Hydrodynamical Scaling Laws for Astrophysical Jets

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The idea of a unified model for all astrophysical jets has been considered for some time now. We present here some hydrodynamical scaling laws relevant for all type of astrophysical jets, analogous to those of Sams et al. [1996]. We use Buckingham's Π theorem of dimensional analysis to obtain a family of dimensional relations among the physical quantities associated with the jets.

1. Introduction

Although the first report of an astrophysical jet was made by Curtis [1918], these objects were extensively studied much later with radio astronomy techniques [Reber 1940]. Quasars, and radiogalaxies were discovered and later gathered in a unified model which proposed a dusty torus around the nucleus of the source [Antonucci and Miller 1985]. Years later, some galactic sources showed similar features to the ones presented by quasars and radiogalaxies, i.e. relativistic fluxes, a central engine, symmetrical collimated jets, radiating lobes, and apparent superluminal motions [Sunyaev et al. 1991]. Optical and X-ray observations showed other similar non–relativistic sources in the galaxy associated to H-H objects [Gouveia Dal Pino 2004]. Lately the strong explosions found in long Gamma Ray Bursts, had been modelled as collapsars, in which a jet is associated to the observed phenomena, in order to explain the observations [Kulkarni et al. 1999, Castro-Tirado et al. 1999].

The similarities between all astrophysical jets, mainly those between quasars and micro–quasars, and the scaling laws for black holes proposed by Sams et al. [1996] and Rees [1998] made us search for the possible existence of some hydrodynamical scaling laws for astrophysical jets.

The present work presents a few mathematical relations that naturally appear as a consequence of dimensional analysis and Buckingham's Π theorem. We begin by considering some of the most natural physical dimensional quantities that have to be included in order to describe some of the physical phenomena related to all classes of jets. With this and the use of dimensional analysis we then calculate the dimensional relations associated to these quantities. Finally, we briefly discuss these relations and their physical relevance to astrophysical jets.

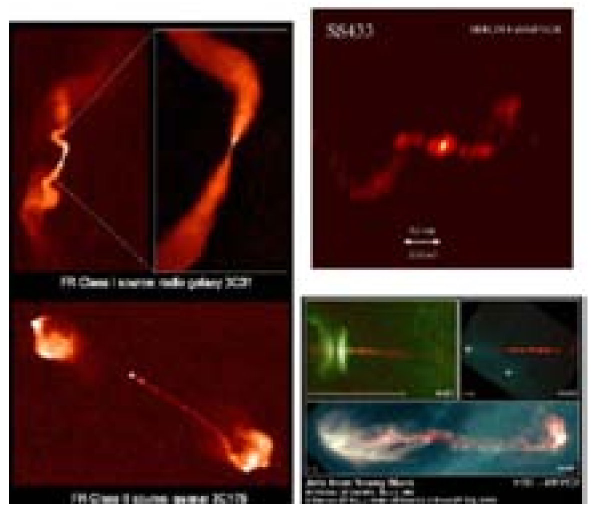

Figure 1: Astrophysical jets are very common and exist in many different sizes. On the left, extending $\sim 10^5$pc, FR I and FR 2 sources are shown. The upper right panel shows the micro–quasar SS 433. It presents relativistic fluxes and apparent superluminal motions analogous to those in quasars. The lower right panel shows jets associated to Herbig–Haro objects with lengths $\sim 10^{-1} - 10\,$pc. All jets have a condensed (sometimes compact) object accreting matter from their surroundings. There is also an accretion disc around the central condensed object and a pair of symmetrical collimated jets that end up in radiating lobes. The images were taken from Bridle [1998], Paragi et al. [2001] and and hubblesite.org.

2. Analysis

A complete description for the formation of an astrophysical jet is certainly complicated. However, there are some essential physical ingredients that must enter into the description of the problem. To begin with, the mass M of the central object must accrete material from its surroundings at an accretion rate $\dot{M}$. Now, because gravity and magnetic fields B are necessary in order to generate jets, Newton's constant of gravity G and the velocity of light c must be taken into account. If in addition there is some characteristic length l (e.g. the jet's length), a characteristic density ρ (e.g. the density of the surrounding medium) and a characteristic velocity v (e.g. the jet's ejection velocity), then the jet's luminosity (or power) L is a function related to all these quantities in the following manner

$$L = L(\dot{M},\, M,\, c,\, G,\, B,\, l,\, v,\, \rho). \qquad (1)$$

Using Buckingham's Π theorem of dimensional analysis [Buckingham 1914, Sedov 1993] the following non–trivial dimensionless parameters are found

$$\Pi_1 = \frac{L}{\dot{M}c^2}, \quad \Pi_2 = \frac{G\dot{M}}{c^3}, \quad \Pi_3 = \frac{Bc^{1/2}M}{\dot{M}^{3/2}}, \qquad (2)$$
$$\Pi_4 = \frac{l\dot{M}}{Mc}, \quad \Pi_5 = \frac{\rho\, c^3 M^2}{\dot{M}^3}.$$

From the parameter Π_2 it follows that

$$\Pi_2 = \left(\frac{GM}{c^2}\right)\left(\frac{\dot{M}}{M}\right)\frac{1}{c}.$$

Since the quantity

$$\tau \equiv \frac{M}{\dot{M}}, \qquad (4)$$

defines a characteristic time in which the central object doubles its mass, then using equation (4) we can write Π_2 as

$$\Pi_2 = \frac{r_{\mathrm{s}}}{2\tau c}, \qquad (5)$$

where r_{S} is the Schwarzschild radius. This relation naturally defines a length

$$\lambda \sim c\tau, \qquad (6)$$

which can be thought of as the maximum possible length a jet could have, since τ is roughly an upper limit to the lifetime of the source.

In what follows we will use the following typical values

$$M \approx 10^{8-9}\,\mathrm{M}_\odot,\ B \approx 100\,\mathrm{G},\ \dot{M} \approx 1\mathrm{M}_\odot\,\mathrm{yr}^{-1},$$
$$L \approx 10^{7-10} L_\odot,\ r_{\mathrm{j}} \approx 10^{4-5}\,\mathrm{pc}, \qquad (7)$$

and

$$M \approx 10^{0-1}\mathrm{M}_\odot,\ B \sim 100\,\mathrm{G},\ \dot{M} \approx 10^{-(8-6)}\mathrm{M}_\odot\mathrm{yr}^{-1},$$
$$L \approx 10^{2-4}\,L_\odot,\ r_{\mathrm{j}} \approx 10^{0-1}\mathrm{pc}, \qquad (8)$$

for quasars and μ–quasars respectively [Meier 2002, Trimble and Aschwanden 2004, Blandford 1990, Carilli et al. 1996, Lovelace and Romanova 1996, Robson 1996, Reipurth et al. 1997, Ferrari 1998, Koide et al. 1998, Camenzind 1999, Ford and Tsvetanov 1999, Vilhu 2002, Wu et al. 2002, Cherepashchuk et al. 2003, Smirnov et al. 2003, Calvet et al. 2004, Mendoza et al. 2004, Mirabel 2004].

¿From equation (2) it is found that

$$\Pi_6 := \Pi_2^{3/2}\Pi_3 = \left(\frac{GM}{c^2}\right)^{3/2} \Bigg/ \ \frac{\sqrt{Mc^2}}{B}. \qquad (9)$$

This relation defines a length r_{j} given by

$$r_{\mathrm{j}} \propto \frac{M^{1/3}c^{2/3}}{B^{2/3}} \approx 10^2 \left(\frac{M}{\mathrm{M}_\odot}\right)^{1/3} \left(\frac{B}{1\mathrm{G}}\right)^{-2/3} \mathrm{pc}. \quad (10)$$

For typical extragalactic radio sources and μ–quasars it follows from equations (7) and (8) that $r_{\mathrm{j}} \propto 10^4\,\mathrm{pc}$ and $r_{\mathrm{j}} \propto 10\,\mathrm{pc}$ respectively. These lengths are fairly similar to the associated length of these jets. In other words, if we identify the length r_{j} as the length of the jet, then a constant of proportionality ~ 1 is needed in equation (10), and so

$$r_{\mathrm{j}} \approx 100 \left(\frac{M}{\mathrm{M}_\odot}\right)^{1/3} \left(\frac{B}{1\mathrm{G}}\right)^{-2/3} \mathrm{pc}. \qquad (11)$$

Since equation (9) is roughly the ratio of the Schwarzschild radius r_{S} to the jet's length r_{j}, then $\Pi_6 << 1$, i.e.

$$\Pi_6 = \frac{\left(Bl^{3/2}\right)\left(GM^2/l\right)^{3/2}}{(Mc^2)^2} << 1, \qquad (12)$$

which in turn implies that

$$B << \frac{c^4}{G^{3/2}M} \approx 10^{23}\left(M/\mathrm{M}_\odot\right)^{-1}\,\mathrm{G}. \qquad (13)$$

The right hand side of this inequality is the maximum upper limit for the magnetic field associated to the accretion disc around the central object. For "extreme" micro–quasars like SS 433 and GRB's the magnetic field $B \gtrsim 10^{16}\,\mathrm{G}$, so that this upper limit works better for those objects [Meier 2002, Trimble and Aschwanden 2004].

From equation (2) it follows that

$$\Pi_7 \equiv \frac{\Pi_1}{\Pi_2\Pi_3^2} = \frac{L\dot{M}}{B^2M^2G},$$

and so

$$L \propto 10^{-7}\left(\frac{B}{1\,\mathrm{G}}\right)^2 \left(\frac{M}{\mathrm{M}_\odot}\right)^2 \left(\frac{\dot{M}}{\mathrm{M}_\odot yr^{-1}}\right)^{-1} L_\odot. \quad (15)$$

For the case of quasars and μ–quasars, using the typical values of equations (7) and (8) it follows that the power $L \propto 10^{15}\, L_{\odot}$ and $L \propto 10^{8}\, L_{\odot}$ respectively. In order to normalise it to the observed values, we can set a constant of proportionality $\sim 10^{-6}$ in equation (15). With this, the jet power relation is given by

$$L \approx 10^{-13} \left(\frac{B}{1\,\mathrm{G}} \right)^{2} \left(\frac{M}{\mathrm{M}_{\odot}} \right)^{2} \left(\frac{\dot{M}}{\mathrm{M}_{\odot} yr^{-1}} \right)^{-1} L_{\odot}. \tag{16}$$

3. Conclusion

Astrophysical jets exist due to a precise combination of electromagnetic, mechanic and gravitational processes, independently of the nature and mass of their central objects.

Here we report the dimensional relation between a few important parameters that enter into the description of the formation of an astrophysical jet.

Of all our results, it is striking the fact that the jet power is inversely proportional to the accretion rate associated with it. This is probably due to the following. For a fixed value of the mass of the central object (in any case, for the time that accretion takes place, the mass of the central object does not increase too much) when the accretion mass rate increases, then the magnetic field lines anchored to the plasma tend to pack up, meaning that the field intensity increases in such a way as to get the correct result given by equation (16).

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