Estimate of the CMB Fluctuation Amplitude from Dark Energy De-coherence

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Homogeneity and correlations in the observed CMB are indicative of some form of cosmological coherence in earlier times. Cosmological dark energy de-coherence is assumed to occur when the rate of expansion of the microscopically relevant cosmological scale parameter in the Friedmann-Lemaitre equations at early times is no longer supra-luminal. This choice of the scale parameter in the FL equations directly relates the scale of dark energy de-coherence to the De Sitter scale (associated with the positive cosmological constant) at late times. It is shown that the class of dynamical models so defined necessarily requires a spatially flat cosmology in order to be consistent with observed structure formation. Prior to de-coherence, the coherence which preserves the uniform density needed to make the FL dynamical equations meaningful must be maintained by supra-luminal (cosmological) correlations and not by the luminal or sub-luminal microscopic exchanges available after de-coherence. The basic assumption is that the dark energy density which is fixed during de-coherence is to be identified with the cosmological constant. This approach makes no assumption about the constancy of dark energy density outside of the finite time interval when the expansion rate is not supra-luminal. It is shown for the entire class of models that the expected amplitude of fluctuations driven by the dark energy de-coherence process is of the order needed to evolve into the fluctuations observed in cosmic microwave background radiation and galactic clustering.

1. INTRODUCTION AND GENERAL APPROACH

The luminosities of distant Type Ia supernovae and analysis of the CMB radiation show that the rate of expansion of the universe has been accelerating for several giga-years, in quantitative agreement with a (positive) cosmological constant fit to the data. When the dynamics of the cosmology is made consistent with cosmological scales so defined, it is expected that the usual microscopic interactions of relativistic quantum mechanics (QED, QCD, etc) cannot contribute to cosmological (gravitational) equilibrations when the relevant FRW scale expansion rate is supra-luminal, $\dot{R} > c$. The cosmological dark energy density is expected to decouple from the energy density in the Friedmann-Lemaitre(FL) equations when the FRW scale expansion is no longer supra-luminal, at which time the microscopic interactions open new degrees of freedom. The general approach used here is to start from well understood macrophysics, assume that the physics of de-coherence defines a cosmological scale parameter, and end the examination of the backward extrapolation of cosmological physics at the time when the rate of expansion of that scale parameter is the velocity of light. The process that takes the cosmology from the very early universe (i.e. prior to de-coherence) through this boundary will be called *gravitational dark energy de-coherence*. An understanding of the physics of de-coherence allows one to use the known value of the cosmological constant (dark energy) to determine the behavior of the scale parameter during early times.

2. COSMOLOGICAL EVOLUTION

2.1. The Friedmann-Lemaitre Equations

Global gravitational coherence prior to de-coherence (i.e. the assumption that the FL equations still apply in the very early universe) solves the horizon problem because the gravitational correlations implied by the FL equations are supra-luminal; it is hypothesized that the same will be true of any type of dark energy to be considered. Prior to the de-coherence scale condition $\dot{R}_{DC} = c$, gravitational influences on scales $R>R_{DC}$ must propagate (at least) at the rate of the gravitational scale expansion, and microscopic interactions (which are assumed to propagate no faster than c) cannot contribute to cosmological scale equilibrations. If the expansion rate is supra-luminal $\dot{R} > c$, scattering states cannot form decomposed (de-coherent) clusters on cosmological scales.

The Friedmann-Lemaitre(FL) equations are assumed valid during de-coherence.

$$H^2(t) \equiv \left(\frac{\dot{R}}{R}\right)^2 = \frac{8\pi G_N}{3c^2}\left(\rho + \rho_\Lambda\right) - \frac{kc^2}{R^2},$$

$$\left(\frac{\ddot{R}}{R}\right) = -\frac{4\pi G_N}{3c^2}\left(\rho + 3P - 2\rho_\Lambda\right)$$

where $H(t)$ is the Hubble expansion rate, the dark energy density is given by ρ_Λ, ρ represents the FL matter-energy density, and P is the pressure. The term which involves the spatial curvature k has explicit scale dependence on the FRW parameter R. The dark energy density is assumed to make a negligible contribution to the FL expansion during de-coherence, but will become significant as the FL energy density decreases due to the expansion of the universe.

2.2. Single Parameter for De-coherence

The energy density during dark energy de-coherence ρ_{DC} satisfies

$$H_{DC}^2(t) \equiv \left(\frac{c}{R_{DC}}\right)^2 = \frac{8\pi G_N}{3c^2}\left(\rho_{DC} + \rho_\Lambda\right) - \frac{kc^2}{R_{DC}^2}.$$

Assuming that the relevant scale R_{DC} describes the evolution of the cosmology, a so called "open" universe $k=-1$ is excluded from undergoing this transition, since the positive dark energy density term ρ_Λ already excludes a solution with $\dot{R} \leq c$. For a "closed" universe that is initially radiation dominated, the scale factors corresponding to de-coherence $\dot{R}_{DC} = c$ and maximal expansion $\dot{R}_{\max} = 0$ can be directly compared:

$$\left(\frac{c}{R_{\max}^2}\right)^2 = \frac{8\pi G_N}{3c^2}\left(\rho(R_{\max}) + \rho_\Lambda\right)$$

$$\cong \frac{8\pi G_N}{3c^2}\rho_{DC}\left(\frac{R_{DC}}{R_{\max}}\right)^4$$

$$\Rightarrow R_{\max}^2 \cong 2R_{DC}^2 .$$

Since the relevant scale has a value R_{DC} early enough for the observed structure of the CMB at last scattering (and the subsequent galactic clustering now observed) to develop, a "closed" ($k=+1$) universe must be ruled out. It follows that this de-coherence scenario necessarily requires a spatially flat cosmology in order to be consistent with structure formation.

Setting the expansion rate to c in the Lemaitre equation with k=0 defines the energy density during de-coherence ρ_{DC} as

$$\rho_{DC} = \frac{3c^2}{8\pi G_N}\left(\frac{c}{R_{DC}}\right)^2 - \rho_\Lambda ,$$

in terms of the single parameter R_{DC}.

As is often assumed, if the cosmology remains radiation dominated in the standard way down to t=0, then the scale parameter satisfies $R(t) = \left(\frac{t}{t_{DC}}\right)^{1/2}$ which gives the time at de-coherence as $t_{DC} = \frac{R_{DC}}{2c}$. The assumption of radiation dominance during de-coherence corresponds to a thermal temperature of

$$g(T_{DC})\left(k_B T_{DC}\right)^4 = \frac{90}{8\pi^3}\left(M_P c^2\right)^2\left(\frac{\hbar c}{R_{DC}}\right)^2 ,$$

where $g(T_{DC})$ counts the number of degrees of freedom associated with particles of mass $mc^2 << k_B T_{DC}$, and M_P is the Planck mass.

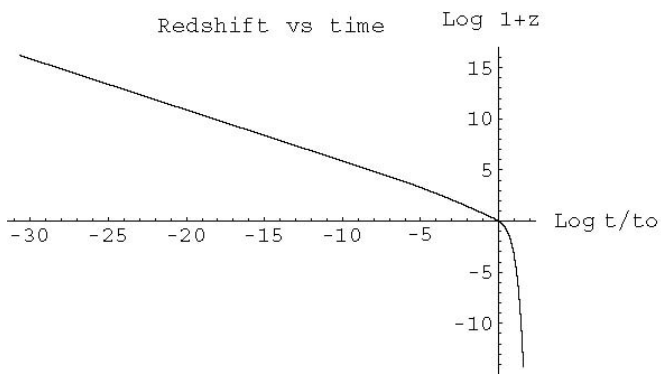
Using the FL densities at radiation-matter (dust) equality $\rho_M(z_{eq}) = \rho_{rad}(z_{eq})$ one can extrapolate back to the de-coherence period. Ignoring threshold effects (which give small corrections near particle thresholds while they are non-relativistic), this gives

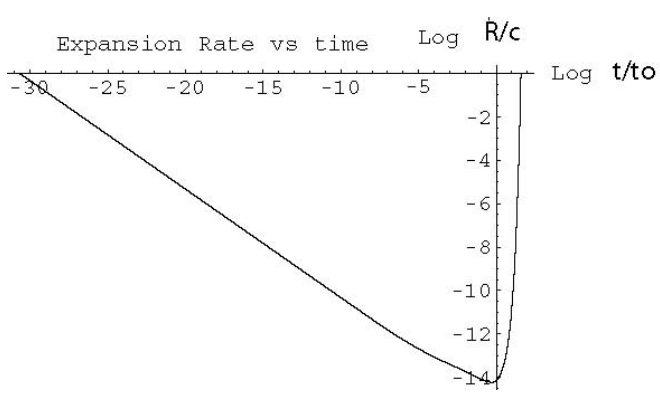
$$1 + z_{DC} = \left(\frac{\rho_{DC}}{\rho_{Mo}}\left(1 + z_{eq}\right)\right)^{1/4}$$

$$\cong \left(\frac{c}{H_o R_{DC}}\right)^{1/2}\left(\frac{1 + z_{eq}}{\Omega_{Mo}}\right)^{1/4}$$

Here, Ω_{Mo} is the present normalized non-relativistic mass density. The scale parameter at the present time is then expressed in terms of this redshift using the usual definition $R_o = \left(1 + z_{DC}\right)R_{DC}$.

The evolution of the cosmology during the period for which the dark energy density is de-coupled from the FL energy density is modeled using the FL equations. There is a period of deceleration, followed by acceleration towards a De Sitter expansion. The rate of scale parameter expansion is sub-luminal during a finite period of this evolution, as shown in the following figures.





The particular value for the scale at de-coherence is determined by the microscopic makeup of the dark energy. Present time corresponds to the origin on both graphs. The value of the expansion rate is c for $R=R_{DC}$, as well as when

the expansion scale reaches the De Sitter radius $R_\Lambda \equiv \sqrt{\frac{3}{\Lambda}} \approx 10^{28}\,cm \approx 1.6\times 10^{10}\,ly$.

2.3. Estimate of CMB Fluctuations

Adiabatic perturbations (those that fractionally perturb the number densities of photons and matter equally) grow according to

$$\Delta = \begin{cases} \Delta_{DC}\left(\frac{R(t)}{R_{DC}}\right)^2 & \text{radiation} - \text{dominated} \\ \Delta_{eq}\left(\frac{R(t)}{R_{eq}}\right) & \text{matter} - \text{dominated} \end{cases}.$$

An estimation of the scale of fluctuations at last scattering is given by

$$\Delta_{LS} = \left(\frac{R_{LS}}{R_{eq}}\right)\left(\frac{R_{eq}}{R_{DC}}\right)^2 \Delta_{DC} = \frac{(1+z_{DC})^2}{(1+z_{eq})(1+z_{LS})}\Delta_{DC}.$$

As will be shortly justified, the energy available for fluctuations in the two point correlation function during dark energy de-coherence is expected to be given by the cosmological dark energy. This means that the amplitude of relative fluctuations $\delta\rho/\rho$ is expected to be of the order

$$\Delta_{DC} \equiv \left(\frac{\rho_\Lambda}{\rho_{DC}}\right)^{1/2} = \frac{R_{DC}}{R_\Lambda}.$$

Using the previous equations, this amplitude at last scattering is given by

$$\Delta_{LS} = \frac{(1+z_{DC})^2}{(1+z_{eq})(1+z_{LS})}\frac{R_{DC}}{R_\Lambda} \cong \frac{1}{1+z_{LS}}\sqrt{\frac{\Omega_{\Lambda o}}{(1-\Omega_{\Lambda o})(1+z_{eq})}} \approx 2.6\times 10^{-5}$$

where a spatially flat cosmology has been assumed. This estimate is independent of the scale parameter during de-coherence R_{DC}, and is of the order observed for the fluctuations in the CMB. Fluctuations in the CMB at last scattering of this order are consistent with the currently observed clustering of galaxies.

2.4. De-coherence during Matter/Plasma Domination

The previous results have demonstrated NO dependency on the energy density during the transition period if de-coherence occurs during radiation domination. For completeness, the amplitude of expected fluctuations if the phase transition occurs during the matter/dust/plasma dominated era is next examined. The acoustic wave has coherent phase information that is transmitted to the CMB at last scattering. There must have been a significant enough passage of time from the creation of the acoustic wave to the time of last scattering such that peaks and troughs of the various modes should be present $\delta t > D_s/v_s$, where D_s is the distance scale of the longest wavelength (sound horizon), and $v_s \sim c/\sqrt{3}$ is the speed of the acoustic wave.

Generally, if the phase transition occurs while the energy density is dominated by dust/plasma, prior to last scattering then the amplitude satisfies

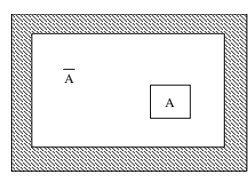
$$\Delta_{LS} \cong \left(\frac{1+z_{DC}}{1+z_{LS}}\right)\sqrt{\frac{\Omega_{\Lambda o}}{(1-\Omega_{\Lambda o})(1+z_{DC})^3\left(1+\frac{1+z_{DC}}{1+z_{eq}}\right)}}$$

which varies from 2×10^{-5} if the phase transition occurs at radiation dust equality, to 4×10^{-5} if it occurs at last scattering.

3. STATISTICAL DARK ENERGY DE-COHERENCE

One type of physical system for which vacuum energy density directly manifests is the set that exhibit the Casimir effect. Lifshitz and his collaborators demonstrated that the Casimir force can be thought of as the superposition of the van der Waals attractions between individual molecules that make up the attracting media resulting from the zero-point motions of the sources. Since these motions are inherently a quantum effect for systems which manifest vacuum energy, one expects space-like correlations consistent with a quantum phenomenon.

A weakly interacting sea of the quantum fluctuations due to zero point motions should establish statistical variations in this ``dark energy" density during de-coherence. One should be able to use simple counting arguments to quantify these variations. Consider a partitioning of the system as demonstrated in the figure.



If the zero-point motions of the sources have a statistical weight $\Omega(E_A)$ associated with a partition A having energy E_A while holding total energy fixed, then the probability of such a partitioning is given by

$$P(E_A) = \frac{\Omega(E_A)}{\Omega_{tot}} = \frac{\Omega_A(E_A)\Omega_{\bar{A}}(E_{tot} - E_A)}{\Omega_{tot}}$$

where $\overline{A}$ represents all external to the A partition.

Requiring then that the most likely configuration of energy partitions results when (the log of) this probability is maximized, this distribution gives a uniform dark energy distribution ($E_\Lambda{}^A = E_\Lambda{}^{\overline{A}}$) if the dark energy E_Λ is given by

$$\frac{1}{E_\Lambda} \equiv \frac{d}{dE}\log\Omega(E).$$

Here E_Λ is an intensive energy associated with the statistical bath and boundary conditions. This result is of course analogous to the zeroth law of thermodynamics.

If one next examines a "small" partition A for which the reservoir has energy E_{tot}-E_A, one can examine the (log of the) lowest order partitioning of energy from the reservoir to the partition A to show

$$\Omega_{\overline{A}}(E_{tot} - E_A) \cong \Omega_{\overline{A}}(E_{tot})\, e^{-E_A/E_\Lambda},$$

thus defining a probability distribution

$$P(E) = \frac{e^{-E/E_\Lambda}}{\sum_{E'} e^{-E'/E_\Lambda}}.$$

For such an ensemble, one can immediately show that

$$\left\langle (\delta E)^2 \right\rangle = E_\Lambda^2 \frac{d}{dE_\Lambda}\left\langle E \right\rangle.$$

A typical equation of state will connect the extensive variable $\langle E \rangle$ to a dimensionless extensive variable that counts the available degrees of freedom N_{DoF}. On dimensional grounds, the terms in a typical equation of state which depend on E_Λ should take the general form $E=N_{DoF}\,(E_\Lambda)^a / \varepsilon^{a-1}$, where ε is a system dependent constant with dimensions of energy. The expected fluctuations are then given by

$$\frac{\left\langle (\delta E)^2 \right\rangle}{\left\langle E \right\rangle^2} = a\frac{E_\Lambda}{E} \sim \frac{1}{N_{DoF}}.$$

In terms of the densities, one can directly write

$$\frac{\left\langle (\delta E)^2 \right\rangle}{\left\langle E \right\rangle^2} = \frac{\left\langle (\delta \rho)^2 \right\rangle}{\left\langle \rho \right\rangle^2} \sim \frac{\rho_\Lambda}{\rho}.$$

This means that the amplitude of relative fluctuations δρ/ρ is expected to be of the order

$$\Delta \equiv \left(\frac{\rho_\Lambda}{\rho}\right)^{1/2}.$$

4. AN EXAMPLE: COLD DARK BOSONIC MATTER

As an example of such a phase transition, consider cold dark bosonic matter made up of particles of mass m. For non-relativistic bosonic dark matter, the relationship between number density and critical density for a free bose gas is given by

$$\frac{N}{V} = \frac{\zeta(3/2)\Gamma(3/2)}{(2\pi)^2\hbar^3}\left(2mk_B T_{crit}\right)^{3/2}.$$

Since the dynamics is assumed non-relativistic, $\rho_m \cong \frac{N}{V}mc^2$, giving the following requirement for a macroscopic quantum system made up of Bose condensed cold dark matter:

$$\left(mc^2\right)^{5/2} < \frac{\rho_m}{\left(2k_B T_{crit}\right)^{3/2}}\frac{(2\pi)^2(\hbar c)^3}{\zeta(3/2)\Gamma(3/2)}.$$

In order for the macroscopic space-like quantum coherent state to persist, the ambient temperature must be less than this critical temperature. If the phase transition occurs while the dark matter is cold (i.e. non-relativistic), its density can be assumed to depend on the redshift by $\rho_m = \rho_{mo}(1+z)^3$. The temperature of the photon gas is expected to likewise scale with the redshift when appropriate pair creation threshold affects are properly incorporated. Substitution into the critical equation gives

$$mc^2 < (1+z)^{3/5}\left(\frac{g(z)}{g(0)}\right)^{3/20} \times 1.2\times 10^{-11}\,\text{GeV}.$$

where $g(z)$ counts the number of low mass degrees of freedom available at redshift z. If the transition occurs at last scattering, this mass must be as low as 0.8 eV.

5. CONCLUSION

When global gravitational coherence of the dark energy is lost, only local coherence of microscopic degrees of freedom within independent clusters is expected to remain, and the dark energy scale coherence with the clusters is lost as the new degrees of freedom become available. The effect of dark energy density at de-coherence is ``frozen out" as a positive cosmological constant. The predicted order of magnitude for the amplitude of CMB fluctuations has been shown to be independent of this scale (and by inference, independent of the energy density) at de-coherence. The dark energy de-coherence hypothesis defines a class of cosmological models all of which give an amplitude of density fluctuations in the CMB expected to be of the order 10^{-5}.

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For more on this poster, see

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