Fractal Dimensions of the Galaxy Distribution Varying by Steps?

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The structure of the large scale distribution of the galaxies have been widely studied since the publication of the first catalogs. Since large redshift samples are available, their analyses seem to show fractal correlations up to the observational limits. The value of the fractal dimension(s) calculated by different authors have become the object of a large debate, as have been the value of the expected transition from fractality to a possible large scale homogeneity. Moreover, some authors have proposed that different scaling regimes might be discerned at different lenght scales. To go further on into this issue, we have applied the correlation integral method to the wider sample currently available. We therefore obtain a fractal dimension of the galaxy distribution which seems to vary by steps whose width might be related to the organization hierarchy observed for the galaxies. This result could explain some of the previous results obtained by other authors from the analyses of less complete catalogs and maybe reconcile their apparent discrepancy. However, the method applied here needs to be further checked, since it produces odd fluctuations at each transition scale, which need to be thoroughly explained.

1. Introduction

Standard cosmology is based on the assumption that the Universe is spatially homogeneous, at least on scales sufficiently large to justify its approximation by a FLRW model.

However, the consensus on a homogeneous feature of structures, even on very large scales, has never been complete. At small separations, data worked out using the correlation function method show a correlation lenght $r_0 = 5h^{-1}$ Mpc and the galaxy distribution exhibits a fractal structure with dimension $D_2 \sim 1.2$ [1]. The correlation integral method [2] gives a fractal dimension slightly larger, $D_2 \sim 1.3$ to 1.5. At larger scales, the value $D_2 \sim 2$ has been proposed [3] up to scales of at least $150h^{-1}$Mpc [4]. For review articles see Martinez, 1999 [5] or Wu, Lahav and Rees, 1999 [6]. It has also been suggested that three scaling regimes might be discerned [7].

At least, the value of the transition scale from inhomogeneity to homogeneity needs to be tested [8, 9].

We use the more recent and complete three-dimensional galaxy catalog, the Sloan Digital Sky Survey (SDSS), to repeat older calculations and hope to obtain more reliable results.

We leave out of the account some other issues related to the galaxy case. These are source evolution and cosmological effects [10–12]. They should be considered in detail in some further study.

2. The correlation Integral method

As a first approach, we have chosen to use a characterization of the structures of point sets which is given by the correlation integral [13], defined as:

$$C_2(r) = \frac{1}{N'(N-1)} \sum_i \sum_{j \neq i} \Theta(r - |\mathbf{X}_i - \mathbf{X}_j|) \qquad (1)$$

where Θ is the Heaviside function. The inner summation is over the whole set of $N-1$ galaxies with coordinates $\mathbf{X}_j$, $j \neq i$, and the outer summation is over a subset of N' galaxies, taken as centers, with coordinates $\mathbf{X}_i$. By taking only the inner N' galaxies as centers we allow for the effect of the finiteness of the sample [4].

This characterization is also valid when the set is not fractal. Therefore it seems appropriate to use this approach to analyse galaxies considered as point sets, provided the spaned scale range shows either a fractal behavior or not.

We may interpret $C_2(r)$ as $\mathcal{N}(r)/N$ where $\mathcal{N}(r)$ is the average number of galaxies within a distance r of a typical galaxy in the set. As r goes to zero, C_2 should vanish as $C_2 \propto r^{D_2}$. For computational purposes it is more convenient to use the form:

$$\log(C_2) = CONST. + D_2 \log(r) \qquad (2)$$

The exponent D_2 is the fractal dimension, necessarily ≤ 3 for an embedding space of dimension three. When D_2 is different from three, the distribution is fractal [14].

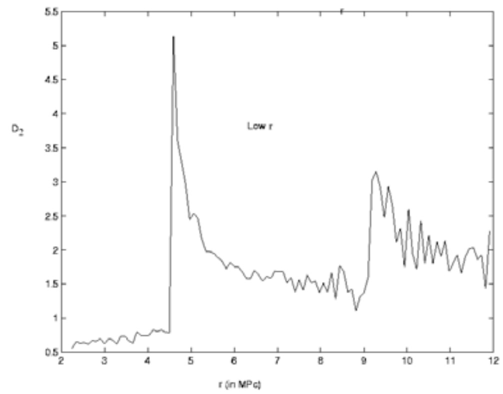

Figure 1: D_2 as a function of r, for small scales.

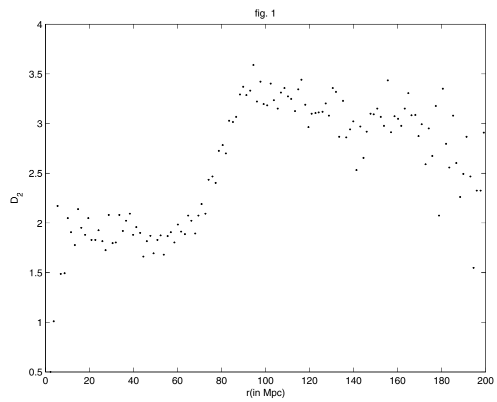

Figure 2: D_2 as a function of r, for larger scales.

3. The analysis

We use the publicly available SDSS data. After appropriate eliminations we are left with 84k galaxies. The rms galaxy redshift errors are estimated to be about 30km/sec [15], therefore they are small for the overall density fluctuations.

We assume $H_0 = 65$ km/s/Mpc. Since our results exhibit large error bars, the value of H_0 is not too important. Another choice would only shift h^{-1} the transition scales.

Because of the large differences between the transitions from one dimension to another, we devide the appropriate range into a large number of segments and calculate D_2 by the Procaccia-Grassberger method [13]. To obtain a very rough estimate on the region of transition, we check three points at a while. It gives the values for D_2 which are shown in Figures 1 and 2.

In the above figures, a step variation of the fractal dimension D_2 is obvious. This could explain why previous studies, limited in scale range, concluded to constant fractal dimensions with different values. However, large fluctuations appear at each transition scale. Then D_2 decreases towards a fluctuating value and rises again with large fluctuations at the next transition scale. The fluctuations are such that the fractal dimension becomes larger than the limiting dimension three at each transition. This seems to point out to some artefacts or bias due either to the employed method or to the data sample. The interpretations we propose are twofold.

3.1. First interpretation (tentative)

If one puts aside the low r results of Figure 1 because of the too unphysical values attained by D_2, one can use only the results of Figure 2 and proceed to their following processing.

One devides the range from 0 to 200 Mpc into 256 segments. To obtain a rough estimate of the region of transition, one first checks three points at a while. This gives 128 values for D_2 (see Figure 2). The error on the average of the so obtained D_2 is taken as the error (more reliable than the (smaller) error obtained from the direct least square fit), because the distribution of these segment values is more Gaussian than the total least square fit. This method tends to give large errors on each segments but enables to observe more clearly the trend. Then one calculates the dimensions by using all the points to perform a least square calculation.

To check these results, we used two different amounts of inner points in the evaluation. Still we kept in mind that all the points, including those outer points which are at a distance used in our calculations, should be within the measured range. We also used, as the maximal distance considered, three different values. In all the cases the transition area remained the same. We then took an inner region which is much larger so that some of the points of the outer region are already in an area not covered by the catalogue. For this case the results differed considerably. We do not get a clear transition and the dimension was on the average 2.3 ± 0.3. The trend is obvious as less and less points are counted from the total available points set we expect to get more and more distorted results.

In Figure 3, a transition seems to appear between two scaling regimes:
1.90 ± 0.03 , for 6 Mpc to 80 Mpc.
3.01 ± 0.04 , for 100 Mpc to 200 Mpc.

Between 80 Mpc and 100 Mpc the picture is not clear, so that the transition might be somewhere around 90 Mpc. In Figure 4, which is obtained by the

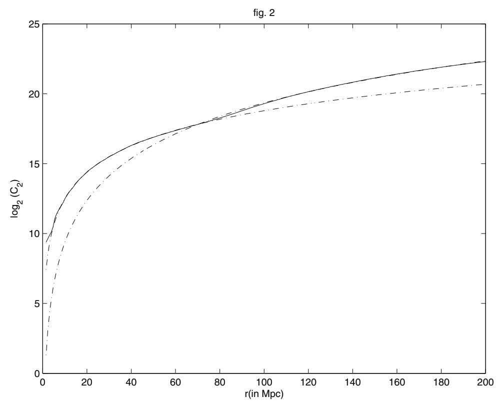

Figure 3: The tentative $D_2 = 2$ to $D_2 = 3$ transition. Taking the parameters of the least square fit for the lower part and upper part, compared to the experimental results (solid line).

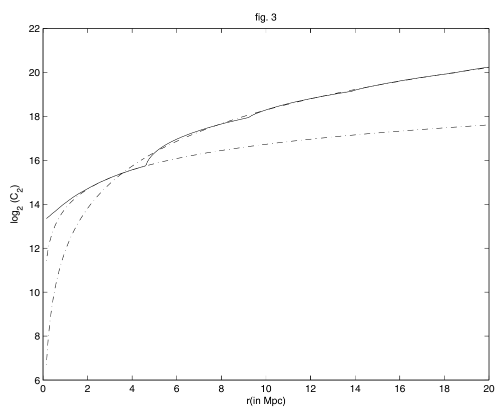

Figure 4: The tentative $D_2 \approx 1$ to $D_2 = 2$ transition with the same procedure as for Figure 3.

same method applied to the low r results of Figure 1, another transition seems to appear between a $D_2 \sim 1$ and $D_2 \sim 2$ regime:
0.89 ± 0.01 for 2 Mpc to 4.5 Mpc.
1.93 ± 0.05 for 6 Mpc to 20 Mpc.

Between 4.5 Mpc and 6 Mpc the picture is also not too clear. At the transition, the dimension increases very dramatically. This might be interpreted as an artefact resulting from the small difference in r of neighbouring bins. But these low r analyses can also be viewed as some confirmation of the preliminary results obtained by [7] who found, for galaxy separations up to about 5 Mpc a D_2 dimension about 1.2 and for larger scales (out to about 30 Mpc, which was the limiting scale of their study) a dimension of about 1.8.

3.2. Second interpretation (seemingly more robust, but to be confirmed)

Figures 1 and 2 show a similar behavior, with a peak at the transition, then an (e^{-r})-like decrease toward a constant value (with fluctuations above the allowed value $D_2 = 3$). When more data become available, and with an analysis method more adapted to the study of multi-fractal distributions, we might be able to check whether this peak is actually an artefact coming from our data analysis method (some step behavior at each transition scale) or if it is due to the data sample.

The $D_2 = 3.3$ peak at 100 Mpc in 2 and the decrease farther could be of the same nature: in this case, the "true" value of D_2 beyond 100 Mpc would not yet have been reached at the limit of the study (200 Mpc) (it might be ≤ 2.6), which could mean that the transition to uniformity ($D_2 = 3$) has not yet been reached at these scales.

4. Conclusion and discussion

We use the publicly available data from the SDSS, to complete an analysis of the fractal dimension of the galaxy distribution, with the correlation integral method.

We check scales up to 130 h^{-1} Mpc. We obtain an obvious step variation of the fractal dimension D_2. This could explain why previous studies, limited in scale range, concluded to constant fractal dimensions, with different values.

Two possible interpretations of these results are proposed:

- A rough mean square fit method gives i) up to 4.5 to 6 Mpc, a dimension of the order one, ii) then a transition to a dimension of the order two, iii) and between 80 and 100 Mpc, another transition to a dimension around three.

- However, the variation of D_2 with scales show a peak at each transition (the fractal dimensions at the peaks become larger than 3), then an (e^{-r})-like decrease toward a constant value (with fluctuations). This could be due to the step behavior at the transition scale. The transition to uniformity ($D_2 = 3$) would thus not yet have been reached at the largest studied scales. Moreover, a transition to homogeneity at 130 h^{-1} Mpc would be inconsistent with the sizes of the largest structures seen in the universe [16, 17]. Last, it would be interesting to check if a variable with scale fractal dimension of the galaxy distribution might be related to theoretical predictions proposed as a consequence of a principle of relativity of scales [18]: a transition to homogeneity predicted around 750 Mpc and a multifractal distribution with a dimension varying by steps whose width might be related to the organization hierarchy observed for the galaxies.

However, these large fluctuations, which appear at each transition scale, seem to point out to some artefacts due either to the employed method or to the data sample. A way to discriminate between the two possible reasons of the appearance of these odd fluctuations would be to test the validity of the application of the Grassberger and Procaccia's correlation integral method to a mutifractal distribution by applying it to a set of mock catalogues of galaxies, artificially constructed, with a known fractal dimension varying by known steps. If the runs of the here employed code reproduce the known features of the distributions, it would suggest that the artefacts might be due to the data sample analysed here. We would therefore need a better sample to go further on and, e. g., apply our method to the next SDSS catalogue, presumably publicly available in a very near future. If, on the contrary, those runs are not able to reproduce the known features of the distributions, it would suggest that the artefacts might be due to the employed method, and we would have to test other ones and find a code better fitted to the study of multifractal distributions.

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