

Cosmological Constraints from 2D SZ Catalogs: Number Counts and the Angular Correlation Function

S. Mei

Johns Hopkins University, 3400 N. Charles Street, 21218 Baltimore, MD, USA

J.G. Bartlett

APC, 11, place Marcelin Berthelot, F-75231 Paris Cedex 05, France

UMR 7164 (CNRS, Université Paris 7, CEA, Observatoire de Paris)

Using a Fisher matrix analysis we quantify the cosmological constraints attainable with the counts and the angular correlation function from a 2-dimensional Sunyaev-Zel'dovich (SZ) cluster catalog. Three kinds of SZ survey are considered: the almost all-sky Planck survey and two deeper ground-based surveys, one with 10% sky coverage, the other one with a coverage of 250 square degrees. With the counts and angular function, and adding the constraint from the local X-ray cluster temperature function, joint 10% to 30% errors (1σ) are achievable on the cosmological parameter pair (σ_8, Ω_M) in the flat concordance model. Constraints from a 2D distribution remain relatively robust to uncertainties in possible cluster gas evolution for the case of Planck; alternatively, we examine constraints on cluster gas physics when assuming priors on the cosmological parameters (e.g., from cosmic microwave background anisotropies and SNIa data), finding a poor ability to constrain gas evolution with the 2-dimensional catalog. From just the SZ counts and angular correlation function we obtain, however, a constraint on the product between the present-day cluster gas mass fraction and the normalization of the mass-temperature relation, T_* , with a precision of 15%. This is particularly interesting because it would be based on a very large catalog and is independent of any X-ray data.

1. Introduction

In the coming years, surveys of galaxy clusters observed with the Sunyaev-Zel'dovich (SZ) effect (Sunyaev & Zel'dovich 1970, 1972; Birkinshaw 1999; Carlstrom et al. 2002) will open a new observational window onto large-scale structure formation and evolution (Barbosa et al. 1996; Eke et al. 1996; Colafrancesco et al. 1997; Diego et al 2002; Haiman et al. 2001; Holder et al. 2001; Kneissl et al. 2001; Weller et al. 2002; Benson et al. 2002). The advantages offered by this window, compared to either the X-ray or the optical, are intrinsic to the SZ effect (Bartlett 2000). They include the ability to detect clusters at high redshift, due to the lack of surface brightness dimming in the SZ, and a “clean” selection on cluster gas thermal energy, a robust quantity expected to have a tight relationship to cluster mass. These properties are particularly advantageous for evolutionary studies because they permit the selection of similar mass clusters over a large range of redshifts. The evolution of cluster abundance with redshift, for example, is sensitive to the cosmological parameters σ_8 and Ω_M , and also to Ω_Λ and the dark energy equation-of-state (Oukbir & Blanchard 1997; Barbosa et al. 1996; Haiman et al. 2001).

This scientific potential is currently motivating a number of observational efforts aimed at realizing SZ surveys with dedicated, optimized interferometers

(AMI¹, AMiBA², SZA³), and large-format bolometer arrays (APEX⁴, ACT⁵, BOLOCAM⁶, ACBAR⁷, SPT⁸). The Planck⁹ satellite, to be launched in 2007, will provide a full-sky catalog of galaxy clusters detected by their SZ signal, one of the largest galaxy cluster catalogs ever constructed, and in the more distant future one may look forward to an even larger catalog from a fourth generation CMB mission, such as the *Inflation Probe* proposed by NASA in the context of the Beyond Einstein Program¹⁰ or a similar mission under study by ESA in the Cosmic Vision programme¹¹.

Follow-up in other wavebands of a SZ catalog is obviously essential for many scientific goals, for instance constraining cosmology and cluster evolution with the redshift distribution and X-ray properties (e.g., Holder et al. 2001; Bartelmann & White 2002; Diego et al. 2002; Weller et al. 2002; Hu 2003; Majumdar & Mohr 2003; Majumdar & Mohr 2004). Ex-

¹<http://www.mrao.cam.ac.uk/telescopes/ami/index.html>

²<http://www.asiaa.sinica.edu.tw/amiba>

³<http://astro.uchicago.edu/sze>

⁴<http://bolo.berkeley.edu/apexsz>

⁵<http://www.hep.upenn.edu/~angelica/act/act.html>

⁶http://astro.caltech.edu/~lbg/bolocam_front.htm

⁷<http://cosmology.berkeley.edu/group/swlh/acbar/>

⁸<http://astro.uchicago.edu/spt/>

⁹<http://astro.estec.esa.nl/Planck/>

¹⁰<http://universe.nasa.gov/program/probes.html>

¹¹http://www.esa.int/esaSC/SEMA7J2IU7E_index_0.html

Table I The surveys that have been considered in this analysis.

Survey	Y limit (arcmin^2)	Coverage (sq.deg.)	Average redshift	Expected number of clusters (for $\Omega_M = 0.27$, $\Omega_\Lambda = 0.73$, $h = 0.72$, $\sigma_8 = 0.84$)
Planck	3×10^{-4}	40000	0.3	36000
SPT	5×10^{-5}	4000	0.6	33000
APEX	2.5×10^{-5}	250	0.7	5000

tensive follow-up will be limited to only small subsets of the larger SZ catalogs. In particular, the follow-up of the Planck all-sky catalog will represent a significant effort. It is therefore interesting to ask the question, what science can be done with a two-dimensional SZ catalog – what we refer to as the SZ *photometric catalog*.

Mei & Bartlett (2003) studied the counts and the angular correlation function of SZ clusters to see how these two statistics could be combined to extract cosmological information before any subsequent follow-up. The angular function has been extensively studied by Diaferio et al. (2003), while three dimensional clustering issues are elaborated by Moscardini et al. (2002). Specifically, we explored how joint measurements of the counts and angular function could be used to constrain the cosmological parameters σ_8 and Ω_M , when the normalization of the Mass-Temperature relation for clusters is known. This work focused on the influence of various cosmological parameters and cluster gas physics on both the counts and the angular function. In previous work, Fan & Chiueh (2001) examined constraints in the σ_8 - Ω_M plane obtained by combining a SZ catalog with limited redshift information (e.g., only two redshift bins) and the local abundance of X-ray clusters.

Mei & Bartlett (2004) extended this work by quantifying the attainable constraints with a standard Fisher analysis, working in the context of the so-called concordance model ($\Omega_M = 0.27$, $\Omega_\Lambda = 0.73$, $h = 0.72$; e.g., Spergel et al. 2003). In this paper we discuss our results from this analysis.

2. SZ cluster physics

The total SZ flux from a cluster (relative to the mean sky brightness, i.e., the unperturbed cosmic microwave background [CMB]) is measured by the integrated Compton y -parameter, which may be expressed in terms of cluster quantities as

$$Y(M, z) = \frac{k\sigma_T}{mc^2} \frac{N_e T}{D_{\text{ang}}^2(z)} \propto \frac{f_{\text{gas}}(M, z) T(M, z) M}{D_{\text{ang}}^2(z)} \quad (1)$$

where k is the Boltzmann constant, m is the electron mass, and N_e is the total number of electrons in

the cluster. In this expression, $f_{\text{gas}}(M, z)$ is the cluster gas mass fraction, $T(M, z)$ is the mean *particle weighted* gas temperature, M is the total virial mass and $D_{\text{ang}}(z)$ is the angular diameter distance in a homogeneous background. The gas mass fraction and temperature are in general functions of cluster mass and redshift.

Scaling arguments lead one to expect

$$T(M, z) = T_* (M_{15} h)^{2/3} [\Delta(z) E(z)^2]^{1/3} \left[1 - 2 \frac{\Omega_\Lambda(z)}{\Delta(z)} \right] \quad (2)$$

where T_* is a normalization constant (expressed in keV), M_{15} is the cluster total mass in units of $10^{15} M_\odot$, $\Delta(z)$ is the non-linear density contrast on virialization (≈ 178) and $h \equiv H_0 / 100 \text{ km/s/Mpc}$. The quantity $E^2(z) = [\Omega_\Lambda + (1 - \Omega_M - \Omega_\Lambda)(1 + z)^2 + \Omega_M(1 + z)^3]$ (the dimensionless Hubble parameter) with the definitions $\Omega_M(z) \equiv \Omega_M(1 + z)^3/E^2(z)$, $\Omega_\Lambda(z) \equiv \Omega_\Lambda/E^2(z)$; notice that Ω_M and Ω_Λ written without an explicit redshift dependence will indicate present-day values ($z = 0$). The gas mass fraction $f_{\text{gas}}(M, z)$ is, on the other hand, constant in the simplest self-similar model, independent of cluster mass and redshift (e.g., Arnaud et al. 2002).

Putting all this together, we express the relation between cluster SZ flux and mass and redshift as

$$Y(M, z) = Y_{15}(z) M_{15}^{5/3+\alpha} (1 + z)^\gamma \quad (3)$$

where $Y_{15}(z)$ incorporates the various constants and redshift dependence of the self-similar model. The exponents α and γ describe any deviations from pure self-similarity, in other words gas evolution, such that the self-similar model is defined by $\alpha = \gamma = 0$.

The explicit expression for $Y_{15}(z)$ is

$$\begin{aligned} Y_{15}(z) &= (7.4 \times 10^{-5} h^{7/6} \text{ arcmin}^2) \left(\frac{T_*}{\text{keV}} \right) \left(\frac{f_{\text{gas}}}{0.07 h^{-3/2}} \right) \times \\ &\quad \left(\frac{\Delta(z) E(z)^2}{178} \right)^{1/3} \left[1 - 2 \frac{\Omega_\Lambda(z)}{\Delta(z)} \right] \frac{1}{d_{\text{ang}}^2(z)} \\ &\equiv Y_* \left(\frac{\Delta(z) E(z)^2}{178} \right)^{1/3} \left[1 - 2 \frac{\Omega_\Lambda(z)}{\Delta(z)} \right] \frac{1}{d_{\text{ang}}^2(z)} \\ &\equiv (1.06 \times 10^{-3} h^{8/3} \text{ arcmin}^2) Y'_* \times \\ &\quad \left(\frac{\Delta(z) E(z)^2}{178} \right)^{1/3} \left[1 - 2 \frac{\Omega_\Lambda(z)}{\Delta(z)} \right] \frac{1}{d_{\text{ang}}^2(z)} \end{aligned} \quad (4)$$

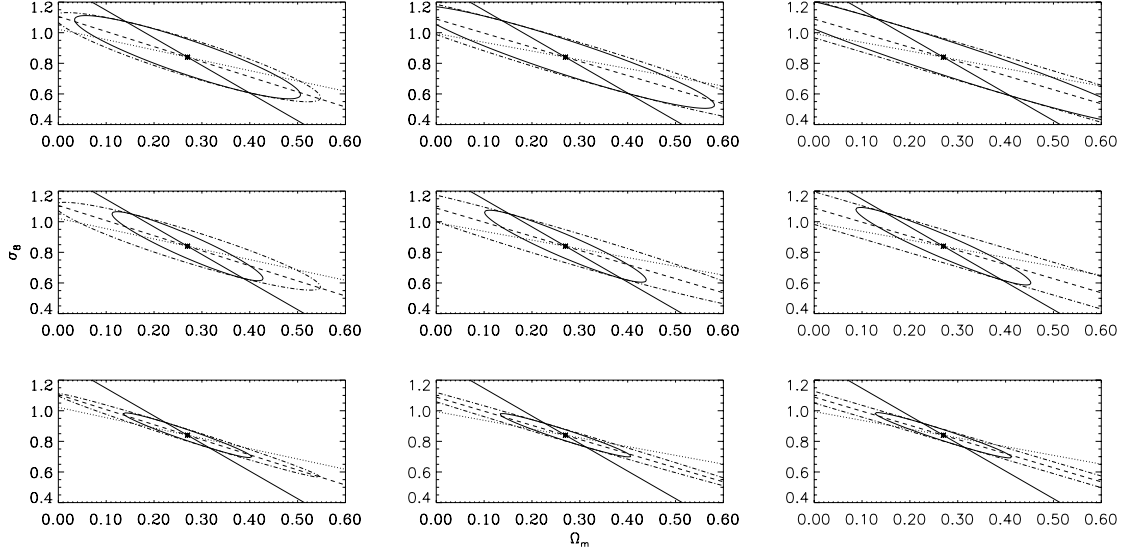


Figure 1: Constraints on σ_8 and Ω_M from a joint analysis of the counts, the angular function and the local X-ray temperature function (constraint from Pierpaoli et al. 2003) are shown at one σ (continuous ellipse). The constraints from the joint counts and angular correlation function are shown by the dashed-dotted ellipse. The dashed line represents the degeneracy line from the singular value decomposition of the counts Fisher matrix. The dotted line represents the degeneracy line from the singular value decomposition of the angular correlation function Fisher matrix. The continuous line crossing the contours represents the degeneracy line from the singular value decomposition of the Fisher matrix for the constraints from the local X-ray temperature function. In the top panel, priors of 30% on T_* and 50% on f_{gas} are assumed; in the middle panel, priors of 10% on T_* and 50% on f_{gas} , and in the bottom panel, priors of 10% on T_* and 15% on f_{gas} are assumed. From left to right, we show the constraints for Planck-like, SPT-like and APEX-like surveys. These constraints are summarized in Table II.

Table II One sigma constraints on σ_8 and Ω_M from a joint analysis of the counts, the angular function and the local X-ray temperature function. For each survey, the prior uncertainties on T_* and f_{gas} , and the expected final constraints on σ_8 , Ω_M , and f_{gas} are given. From the joint analysis we derive constraints σ_8 , Ω_M , but also gain precision on f_{gas} . This table summarizes the results of Figure 1.

Survey	T_* Prior Unc. (%)	f_{gas} Prior Unc. (%)	σ_{σ_8} (%)	σ_{Ω_M} (%)	$\sigma_{f_{\text{gas}}}$ (%)
Planck	30	50	20	60	30
	10	50	20	40	30
	10	15	10	30	10
SPT	30	50	30	80	35
	10	50	20	40	30
	10	15	10	30	10
APEX	30	50	35	100	35
	10	50	20	45	35
	10	15	10	35	10

where $D_{\text{ang}} \equiv H_0^{-1} d_{\text{ang}}$, and we use $Y'_* \equiv f_{\text{gas}} T_*$ to indicate our normalization of this relation.

In our Fisher analysis we consider Y'_* , σ_8 and Ω_M , and adapt a fiducial model with $Y'_* = 0.17$ keV – for example, $f_{\text{gas}} = 0.07 h^{-1.5}$ (Mohr et al. 1999, see also Grego et al. 2002) and $T_* = 1.5$ keV (Pierpaoli et al. 2003) – and a flat cosmological model with $\sigma_8 = 0.84$ and $\Omega_M = 0.27$ (Spergel et al. 2003).

3. Constraints on cosmological parameters

We study two situations to illustrate the use of a SZ photometric catalog: constraints on the cosmological parameter pair (σ_8, Ω_M) in the presence of possible cluster gas evolution, and constraints on cluster gas physics assuming strong cosmological priors (e.g., from cosmic microwave background anisotropies and

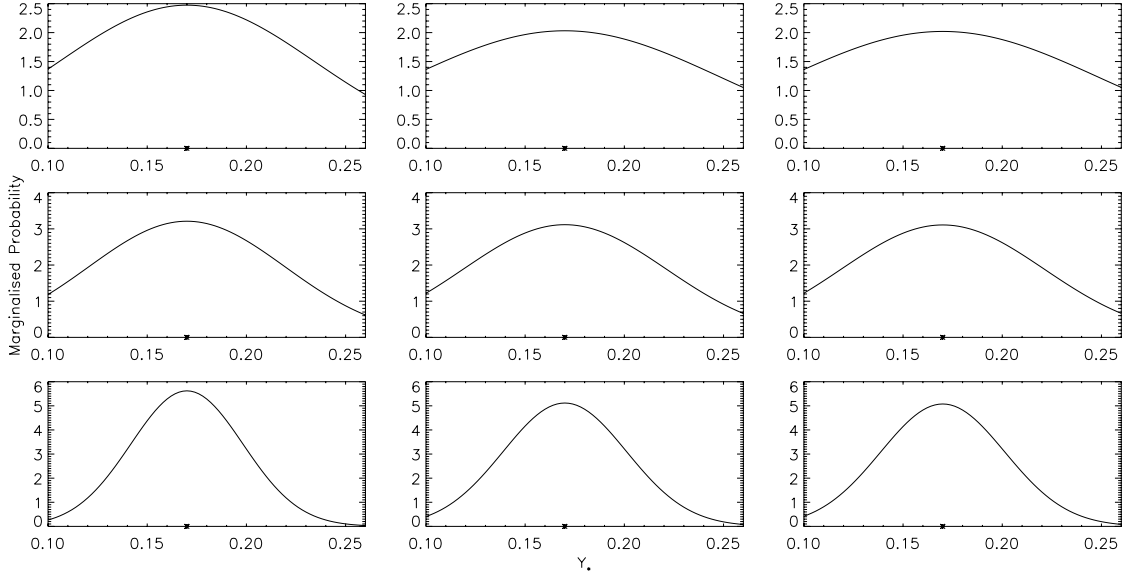


Figure 2: Constraints on Y'_* from a joint analysis of the counts and the angular function when the cosmological parameters are known. In the top panel, priors are taken as 10% on σ_8 and Ω_M , and ± 1 for α and γ . In the center panel, the prior on α and γ are dropped to ± 0.1 . In the bottom panel, priors are 5% on σ_8 and Ω_M , and ± 0.1 for α and γ . From left to right, we show the constraints for Planck-like, SPT-like and APEX-like surveys. These constraints are summarized in Table III.

Table III One sigma constraints on Y'_* from a joint analysis of the counts and the angular function when the cosmological parameters and T_* are known. This table corresponds to Figure 2.

Survey	α and γ Prior Unc.	σ_8 and Ω_M Prior Unc. (%)	$\sigma_{Y'_*}$
Planck	1	10	0.5
	0.1	10	0.3
	0.1	5	0.16
SPT	1	10	0.6
	0.1	10	0.3
	0.1	5	0.18
APEX	1	10	0.6
	0.1	10	0.3
	0.1	5	0.18

SN Ia distance measurements). We furthermore examine the gain obtained by incorporating the constraint from the local X-ray cluster temperature distribution function. Our results are shown in Fig. 1 and Fig. 2 and summarized in Tables II and III.

Our present study takes as examples the almost full-sky SZ catalog expected from Planck, (Aghanim et al. 1997; Bartelmann 2001; Diego et al. 2002; and references therein), and two deeper ground-based experiments, one covering 4000 square degrees (e.g., Haiman et al. 2001; Holder et al. 2001; Majumdar & Mohr 2004), representative of the South Pole Telescope (SPT) survey, and one covering 250 square degrees, representative of the APEX survey. The characteristics of these surveys are summarized in Table 1 with flux limits quoted at a signal-to-noise of better

than three.

The local abundance of X-ray clusters, as measured by the present-day X-ray temperature distribution function, adds additional information that can be usefully combined with the SZ counts and angular function. With prior information on T_* , all three parameters (σ_8, Ω_M, Y'_*) may be constrained, which also yields a constraint on the cluster gas mass fraction f_{gas} . This determination of f_{gas} would be truly representative of the cluster population, as it is an average over a potentially very large number of objects.

Constraints on the order of 10% to 30% (around the concordance values) are obtained on the cosmological parameters (σ_8, Ω_M) with both an all-sky survey to $Y \sim 10^{-4}$ arcmin² and a deep ground-based survey to $Y \sim 10^{-5}$ arcmin² (see Figure 1 and Table II). To

achieve these results, one must have external information equivalent to a 10% prior on the value of the normalization of the $T - M$ relation (T_*).

If the cosmological parameters are known, we are able, by constraining the normalization of the $Y(M, z)$ relation, to constrain the present day ($z = 0$) gas mass fraction f_{gas} to about 20% (with a prior of 10% on the normalisation of the mass/temperature relation T_*); or, vice versa, the normalization of the mass/temperature relation T_* to about 20% (with a prior of 10% on f_{gas}); this is shown in Fig 2 and Table III. This would represent a measurement over a very large number of clusters.

4. Discussion & conclusion

Our general results are not greatly affected by non-standard (i.e., non self-similar) gas evolution, in particular in the case of Planck. The corollary is that we are unable to turn the argument around in the sense that even if the cosmological parameters are taken as fixed, very little restriction is placed on gas evolution.

In conclusion, an angular SZ catalog in which both the counts and angular correlation function are measured can provide useful cosmological constraints, permitting an immediate return on a SZ survey before subsequent follow-up observations.

Acknowledgments

S. Mei thanks Matthias Bartelmann, Frank Bertoldi and Saleem Zaroubi for useful discussions, and acknowledges support from the European Space Agency External Fellowship program. Some of this work was performed at the Lawrence Berkeley National Laboratory and at the University of California, Berkeley, thanks to funding from the France Berkeley Fund.

References

Aghanim, N., de Luca, A., Bouchet, F.R. *et al.* 1997, A&A, 325, 9
 Arnaud, M., Aghanim, N., Neumann, D. M. 2002, 389, 1
 Barbosa D., Bartlett J.G., Blanchard A. *et al.*, 1996, A&A, 314, 13
 Bartelmann, M. 2001, A&A, 370, 754
 Bartelmann, M. & White, S.D.M. 2002, A&A, 388, 732
 Bartlett J.G. 2000, astro-ph/0001267
 Benson, A.J., Reichardt, C., Kamionkowski, M. 2002, MNRAS, 331, 71
 Birkinshaw M. 1999, Phys Rep, 310, 97

Bryan G.L. & Norman M.L. 1998, ApJ, 495, 80
 Carlstrom, J.E., Holder, G.P., Reese, E.D. 2002, ARA&A, 40, 643
 Carroll, S.M.; Press, W.H., Turner, E. L. 1992, ARA&A, 30, 499
 Colafrancesco, S., Mazzotta, P., Vittorio, N. 1997, ApJ, 488, 566
 da Silva, A.C., Kay, S.T., Liddle A.R. *et al.* 2004, MNRAS, 348, 1401
 Diaferio, A., Nusser, A., Yoshida, N. *et al.* 2003, MNRAS, 338, 433
 Diego, J.M., Martínez-González, E., Sanz J.L. *et al.* 2002, MNRAS, 331, 556
 Eke, V.R., Cole, S., Frenk, C.S. 1996, MNRAS, 282, 263
 Eisenstein, D.J., Hu W., Tegmark M. 1999, ApJ, 518, 2
 Ettori, S., Tozzi, P., Borgani, S. *et al.* 2004, A&A, 417, 13
 Fan, Z. & Chiueh, T. 2001, ApJ, 550, 547
 Finoguenov A., Reiprich T.H. & Böhringer H. 2001, A&A, 368, 749
 Grego, L., Carlstrom, J.E., Reese, E.D. *et al.* 2002, ApJ, 552, 2
 Haiman, Z., Mohr, J.J., Holder, G. 2001, ApJ, 553, 545
 Holder, G., Haiman, Z., Mohr, J.J. 2001, 560, L111
 Horner D.J., Mushotsky R.F. & Scharf C.A. 1999, ApJ, 520, 78
 Hu, W. 2003, PhRvD, 67, 1304
 Huterer, D. & White, M. 2002, ApJL, 578, L95
 Kaiser N. 1986, MNRAS, 222, 323
 Kneissl, R., Jones, M.E., Saunders, R. *et al.* 2001, MNRAS, 328, 783
 Landy, S.D. & Szalay, A.S. 1993, ApJ, 412, 64
 Levine, E.S., Schulz, A.E., White, M., 2002, ApJ, 577, 569
 Majumdar S. & Mohr J.J. 2003, ApJ, 585, 603
 Majumdar S. & Mohr J.J. 2004, ApJ, 613, 41
 Mei, S. & Bartlett, J.G. 2003, A&A, 410, 767
 Mei, S. & Bartlett, J.G. 2004, A&A, 425, 1
 Mohr, J.J., Mathiesen, B., Evrard, A. E. 1999, ApJ 517, 627
 Moscardini, L., Bartelmann, M., Matarrese, S. *et al.* 2002, MNRAS, 340, 102
 Muanwong, O., Thomas P.A., Kay, S.T. *et al.* 2002, MNRAS, 336, 527
 Nevalainen J., Markevitch M. & Forman W. 2000, ApJ, 532, 694
 Oukbir J. & Blanchard A. 1997, A&A, 317, 1
 Peacock J. A., Dodds, S. J. 1996, MNRAS, 280, L19
 Peebles P.J.E. 1980, The Large-Scale Structure of the Universe, Princeton Series in Physics, Princeton University Press (Princeton: New Jersey)
 Pierpaoli E., Borgani S., Scott D. *et al.* 2003, MNRAS, 342, 163
 Press, W.H. *et al.* 1992, *Numerical Recipes*, Cambridge University Press, New York

- Sheth R.K. & Tormen G. 1999, MNRAS, 308, 199
Sheth R.K., Mo H.J. & Tormen G., 2001, MNRAS, 323, 1
Spergel, D.N., Verde L., Peiris H.V. *et al.* 2003, ApJS, 148, 175S
Sunyaev, R. A. & Zel'dovich, Ya. B. 1970, Comments Astrophys. Space Phys., 2, 66
Sunyaev, R. A. & Zel'dovich, Ya. B. 1972, Comments Astrophys. Space Phys., 4, 173
Weller, J., Battye, R.A., Kneissl, R. 2002, PhRvL, 88, 1301
Xu H., Jin G. & Wu X.-P. 2001, ApJ 553, 78