

SALT: a Spectral Adaptive Light curve Template for Type Ia Supernovae

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We present a new method to estimate luminosity distances of Type Ia supernovae (SNe Ia) from their multi-color light curves. The method was developed in view of analyzing the large number of SN Ia multi-color light curves measured in current high-redshift projects such as ESSENCE or SNLS. The technique is based on modeling SN Ia luminosity as a function of phase, wavelength, a shape parameter, and a color parameter. The model is trained with a sample of well measured nearby SNe Ia and then tested with an independent set of supernovae. We obtain luminosity distance estimates using U - and B -band data only, comparable in precision with those obtained with B - and V -band data.

1. INTRODUCTION

Type Ia supernovae (SNe Ia) are a powerful tool to study the evolution of the luminosity distance as a function of redshift and to subsequently constrain the cosmological parameters. SNe Ia are indeed very luminous and “standardizable” candles, and have led to the discovery of the acceleration of the Universe ([12, 17]).

Although often described as an homogeneous class of objects, SNe Ia exhibit variability in light curve shapes, colors, intrinsic luminosity and spectral features. Finding correlations among SN Ia observables is motivated by improving the estimation of their intrinsic luminosity on an event by event basis, in order to reduce the scatter in luminosity distance estimates. Among the observed correlations between observables related to photometric measurements are:

- a width-luminosity (or brighter-slower) relation, which expresses the fact that brighter supernovae have a slower decline rate than fainter ones ([6, 13, 14, 16, 18]).
- a brighter-bluer relation which was made explicit in [11, 25, 26], and assumed to be due to extinction by dust in other works ([1, 8, 12, 17, 19, 24]).

The case of the brighter-bluer relation is interesting. Even if authors do not fully agree on the origin of the effect nor on the strength of the correlation, most, if not all, recent attempts to build a SN Ia Hubble diagram have made use of color in their distance estimator. [20] summarizes previous work on the subject and proposes a way to reconcile divergent interpretations of data by taking into account the correlation between light curve shape and color.

Other methods have also been recently proposed to estimate luminosity distances (see for example [27]). We propose here to parameterize the light curve model

with a minimal parameter set: a luminosity parameter, a decline rate parameter and a single color parameter. Our approach will be to build a phenomenological model of the expected SN flux, continuously varying with phase, wavelength, decline rate and color, in order to capture all these features at once, and that is easily applicable to high-redshift SNe Ia currently measured in projects such as ESSENCE¹ or SNLS². Particularly important for high redshift events, we require that our model predicts light curves for any band located between U and R rest-frame since it is rather common, in current experiments that one of the available bands of high redshift objects falls blue-wards of B rest-frame. Also, since high redshift objects often lack late-time photometric data (or have one of too poor quality), we cannot rely on this data to estimate color, as proposed in [9].

In section 2 we describe the semi-analytic model used. We then describe how the coefficients of the model are determined by iterative training based on a set of well-sampled nearby SNe Ia from the literature, and highlight some properties of the resulting model. At this stage, the aim is to model multi-color light curves and not to estimate luminosity distances. The model is then tested in section 4 with an independent set of SNe Ia in the Hubble flow to check its consistency. A luminosity distance estimate is constructed from the fitted parameters of the light curve model. It is used to build Hubble diagrams successively from (B, V) and (U, B) pairs of light curves in order to assess the precision of this approach.

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²cft.hawaii.edu/SNLS

2. THE LIGHT CURVE MODEL

As already mentioned, we choose to parameterize light curves (more precisely light curve pairs or triplets when available) using a single luminosity, a single shape parameter and a single color. The choice among possible implementations is largely arbitrary. We choose parameters which enable comparisons with previous works:

- f_0 : a global intensity parameter defined below,
- s : a time stretch factor as the decline rate indicator [13]. In Goldhaber et al. [4], this parameter is shown to apply to the rising part of the light curve as well. However, while the stretch paradigm describes well the bright part of the B light curve, it does poorly at late time. It also fails to capture the shape variations in the other bands. This is why our model uses the stretch parameter as an index rather than the stretch paradigm itself. As described below, by construction our model follows exactly the stretch paradigm in the B band.
- $c = (B - V)_{max} + 0.057$, where $(B - V)_{max}$ is measured at B maximum, and -0.057 is the chosen reference color (Vega magnitudes) of a SN Ia.

With these definitions, the expected counting rate f_{SN} at a phase p in a given pass-band T of a supernovae at redshift z can be written:

$$f_{SN}(T) = v_0(1+z) \left(\frac{d}{d_L(z)} \right)^2 \times \int \phi(p, \lambda, s, c) \frac{\lambda}{hc} T(\lambda(1+z)) d\lambda$$

where $\phi(p, \lambda, s, c)$ represents a benchmark supernova energy flux per unit wavelength placed at some reference distance $d \ll c/H_0$ from the observer, which may vary with stretch and color. The transmission $T(\lambda)$ has then the dimension of an area times counts in the detector per photon. v_0 is the absolute luminosity excess (or deficit) of the supernova studied with respect to the benchmark. Note that the potential extinction by dust in the host galaxy is not explicit in the equation. Instead, we choose to incorporate it in the model $\phi(p, \lambda, s, c)$. We will discuss this in more details.

In the following, we use a global normalization parameter $f_0 = v_0(d/d_L(z))^2$. This choice of parameter, hiding the cosmological dependence of fluxes, is well suited for light curve modeling without any consideration on distances (for instance it can be applied to supernovae that are not in the Hubble flow).

Building an average spectral template ϕ as a function of phase, wavelength, color and stretch from observations is complicated because of the inhomogeneity and incompleteness of published data. In order to

implement stretch dependent light curve shapes and colors we therefore used the following approximation:

$$f_{SN}(T) = f_0(1+z) \int \phi(p_s, \lambda) \frac{\lambda}{hc} T(\lambda(1+z)) d\lambda \times \exp[-0.4 \ln(10) \times \mathcal{K}(p_s, \lambda_T, s, c)] \quad (1)$$

where $p_s \equiv p/s$ is a stretch-corrected phase. This functional form defines the light curve model.

In 1, ϕ no longer depends explicitly on s and c , and $\mathcal{K}(p_s, \lambda, s, c)$ is a smooth ‘‘correction’’ function of our four variables. λ_T is the central wavelength of the filter T . \mathcal{K} enables one to implement light curve shape variations that are more complicated than simple dilations of the time scale, along with stretch dependent colors. As described below, \mathcal{K} varies smoothly with λ ; this justifies placing it outside the integral over wavelength.

While the functional form of $T(\lambda)$ is determined by optical transmission measurements its normalization can be determined from the integrated flux f_{ref} (deduced from the zero point) of a known standard spectrum ϕ_{ref} , via the relation

$$\int \phi_{ref}(\lambda) \frac{\lambda}{hc} T(\lambda) d\lambda = f_{ref} \quad (2)$$

The functions ϕ and \mathcal{K} define the model. Once they are determined, one can fit the supernova photometric data points, measured in a minimum of two passbands, to estimate f_0 , s , c , and a date of B maximum light, which is a nuisance parameter. With only one passband, c must be held fixed.

For $\phi(p_s, \lambda)$, we use a template spectrum assembled by P. Nugent (Nugent et al. [10] and private communication) smoothed along the phase (time) axis, and normalized as a function of phase to the B -band light curve template ‘‘Parab -18’’ of Goldhaber et al. [4]. The empirical correction function \mathcal{K} is implemented as a sum of two polynomials:

$$\mathcal{K}(p_s, \lambda, s, c) = \mathcal{K}_s(p_s, \lambda, s) + \mathcal{K}_c(\lambda, c) \quad (3)$$

where we explicitly separate the corrections associated with the parameters s and c to clarify their interpretation. $\mathcal{K}_s(p_s, \lambda, s)$ modifies the shape of light curves and absorbs any stretch-color relation except for the $(B - V)_{max}$ color. Indeed we want c to describe exactly the $(B - V)_{max}$ color. $\mathcal{K}_c(\lambda, c)$ is then a color correction as a function of wavelength and color. The number of independent coefficients in the model is 34. We will call ‘‘training’’ the determination of these coefficients from measurements of nearby SNe Ia.

3. TRAINING THE MODEL

Since f_0 describes the *observed* luminosity in the B band, the model only incorporates stretch-shape and

stretch-color relations, but no correlation involving luminosity. This option was chosen in order to allow us to train the model with objects at unknown distances, in particular the nearby objects in the sample of Jha [7] measured in the U band. If one offsets all magnitudes of each training object by an arbitrary amount, possibly different for each object, the resulting model will not change. One may then consider incorporating high redshift objects into the training, but we did not do it here.

The model was trained and tested using a sample of published nearby supernova light curves. We collected 122 SNe Ia for which B - and V -band light curves are available in the literature, including data from Hamuy et al. [6], Riess et al. [20] and Jha [7] for a total of 94 objects, and 28 additional supernovae collected from various sources.

Objects were then selected based on two main criteria. First, we kept supernovae with at least two measurements before the maximum in the B or the V -band. This is necessary to ensure that the date of maximum is well defined and that the measurements can safely be used as a function of phase. Out of the whole sample, 56 SN satisfied this criterion. Then under-luminous peculiar supernovae were rejected from our sample, but SN 1991T-like events were kept. The resulting sample of 50 SNe was then split into two sets: a training sample and a test sample. The training sample (35 SNe) was used to adjust the coefficients of the polynomial of the model. It contains all the supernovae with redshifts smaller than 0.015 (not in the Hubble flow) and 6 supernovae at redshifts above 0.015, for which with U -band data was available, in order to improve the model in this wavelength region. The test sample contains 26 supernovae. The data was not pre processed in any way prior to fitting. To account for the Milky Way extinction, we incorporate it into the instrument transmission, using the law from (author?) [3] with a color excess $E(B-V)$ obtained from Schlegel et al. [22] dust maps at the position of the object to fit.

All the published nearby supernova magnitudes are expressed in the Johnson-Cousins $UBVR$ system. In Equation (1), we use models of the instrument transmissions as a function of wavelength. We adopted the transmission functions published by (author?) [2], and interpreted them as $\lambda T(\lambda)$ (see equation 1), i.e. counts per unit energy, following a footnote of Suntzeff et al. [23].

Training the model consists in determining the $\mathcal{K}(p_s, \lambda, s, c) = \mathcal{K}_s(p_s, \lambda, s) + \mathcal{K}_c(\lambda, c)$ correction function (Eq. 3) using the training sample data in the UBV and R -bands. We start with a first guess:

$$\begin{aligned} \mathcal{K}_s(p_s, \lambda, s) &= 0 \\ \mathcal{K}_c(\lambda, c) &= c \times (\lambda - \lambda_B) / (\lambda_V - \lambda_B) \end{aligned}$$

and we use an iterative algorithm which can be sketched as follows:

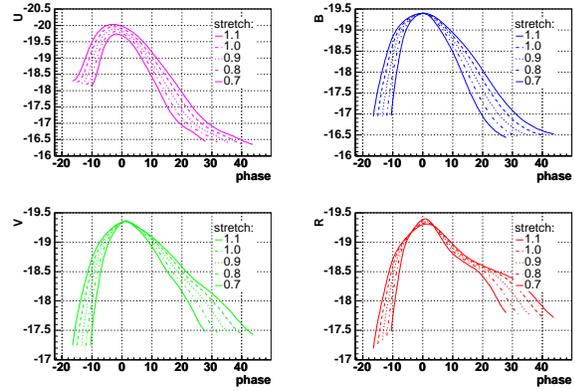


Figure 1: The $UBVR$ template light curves (from top left to bottom right) obtained after the training phase for different values of the stretch and null color.

1. Fit the light curves using the current determination of \mathcal{K} .
2. Fit $\delta\mathcal{K}(p_s, \lambda, s, c)$, an instance of the \mathcal{K} function, on the light curve residuals. During this step, identify and remove the outliers data points.
3. $\mathcal{K} \leftarrow \mathcal{K} + \delta\mathcal{K}$.
4. GOTO step 1, until $\delta\mathcal{K}$ becomes negligible.

3.1. Results of the training

The fit converged after four iterations. 2480 measurement points were fitted, and 39 were discarded as outliers (at the 3σ level). Compared to the number of free coefficients of the model, we can safely conclude that the model is not over-trained. The standard deviation of the residuals to the model in $UBVR$ are respectively of 0.09, 0.09, 0.06, 0.07 magnitudes. Figure (1) shows the final U , B , V and R templates obtained at the end of the process as a function of stretch. By construction, the rest-frame B and V -band magnitudes at maximum do not vary with stretch. We find a strong dependence of $(U - B)_{max}$ with stretch ($\delta(U - B)_{max} \simeq -\delta s$, compatible with [7]), which is an essential feature for the model to reproduce in order to estimate a reliable color in the wavelength range between U and B . The model also manages to reproduce a stretch-dependent secondary shoulder in the R -band. The k , stretch, and color corrected light curves of SNe from the training sample are shown figure 2 along with the model for $(s, c) = (1, 0)$.

Figure 3 represents the color correction $\mathcal{K}_c(\lambda, c)$ for $c = 0.1$ compared to the dust extinction law from (author?) [3]. Interestingly enough, the law we obtain follows pretty well that of Cardelli except in the U -band where we get a slightly stronger dependence on c . As a consequence, we deduce that the relation between $E(B-V)$ and $E(V-R)$ are very similar (i.e. in-

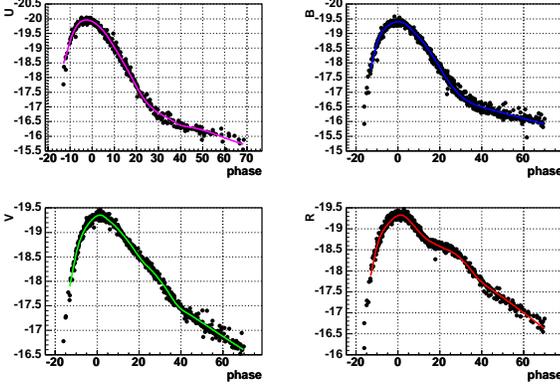


Figure 2: The k , stretch, and color corrected rest-frame $UBVR$ light curves from the training sample. The solid curves represent the template light curves for $(s, c) = (1, 0)$.

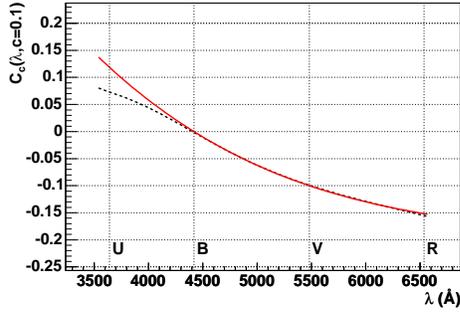


Figure 3: The color correction \mathcal{K}_c as a function of wavelength for a value of c of 0.1. The dashed curve represents the extinction with respect to B -band, $(A_\lambda - A_B)$, from (author?) [3] with $R_V = 3.1$ and $E(B - V) = 0.1$.

distinguishable) to the ones expected from reddening by dust. This similarity, noted by [20], does not prove however that c can be interpreted as reddening by dust; an additional requirement for this hypothesis to be valid would be that the peak B -band magnitude increases with c by a value of $R_B \times c$. We will see that this is not the case in the next section. Let us also emphasize that the stretch dependent part of the $U - B$ and $V - R$ colors are included in the stretch dependent term $\mathcal{K}_s(p_s, \lambda, s)$, and *not* in the color curve of fig. 3.

4. PERFORMANCE STUDY

Once the correction function $\mathcal{K}(p_s, \lambda, s, c)$ is determined, we can fit the model on an independent sample of nearby SN Ia light curves. This allows us to perform various consistency checks, in order to make sure

that the model describes well the UBV photometry of SNe Ia.

One can define a rest-frame B magnitude m_B^* [13]

$$m_B^* = -2.5 \log_{10} \frac{f_{SN}(T_B^*)}{(1+z) f_{ref}(T_B)} \quad (4)$$

where f_{SN} and f_{ref} are respectively defined by equations 1 and 2, T_B is the transmission of the B filter and $T_B^*(\lambda) = T_B(\lambda/(1+z))$ is a redshifted B transmission. One can check that m_B^* varies as $5 \log_{10} d_L(z)$ with redshift and that $m_B^* \rightarrow m_B$ for $z \ll 1$, where m_B is the conventional B magnitude. We incorporate the Hubble parameter dependence of d_L in a constant parameter $M_B^{70} = M_B - 5 \log_{10}(h_{70})$, which is the mean absolute magnitude of a SN Ia with $s = 1$ and $c = 0$, for a value of the Hubble parameter of 70 $\text{km.s}^{-1}.\text{Mpc}^{-1}$. Following Tripp [25], we adopt linear corrections of coefficients α and β respectively for stretch and color to build a distance estimator:

$$m_B^* - M_B^{70} - 43.16 - \alpha(s - 1) + \beta c \quad (5)$$

Its expectation value for a supernova at redshift z is $5 \log_{10}(d_L(z) H_0 c^{-1})$. In what follows we apply this method to build low- z Hubble diagrams using successively (B, V) and (U, B) light curves. Since our goal is here to test the distance estimator rather than actually perform a cosmological fit, we impose the ‘‘concordance’’ cosmological parameters ($\Omega_M = 0.3$ and $\Omega_\Lambda = 0.7$) when fitting M_B^{70} , α and β .

4.1. Hubble diagram in BV

Using all B and V -band light curves of supernovae with redshifts larger than 0.015 from the test sample, we obtain: $M_B^{70} = -19.40 \pm 0.05$, $\alpha = 1.47 \pm 0.28$ and $\beta = 2.11 \pm 0.35$. The standard deviation of residuals is 0.17 ± 0.03 ³. Uncertainties on m_B, s, c along with their covariance were included in the fit⁴, we also considered an uncertainty on redshifts due to peculiar velocities of 300 km.s^{-1} ; an additional ‘‘intrinsic’’ dispersion of 0.15 is needed in order to get a χ^2 per degree of freedom of 1. This value is quite small and comparable with results obtained with other distance estimators. The observed brighter–slower and brighter–bluer relations are shown figure 4.

Our approach to estimating distances easily compares to the one adopted in [25]: the main differences are the light curve model and the brighter–slower

³Note that this number takes into account the number of parameters in the fit. The measured RMS value is 0.15 ± 0.03 .

⁴The uncertainties on the distance estimate formally depend on α and β , and increase with them. As a consequence, the χ^2 minimum is biased toward large values of these parameters. We therefore computed the uncertainties with the initial values, and use the result of the fit at the final iteration.

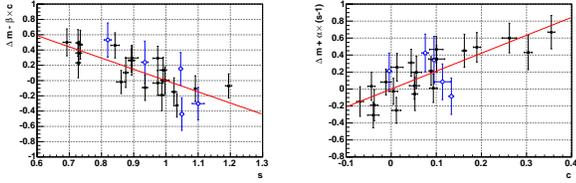


Figure 4: Residuals to the Hubble diagram as a function of stretch s and color c indexes for supernovae of the test sample fitted in B and V bands. SNe with redshifts smaller than 0.015 are labeled with opened symbols.

parameterization. When we fit the same SNe sample (The Calán-Tololo sample from [5]), a value of $\alpha = 1.04 \pm 0.24$ and $\beta = 2.08 \pm 0.27$ are obtained, which compare well to $\alpha = 0.88^5$, $\beta = 2.09$ of [25], based on peak luminosity, color, and decline rate estimates from [5]. We conclude that our model correctly reproduces basic parameter estimations of previous works.

Concerning the interpretation of the brighter-bluer correlation, we find a value of β which is incompatible with $R_B = 4.1$, expected for extinction by dust analogous to the observed law in the Milky Way. The value we find is compatible with those found in previous works (see Tripp [25] and references therein). However, as stressed in Riess et al. [20], the color excess (or deficit) at maximum should not be interpreted as entirely due to extinction but be corrected for the part of this excess that is correlated with stretch. We measure a stretch-color slope of about 0.2, similar to the relation proposed in Phillips et al. [15]⁶ and can redefine our parameters to account for this correlation ($c' = c + 0.2(s - 1)$ and $s' = s$), so that s' and c' are uncorrelated. The correlation coefficients then become $\alpha' = \alpha + 0.2\beta$ and $\beta' = \beta$, which means that redefining the color excess to explicitly assign to stretch the color variations correlated to stretch does not change the brighter-bluer correlation strength.

4.2. Hubble diagram in UB

We applied the same procedure as in the previous section to fit the U and B -band light curves of the test sample for which U -band measurements are available and redshifts larger than 0.015 (7 supernovae). We obtain $M_B^{70} = -19.36 \pm 0.06$, $\alpha = 0.8 \pm 0.4$, $\beta = 3.3 \pm 0.6$, and the standard deviation of residuals is 0.13. Without any additional intrinsic dispersion, the

⁵ $b = 0.52$ translates to $\alpha \simeq 0.88$ when using stretch and the first order relation ($\Delta M_{15} - 1.1) \simeq 1.7(1 - s)$.

⁶The proposed relation is $\frac{dc}{d\Delta M_{15}} = 0.114 \pm 0.037$. With the approximate relation $\frac{d\Delta M_{15}}{ds_B} \simeq -1.7$ (at $s_B = 1$), we expect $dc/ds \simeq -0.2$.

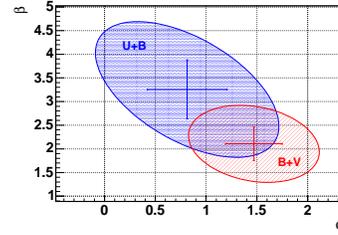


Figure 5: 68% joint confidence regions for (α, β) fitted using either UB or BV light curves of the test sample. The crosses show the best fitted values with 1σ uncertainties.

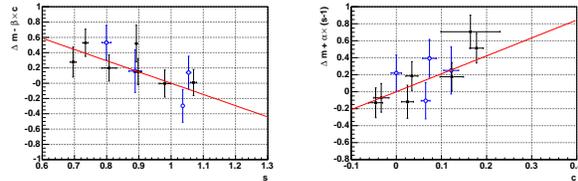


Figure 6: Residuals to the Hubble diagram as a function of stretch s and color c indexes for supernovae of the test sample fitted in U and B bands. The values of M_B, α, β used here are those fitted using B and V bands as described in the text. SNe with redshifts smaller than 0.015 are labeled with opened symbols.

χ^2 per degree of freedom is of 0.7. As expected, these results are consistent with the fit using B and V , as shown by the confidence contours for α and β fitted using either UB or BV light curves shown figure 5. Figure 6 presents the residuals to the Hubble diagram as a function of redshift, stretch and color using the values of M_B', α, β fitted with B and V band light curves in the previous section. Fitting the Hubble diagram with the values of α and β obtained with B and V -band light curves, the standard deviation of residuals is 0.18 ± 0.05 .

5. CONCLUSION

We have proposed a new method to fit broadband light curves of type Ia supernovae. It allows us to determine simultaneously the SN Ia rest-frame B magnitude at maximum, stretch and color excess (or deficit) using any measured multi-color light curve within the wavelength range of rest-frame U to R . In particular, we have been able to estimate distances from rest-frame U - and B -band measurements comparable in precision with those obtained from rest-frame B - and V -bands. This technique is particularly well suited for the treatment of high-redshift SNe Ia for which limited coverage is obtained in both wavelength and phase.

The k -corrections, which allow the observer to transform the observed magnitudes into the standard rest-frame magnitudes are built-in; the model includes the dependence on stretch and color of the spectrum template needed to estimate those corrections. In particular, the well-known correlation between $(U - B)_{max}$ and stretch is reproduced.

We have tested this fitting procedure on an almost independent sample of SNe Ia. Using alternatively BV and UB bands, we managed to retrieve consistent parameters and hence build Hubble diagrams with both sets of data. The dispersions about the Hubble line were found to be 0.17 ± 0.03 and 0.13 ± 0.04 in the BV and UB bands respectively. The reader may be surprised by the UB dispersion being smaller than the BV one, but both dispersions are statistically compatible.

The BV dispersion is larger than the value of 0.12 obtained in Riess et al. [19] with the Multicolor Light-Curve Shapes method (MLCS). However, it is smaller than the value computed from the latest nearby sample in Riess et al. [21] which gives a value of 0.22 (computed with SNe at redshifts $0.015 < z < 0.1$ from the "golden" sample in Table 5 of Riess et al. [21], also analyzed with the MLCS method). This difference may be due to the use of different training samples in those two papers. Wang et al. [27] finds a weighted dispersion of 0.08 with the CMAGIC method for a sub-sample of SNe with $B_{max} - V_{max} < 0.05$. With a weaker cut on color, $B_{max} - V_{max} < 0.5$, the dispersion rises to about 0.15, which is consistent with our result. A more detailed comparison would require us to perform the comparison on the same sample of SNe (due to the limited statistics).

The $(B - V)$ and $(V - R)$ stretch-independent colors we obtain are extremely similar to the ones expected from reddening by dust. The $(U - B)$ color departs from this law. We find a relation between $(B - V)$ color and observed B luminosity incompatible with $R_B = 4.1$, at more than 3 standard deviations, even when accounting for the stretch-color correlation.

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