Gravitationally Lensed Oscillations of a Slender Torus

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Using a 3D ray-tracing numerical code we study rapid variability of radiation originating from accreting compact objects. We construct a toy model of an oscillating torus in the slender approximation assuming thermal bremsstrahlung for the intrinsic emissivity of the medium and we compute observed (predicted) radiation signal including contribution of indirect (higher-order) images and caustics in the Schwarzschild spacetime. We show that the simplest oscillation mode in an accretion flow, axisymmetric up-and-down motion at the meridional epicyclic frequency, may be directly observable when it occurs in the inner parts of accretion flow around neutron stars and black holes. Together with the second oscillation mode, an in-and-out motion at the radial epicyclic frequency, it may then be responsible for the high-frequency modulations of the X-ray flux observed at two distinct frequencies (twin HF-QPOs) in microquasars and low-mass X-ray binaries.

1. INTRODUCTION

In many active galactic nuclei as well as in microquasars, cataclysmic variables and low-mass X-ray binaries we observe rapid temporal changes of the flux and of individual spectral features. In the widely accepted scenario, these sources contain a compact object surrounded by an accretion flow in the form of a disc or a torus (see e.g. [9], [7]). The strong gravity near these objects introduces distinctive deviations from Newtonian physics including bending of light rays, gravitational red shift and existence of the inner-most stable circular orbit. All these effects of general relativity affect profiles of observed light curves and have impact on the power spectra.

Especially, the discoveries made by the Rossi X-ray Timing Explorer (RXTE) of high-frequency oscillations are still attracting attention, because their frequencies are close to orbital frequencies near the neutron star surface or the last stable circular orbit around a black hole. The origin of the modulations, known as quasi-periodic oscillations (QPOs) because they are not quite coherent, still remains unknown (see [7] for a review).

We show that gravitational lensing of the photon trajectories in a vicinity of a black hole (in Schwarzschild metric) suffices to appreciably modulate the flux observed at infinity even if the source is symmetric about the axis of a black hole and moves parallel to it. Previous ray-tracing computations in the context of QPO were performed for hot spots in the plane of the accretion disk ([4], [8]). Our computations are performed for an axi-symetric and optically thin source. Specifically, we show that an otherwise steady toroidal source oscillating in an incompressible $m = 0$ mode about the equatorial plane of the black hole gives rise to a periodically modulated flux. A realistic torus will oscillate in a combination of internal [2] and external [3] modes, rather that at a unique frequency, and in the case of two modes we demonstrate that the observed power ratio exhibits strong dependence on the viewing angle.

2. CALCULATION METHOD

In order to compute the amount of radiation coming from the source we have developed a new three-dimensional ray-tracing code which integrates geodesic and geodesic deviation equations in a given stationary spacetime. Photon trajectories are integrated backward in time from the observer positioned at infinity at some inclination angle i with respect to the z-axis. At certain points along the trajectory the current position, momentum, time delay and magnification are recorded. This information is then used to reconstruct each photon's path and calculate the total amount of incoming flux.

The intensity observed at infinity is an integration of the emissivity f over the path length along geodesics and it can be written down as

$$I_{\rm obs}(t) = \int f(r,\, \theta,\, \phi,\, t{-}\Delta t)\sqrt{-g_{tt}}\, k^t\, g^4\, \mathrm{d}\lambda\ .$$

The integration goes along the light ray parametrized by an affine parameter λ. Here, k^t is the time component of photon's 4-momentum, g is the red-shift factor and Δt is the photon time delay.

3. A SLENDER TORUS

We consider an isolated, luminous, optically thin and geometrically slender torus with a constant angular momentum around a non-rotating black hole of mass M. Such a torus orbits a point mass at a radius that is much larger than its width and these systems have already been studied in various forms ([2], [6]). In this problem, all radii scale with M, and all frequencies scale as $1/M$. For a convenient comparison with the observed frequencies we chose $M = 10\, M_\odot$.

The torus in the slender approximation is assumed to be circular in cross-section with radius $R_0 = 2.0\, M$, which is held constant in time, as it is the intrinsic emissivity per unit co-moving volume of the torus.

The circle of maximum pressure is in the equilibrium position in the equatorial plane ($\tilde{z} = 0$) in distance $\tilde{r}_0 = 10.8\,M$ from the source of gravity, where $\tilde{r}$, $\tilde{z}$ are cylindrical coordinates related to the Schwarzschild coordinates through $\tilde{r} = r\,\sin\theta$, $\tilde{z} = r\,\cos\theta$.

The observed luminosity variations will depend on the properties of the torus. We consider it to be made of an optically thin polytropic medium and we take the emissivity in the local frame to be

$$\begin{aligned} f &\propto \rho^2 T^{\frac{1}{2}} \,, \\ \text{with} \quad T &= K\,\rho^{\gamma-1}\mu m_{\rm u}/k_{\rm B} \,, \end{aligned}$$

where $\gamma = \frac{5}{3}$, $\mu = \frac{7}{4}$, $m_{\rm u}$ and $k_{\rm B}$ are polytropic index, molecular weight, atomic mass unit and the Boltzmann constant, respectively.

In the slender approximation, we construct the density profile by taking the equipotential structure obtained by Taylor expanding in the $\tilde{z}$ direction equilibrium solutions of the relativistic Euler equation [1] of a torus with uniform angular momentum $\ell(\tilde{r}) = \ell_{\rm K}(\tilde{r}_0) = \sqrt{M\,\tilde{r}_0^3}/(\tilde{r}_0 - 2M)$, so that the potential W has the form

$$W(R) = \frac{R_0^2 - R^2}{2\,\tilde{r}_0^2(\tilde{r}_0 - 3M)} + W_0 \,,$$

and the density profile is

$$\rho(R) = \left[\frac{\gamma-1}{K\gamma}\left({\rm e}^{W(R)-W_0} - 1\right)\right]^{1/\gamma-1} \,.$$

4. TORUS OSCILLATIONS

We allow the torus to execute vertical oscillations parallel to its axis as well as the radial oscillations in the perpendicular direction, so that the position of the central circle vary as

$$\begin{aligned} \tilde{r}(t) &= \tilde{r}_0 + \delta\tilde{r}_0\,\sin(\omega_r t) \,, \\ \tilde{z}(t) &= \delta\tilde{z}_0\,\sin(\omega_\theta t) \,. \end{aligned}$$

Here, $\omega_\theta = \Omega_{\rm K} = \sqrt{M/r^3}$ is the vertical epicyclic frequency, in Schwarzschild geometry equal to the Keplerian orbital frequency, and with our choice of $\tilde{r}_0 = 10.8M$ the radial epicyclic frequency is $\omega_r = \frac{2}{3}\,\omega_\theta$. The amplitude of both vertical and radial motion is set to $\delta\tilde{z}_0 = \delta\tilde{r}_0 = 0.1M$.

The radial motion results in a periodic change of volume of the torus. Because the optically thin torus is assumed to be filled with a polytropic gas radiating by bremsstrahlung cooling, there is a corresponding change of luminosity, with a clear periodicity at $2\pi/\omega_r$.

On the contrary, the vertical motion does not change the properties of the torus or its overall luminosity. We find that in spite of this, and although the torus is perfectly axisymetric, the flux observed at infinity clearly varies at the oscillation frequency ω_θ. This is caused by relativistic effects at the source (lensing, beaming and time delay), and no other cause need to be invoked to explain in principle the highest-frequency modulation of X-rays in luminous black-hole binary sources.

The resulting power spectrum of the torus is shown in Fig. 1. Two distinct periodic components are clearly seen at the two oscillation frequencies ω_r and ω_θ. Their relative power depends strongly on the inclination angle (Figs. 1, 2).

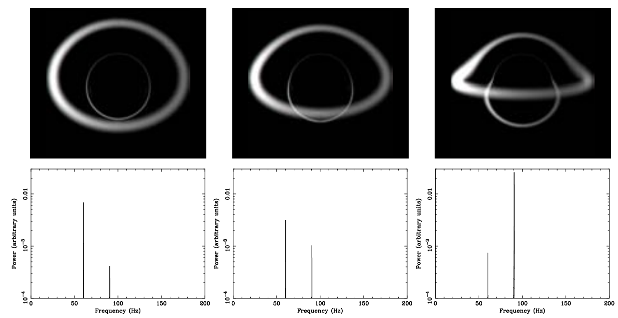

Figure 1: Results of a numerical ray-tracing. Instant snapshots and power spectra of an oscillating torus as viewed by a distant observer for inclinations 45° (*left*), 65° (*middle*) and 85° (*right*).

Since the intrinsic luminosity varies only at the lower of the two frequencies, the lower peak is dominant at low inclinations (on-axis view). At higher inclinations both lensing and beaming are strong and the peak associted with ω_θ gains importance. The correlation with the phase angle of oscillations (Fig. 2 *right*) is extremely small.

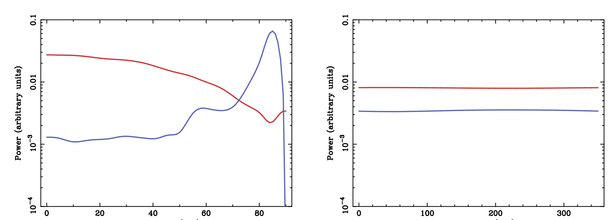

Figure 2: Power of radial (*red line*) and vertical (*blue line*) oscillations as a function of the viewing angle (*left*) and their relative phase (*right*) both for inclination 65°.

5. CONCLUSIONS

We have shown that gravitational lensing at the source will modulate the flux received from an axially symmetric emitter oscillating about the equatorial plane of a black hole. In a torus executing simultaneous oscillations in the radial and vertical directions at frequencies ω_r and $\omega_\theta = (3/2)\omega_r$, as expected in the parametric resonance model [5], both frequencies will show up in the power spectrum, with no other (e.g. harmonic) strong components. The lower of the frequencies may reflect changes in the emissivity of the torus, but the presence of the upper frequency is explained by effects of relativity alone.

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