (time zero) values of orbital eccentricity $e(t)$ and semimajor axis $a(t)$ identical as those reported for S2 star in Sagittarius A*. For the central mass we adopted $M_\bullet = 4\times10^6 M_\odot$. A false-colour plot shows what fraction of time the orbiting star spends with a certain value of the orbital eccentricity. Therefore, this graph illustrates the expected fluctuations of eccentricity and explains the existence of highly elliptic trajectories, assuming that an accretion disc or a torus was present at the centre of our Galaxy. The required mass of the disc M_d was of the order of fraction of percent of the central black hole mass, consistent with the present-day upper limit. Characteristic time-scale t_c of the oscillations depends on $M_\mathrm{d}/M_\bullet$ ratio, and is indeed much longer that the dynamical time at corresponding radius (of the order of thousand orbital revolutions in our case, $M_\mathrm{d}/M_\bullet \lesssim 0.01$).

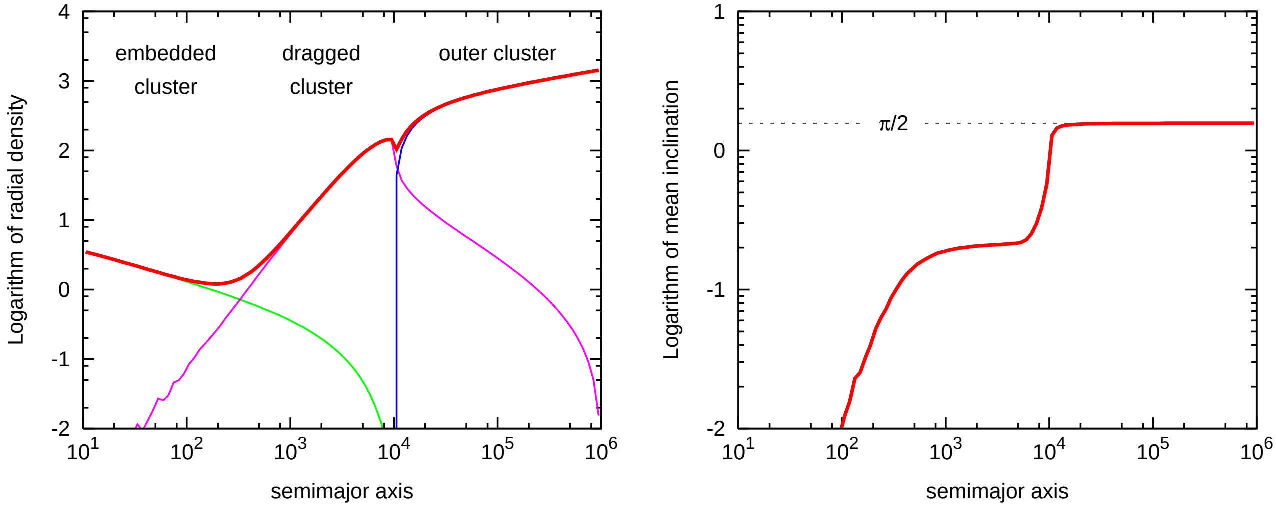

Figure 2: Left panel: distribution of semi-major axes throughout the stellar cluster, modified by interaction with the gas-pressure dominated standard accretion disc. The broken power-law profile is established by different types of interaction governing different regions of phase space. The three stellar sub-samples are present in our model and they can be easily distinguished in the plot: the outer reservoir (magenta), the inner part of the cluster which is dragged by the disc (blue), and a subsample of stars fully embedded in the disc (green). Right panel: Graph of mean inclination $\langle i \rangle$ in the cluster. The reservoir is spherically symmetric ($\langle i \rangle = \pi/2$), the dragged cluster is somewhat flattened ($0 < \langle i \rangle < \pi/2$), and the embedded population is located in the disc plane ($\langle i \rangle = 0$).

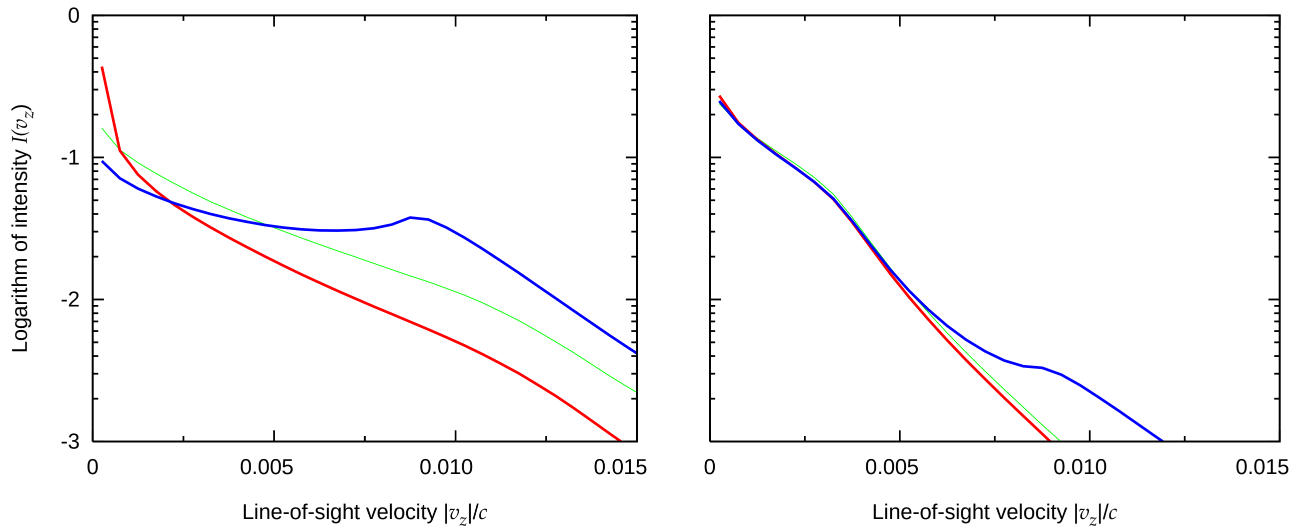

Figure 3: Velocity profile along the line of sight of the inner cluster, integrated across a column of cross-sectional radius $R_{\rm s} = 10^4 R_{\rm g}$ (left panel) and $R_{\rm s} = 10^5 R_{\rm g}$ (right panel). Blue/red lines represent different view angles of the observer: $\xi = 0^\circ$ and 60°, respectively. Growing anisotropy of the modified cluster produces the dependence of measured line profile on ξ, i.e. $I \equiv I(v_z; \xi)$. The green line is for the reference Bahcall–Wolf distribution, which exhibits spherical symmetry. See ref. [11] for further details.

The cluster

Now we construct a steady-state cluster assuming that it is supplied with fresh stars from the reservoir at a rate inversely proportional to the relaxation time. The effects introduced in the previous section are taken into account. Our computational scheme allows us to further distinguish between two subsamples of the inner cluster: the dragged inner cluster consists of stars on orbits crossing the disc periodically; the embedded inner cluster is formed by stars entirely aligned with the disc. See Šubr et al. [11] for details of our approach. For the definiteness of examples we assumed that the central mass ($M_\bullet = 10^8 M_\odot$) is surrounded by the gas-pressure dominated Shakura-Sunyaev disc ($s = -3/4$) with $\dot{M} = 0.1 M_{\rm Edd}$ and

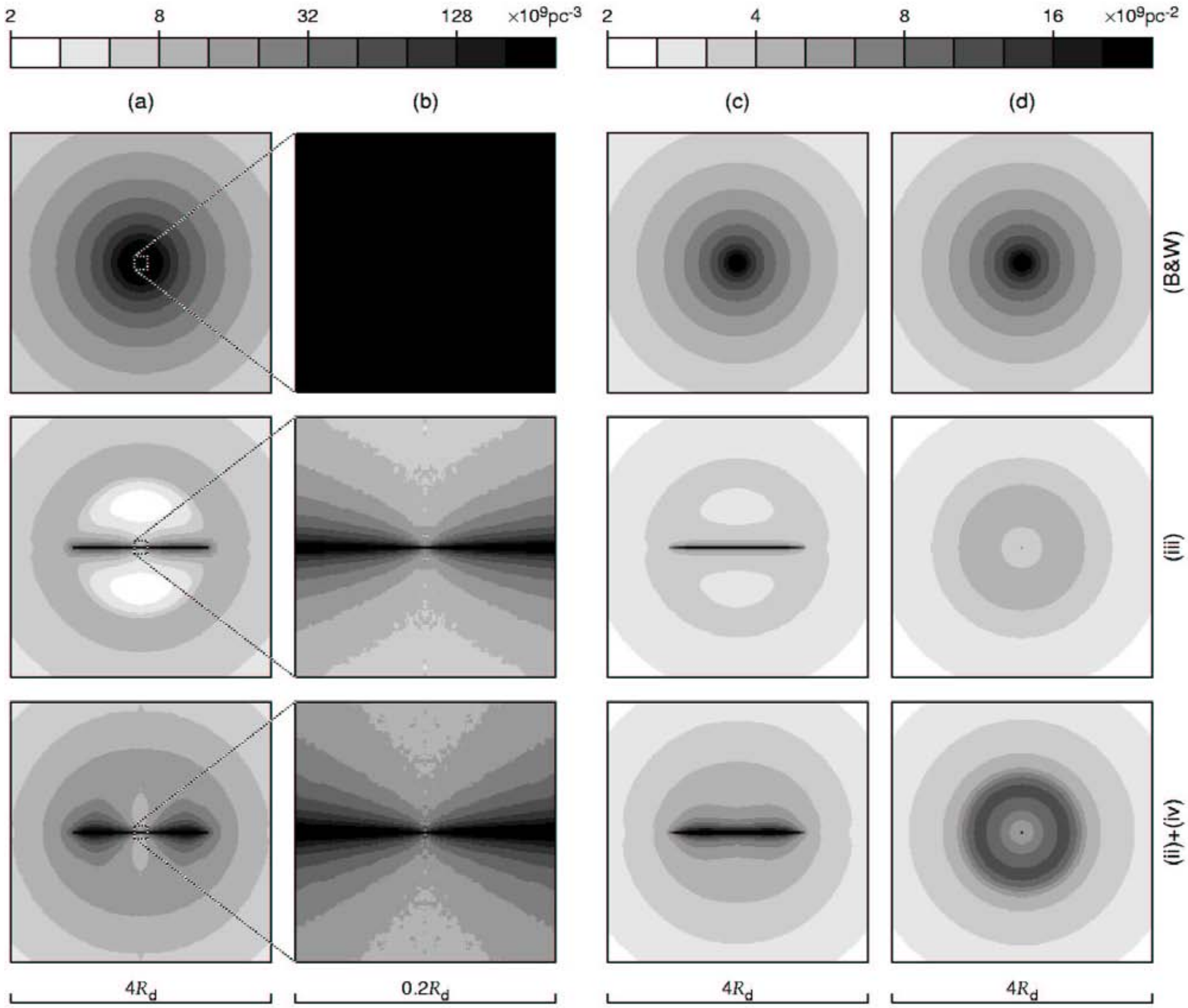

Figure 4: The spatial density $n_*(r)$ and the corresponding projected density of the cluster are shown using logarithmically spaced levels of shading. Columns (a) and (b) represent the meridional section at two different scales, namely, $4R_\mathrm{d}$ and $0.2R_\mathrm{d}$ across (radii are expressed in terms of the disc outer radius, R_d). Next columns are the edge-on (c) and the face-on (d) projections of the cluster. Across columns, the upper row shows the referential cluster [1] ($n_* \propto r^{-7/4}$). In subsequent rows, the system has been already modified via the interaction with two types of discs, case (iii) and case (ii)+(iv), as discussed in ref. [11].

viscosity parameter $\alpha = 0.1$ (so these values are adequate for an AGN). The outer stellar cluster can be characterized by the number density $n_0 = 10^6\mathrm{pc}^{-3}$ and velocity dispersion $\sigma_\mathrm{c} = 200\,\mathrm{km/s}$.

Structure of the cluster modified by the interaction with an accretion disc

Figure 2 shows the density structure of the modified cluster. Majority of stars forming the embedded cluster sink to the centre in the regime of density waves, hence $v_r \propto r^{1/2}$ and $n(a) \propto a^{-1/2}$. In the dragged cluster, the orbital decay leads to the governing index given by $s - 1/2 = -5/4$, and the corresponding number density $n(a) \propto a^{5/4}$. The asymptotic profile $\propto a^{1/4}$ of the outer cluster is determined by the initial distribution. Isotropy of the outer cluster is violated in the inner regions where the mean inclination saturates at ≈ 0.2 (dragged cluster). The influence of the disc manifests itself in different characteristics of the inner cluster. For example, dependence of the drag on size and mass of stars causes gradual segregation of different stellar types present in the cluster.

Figure 3 shows the integrated properties of the cluster that can be compared with observation. We plot the shape of a synthetic spectral line $I(v_z)$, i.e. intensity in the line as a function of line-of-sight velocity v_z near the projected center of the cluster. Local maximum of the line occurs around $v \sim \sin(\xi) v_{\rm K}(R_{\rm d})$. For some values of model parameters, this secondary peak exceeds the central maximum and dominates the predicted profile. High-velocity tails of the line profiles are also noticeably affected in comparison with an unperturbed form of the outer cluster [11].

Oblateness of the cluster

Figure 4 shows various two-dimensional sections of the cluster arranged in four columns. One can clearly observe the impact that star–disc collisions have on the cluster structure, namely, an increasing oblateness of the stellar population in the core and, in some cases, the tendency to form an annulus of stars. The reason for different structures is the continuous crashing of stars on the disc plane. Furthermore, in case of stars embedded in the disc, different modes of star-disc interaction occur and facilitate their radial transport to the central hole.

It is worth noticing that the gravity of the oblate cluster adds with the field of the embedded accretion disc. This way the effect of orbital oscillations, discussed in previous sections, may be further enhanced. Indeed, any non-spherical perturbation, for example general-relativity effects of frame-dragging near a rotating black hole, can potentially be responsible for similar kind of oscillations. However, the issue cannot be resolved within the framework of our simple model, because other effects intervene (for example the fragmentation of the disc into selfgravitating clumps) and these may act in the opposite direction.

3. DISCUSSION

The orbits are aligned and circularized at typical radii of $10^2 \div 10^3 R_{\rm g}$, spiralling further on nearly circular orbits towards the centre. This provides limits on gravitational waves emerging from the cluster. The rate of the capture events can be estimated as

$$\dot{M}_{\rm s} \;\approx\; 10^{-2} M_8^{5/4} \left(\frac{n_0}{10^6\,{\rm pc}^{-3}}\right)^2 \times \left(\frac{M_*}{M_\odot}\right)^2 \left(\frac{R_{\rm d}}{10^4 R_{\rm g}}\right) M_\odot\,{\rm yr}^{-1}\,. \qquad (3)$$

The total accretion rate onto the black hole is a sum of $\dot{M}_{\rm s}$ (which involves massive stars in the cluster), and the accretion rate $\dot{M}$ (gas in the disc).

It is worth noticing that the orbital decay of stars near a black hole is indeed relevant for forthcoming gravitational wave experiments, because the gas-dynamical drag should be taken into account with sufficient accuracy. It was estimated [3, 7] that this effect can be safely ignored at late stages, shortly before the star plunges into the hole, if accretion takes place in the mode of a very diluted flow, but the situation is quite different in case of AGN hosting rather dense nuclear discs. Gas-dynamical effects most likely dominate over the gravitational radiation as far as their influence on the cluster structure is concerned.

The average rate of energy losses which the orbiter experiences via gravitational radiation (per revolution) can be written in terms of orbital parameters [9]:

$$\dot{\mathcal{E}} = \frac{32}{5}\frac{G^4}{c^5}\frac{M^3 M_*^2}{a^5\,(1-e^2)^{7/2}}\; f_1(e). \qquad (4)$$

Linked with this $\dot{\mathcal{E}}$ is the change of semi-major axis

$$\dot{a} = -1.28\times10^{-7} c M_8^{-1}\,\frac{M_*}{M_\odot}\left[\frac{a}{r_{\rm g}}\right]^{-3}\; f_2(e) \qquad (5)$$

and the loss of angular momentum

$$\dot{\mathcal{L}} = \frac{32}{5}\frac{G^{7/2}}{c^5}\frac{M^{5/2} M_*^2}{a^{7/2}\,(1-e^2)^2}\; f_3(e). \qquad (6)$$

Functions f_1, f_2 and f_3 are of the order of unity. The above-given formulae (4)–(6) assume the star is on an elliptic trajectory in Schwarzschild geometry. It therefore provides only an order-of-magnitude estimation, because collective effects operating in the cluster are neglected.

Gravitational radiation orbital decay competes with the orbital decay caused by hydrodynamics of star–disc encounters. The relative importance of these two influences depends on the ratio between the hydrodynamical drag (which is roughly proportional to the disc density) versus gravitational-wave losses (which increase with orbital eccentricity). Discussing this interplay is a tricky task once the gravitationally induced oscillations of eccentricity are taken into account. See refs. [2, 4] for a more detailed discussion of these effects. Substantially different results and timescales can be expected for objects with rather dense discs (such as the case of standard discs in AGNs) and those which are inactive and represent a rather clean system (such as our own Galactic Center).

4. CONCLUSIONS

Although the above-described model is intended mainly for active galactic nuclei with a relatively dense accretion disc or a dusty torus, the problem of stars crashing on a gaseous disc may be relevant also for the centre of our Galaxy. The modified cluster structure is relevant for estimating the rate of black-hole feeding and, vice-versa, for the issue of feedback that a super-massive black hole exhibits on the the host galaxy.

The inherent limitations persist in our present discussion, namely, we have not incorporated a fully self-consistent treatment of the disc gravity. Even though we computed the gravitational field across a sufficiently large domain of space, we did not account for the feedback which star–disc collisions exert on the disc structure. Introducing some kind of a clumpy model of the disc will be very interesting, as it may exert a more substantial effect on the cluster structure by elevating the impact of star–disc collisions. For further details we refer the reader to papers [11] and [12], and to references cited therein.

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