# General Relativistic Radiative Transfer in Tori

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We investigate black hole systems where the accretion rate approaches the Eddington limit. This means that the thin disk approximation no longer holds, and the flow puffs up into a three dimensional configuration. The result are thick tori with a wide range of optical depths. We use a new covariant moment method to calculate images of these tori around Kerr black holes where the effects of electron scattering are dominant. We also investigate the relativistic smearing of line emission from these objects.

## 1. INTRODUCTION

The strong X-rays observed in active galactic nuclei (AGN) and some X-ray binaries are believed to be powered by accretion of material into black holes. The curved space-time around the black hole influences not only the accretion hydrodynamics but also the transport of radiation from the accretion flow.

Far from the central black hole, it is generally assumed the material forms a thin accretion disk. [5] A relativistic version of the thin disk model was developed by Novikov and Thorne. [4] However, it is known that the assumptions used in deriving the thin disk model break down at high accretion rates near the black hole. Radiation pressure becomes important, and can no longer be ignored. The accretion disk puffs up out of the equatorial plane, and no longer is thin.

Various models of thick disks and tori exist in the literature. For descriptions of the physics of Newtonian tori, see Frank, King and Raine.[2] General relativity modifies the effective potential the gas in the torus fills. Abramowicz, Jaroszy/'nski and Sikora [1] derived the form of this modified potential. Their formulation can be simplified when the angular momentum is constant, and thus that case has been well explored in the literature.

This paper explores the case where a powerlaw-modified Keplerian profile is used. However, the same formulation can be applied to arbitrary angular momentum and velocity laws.

Using a model of the distribution of material in the tori together with their velocity fields and temperature distributions, it is possible to calculate the emissivity and opacity as a function of position. Combined with the theory of relativistic radiative transfer, it is then possible to determine what will be observed from afar.

We calculate what images would be seen if we had an ultra-high resolution telescope in the X-ray region of the spectrum. We then integrate the flux over the images to produce spectra

## 2. RELATIVISTIC RADIATIVE TRANSFER

The radiative transfer equation in curved space-time without scattering reads

$$\frac{d\mathcal{I}}{d\lambda} = -k_\alpha u^\alpha|_\lambda \left[-\chi_0(x^\beta,\nu) + \eta_0(x^\beta,\nu)\right] \qquad (1)$$

where $\mathcal{I} = I/\nu^3$ is the Lorentz invariant intensity, $\chi_0(x^\beta,\nu)$ is the absorption coefficient in the rest frame of the material, $\eta_0(x^\beta,\nu)$ is the emissivity in the rest frame of the material, and $\lambda$ is an affine parameter for the ray path.

The simplest way to solve the radiative transfer equation is via the method of ray-tracing. Ray tracing works by following the geodesics of the photons backwards in time from the observer to the emission region. If the optical depth is integrated along the path for each frequency, it is possible to calculate the emission seen at the observer for each path element along the ray.[3]

By creating an image, the effects of gravitational lensing are implicitly included. The image can be integrated over to produce a spectrum. These spectra can then be compared with those of AGN observed by satellites such as *XMM-Newton* and *Chandra*.

## 3. RELATIVISTIC TORI

We approximate the angular velocity of the gas orbiting the black hole by assuming the form is Keplerian modified by a power law.

$$\omega = \frac{1}{(r\sin\theta)^{3/2} + a}\left(\frac{r_k}{r\sin\theta}\right)^n = \frac{\dot{\phi}}{\dot{t}} \qquad (2)$$

From this it is possible to calculate the acceleration as a function of position, and thus the equipotential which describes the largest torus with these parameters. There is a maximal size any given velocity profile has a marginally stable orbit around the black hole, setting a lower limit on the radius of the inner edge. Note that it is also possible to parametrise the angular momentum as a function of position instead, where similar results are obtained.

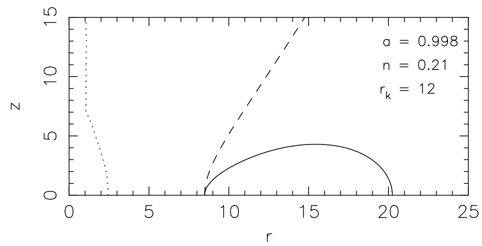


Figure 1: This graph shows the locations of the light cylinder (dotted), the marginally stable orbit (dashed) and the outer surface of the torus. The parameters are $n = 0.21, r = 12$ and the black hole has a spin parameter $a = 0.998$

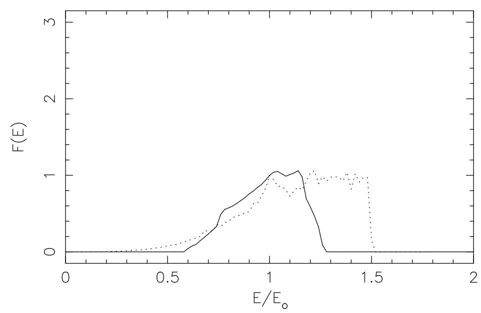


Figure 2: A comparison of the line profiles from a thin accretion disk (dotted), and an optically thick accretion torus (solid) viewed at an inclination angle of 85°.

Using Boyer-Lindquist coordinates, the acceleration in terms of the four velocity components is

$$
\begin{aligned}
-\frac{\Sigma}{\Delta}a^r &= \frac{\Sigma - 2r}{\Sigma^2}\left(\dot{t} - a\sin\theta\dot{\phi}\right)^2 + r\sin^2\theta\dot{\phi} \qquad (3)\\
-\Sigma a^\theta &= \sin\theta\cos\theta\left[\frac{2r}{\Sigma^2}\left(a\dot{t} - (r^2+a^2)\dot{\phi}\right)^2 + \Delta\dot{\phi}^2\right],
\end{aligned}
$$

where $\dot{t}$ is the lorentz factor, and $\dot{\phi}$ is defined in equation 2.

This yields the outer surface for an optically thick torus. Given an emissivity law for a line on the torus surface, such as $I \propto r^{-2}$, it is possible to calculate the observed line profile. These line profiles will be different to those of accretion disks because tori self-obscure. The inner part of the torus, corresponding to the fastest moving gas, is obscured at some inclination angles so the corresponding wings of the line profile are reduced. See fig. 2. This produes a line profile that looks like a broad hump instead of a wedge shape. This may explain why not many sources with asymmetric lines like MCG-6-30-15 are observed.

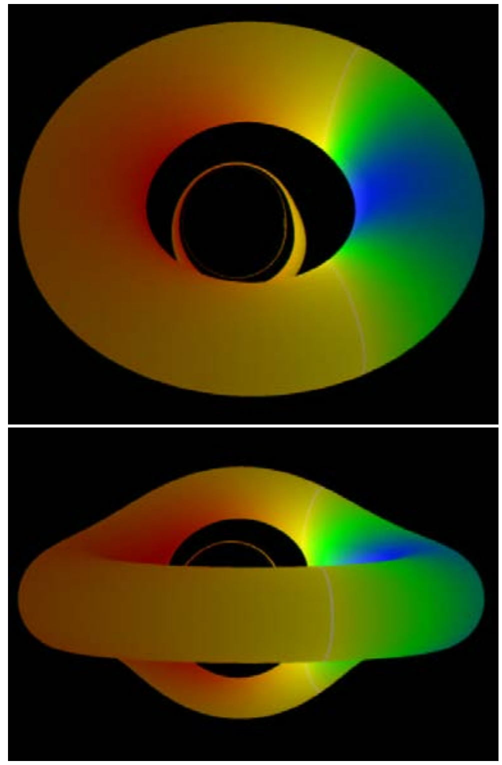

Figure 3: The frequency-shift colour image of a torus around a Kerr black hole viewed at inclination angles of 45° and 85°. At high inclination angles, the inner surface is obscured, altering the observed spectrum.

## 4. A FUZZY TORUS

If an equation of state is used, it is possible to calculate the density of the gas as a function of position. We assume that the relative ratio of radiation pressure to gas pressure is constant throughout the torus, and use this to derive an equation of state.

$$\kappa = \frac{h}{2\pi}\left[\frac{45(1-\beta)}{\pi^2(\mu m_H \beta)^4}\right]^{1/3}, \qquad (4)$$

$\Gamma = 4/3$ and $\beta$ is the fraction of gas pressure relative to the total pressure.

Then, by using the equation of hydrostatic equilibrium derived from the stress-energy tensor of an ideal fluid we obtain,

$$\frac{\partial\rho}{\partial x^\alpha} = a_\alpha\left(\frac{\rho^{2-\Gamma}}{\kappa\Gamma} + \frac{\rho}{\Gamma - 1}\right). \qquad (5)$$

Using the $a^\alpha$ in equation 3, the equation for the density of the gas can be solved numerically. Using the equation of state, we can then calculate the temperature, and thus the emissivity and opacity for the radiative transfer equation.

Figure 4 shows a case where absorption has been included proportional to the density of the gas. This

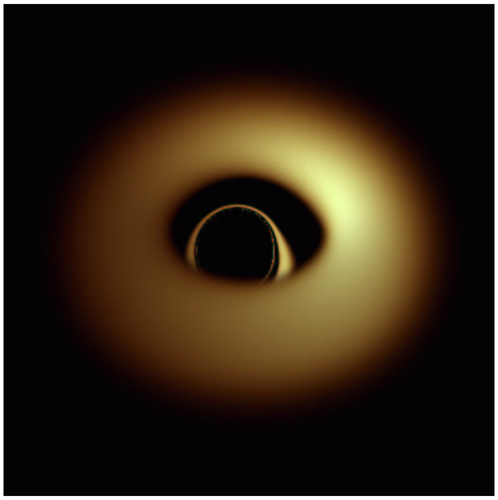

Figure 4: Bolometric flux image of an accretion torus around a Kerr black hole with an inclination angle of 45°. Absorption has been included proportional to the density of the gas. Limb darkening can be seen, and the first two higher order images can be seen in the middle of the image, where the light has orbited the black hole on its way to the observer.

produces limb darkening automatically. Note how the first two higher order images can be seen in the middle of the image, where the light has orbited the black hole on its way to the observer. This is an example of how extreme gravitational lensing alters what is seen.

These simple analytic models of emissivity and opacity as a function of position depend on the arbitrary velocity law chosen for the flow. This degree of freedom can be relaxed by moving to hydrodynamic simulations of accretion flow. However, this requires vastly greater computational resources, especially if the radiation reaction force is included in the calculations.

## 5. TORI WITH SCATTERING

For a more correct model of tori around black holes, the effects of electron scattering must be included, since the scattering opacity dominates over free-free absorption. The ray-tracing method doesn't handle the non-local behaviour of the scattering term very well. It is possible to use a Monte-Carlo technique to model scattering at low optical depths. However, it takes exponentially more rays to model what is seen as scattering opacity rises.

The solution is to use a moment method to appoximate the radiation field as a function of angle at each location. We use a simplified version of the Thorne (1981) method where the series of moments is telescoped into one term. This yields the following covariant version of Eddingtons approximation at first order.[6]

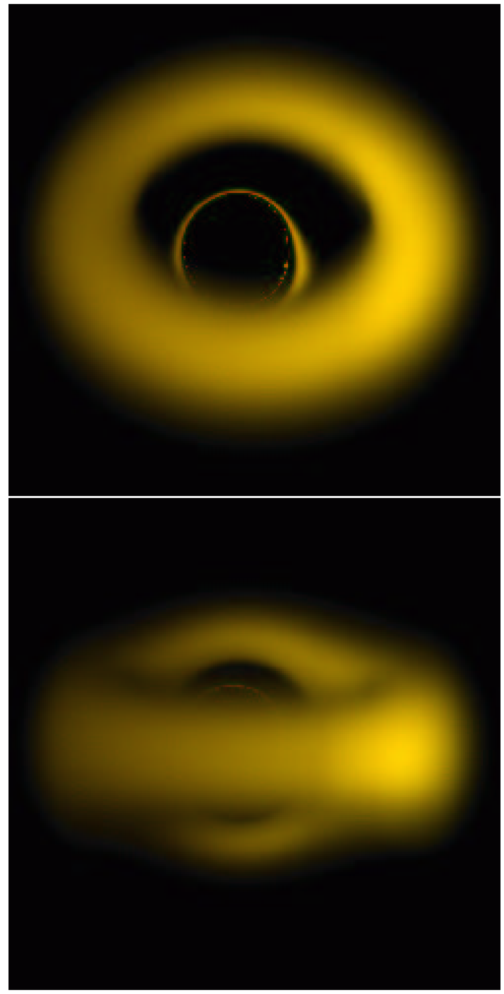

Figure 5: Accretion torus images around a Kerr black hole with an inclination angle of 45° and 85°. These images are of tori where the effects of free-free emission, absorption and electron scattering have been taken into account in the radiative transfer.

$$m_\alpha \left[ J^\alpha_{,\beta} m^\beta + 2\Gamma^\alpha_{\delta\beta} J^\beta m^\delta + \left(\frac{1}{E^2}\frac{\partial E}{\partial \lambda}\right)\left(\frac{\partial J^\alpha}{\partial(\ln E)} - J^\alpha\right)\right] = -\left(\chi_0 + \rho\sigma_T\right) J^\alpha m_\alpha + \eta_0 + \rho\sigma_T J^\alpha u_\alpha, \tag{6}$$

where $J^\alpha(E, x^\alpha)$ is a linear combination of the zeroth and first order moments of the radiation field. $u^\alpha$ is the bulk fluid velocity, and $m^\alpha = k^\alpha/E$, with $E$ the photon energy.

We solve the moment equation using a second order implicit upwind numerical method. The results are then used by a ray-tracer to evaluate the flux scattered into the line of sight along each ray. The moment method works best at high optical depths, and the ray tracer works best in the free streaming limit. By combining them in the same algorithm we obtain the best of both.

Future work will include changing to a 3D numerical scheme, and moving to higher order accuracy in the moment method. Another goal is the addition of this radiative transfer method to hydrodynamics codes so that radiation reaction forces are included in the dynamics. Finally, the moment method can be rederived to apply to polarized radiative transport.

## Acknowledgments

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