Shadows (Mirages) Around Black Holes and Retro Gravitational Lensing

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Recently Holz & Wheeler [1] considered a very attracting possibility to detect retro-MACHOs, i.e. retro-images of the Sun by a Schwarzschild black hole. In this paper we discuss glories (mirages) formed near rapidly rotating Kerr black hole horizons and propose a procedure to measure masses and rotation parameters analyzing these forms of mirages (a detailed description of the problem is given in [2]). In some sense that is a manifestation of gravitational lens effect in the strong gravitational field near black hole horizon and a generalization of the retro-gravitational lens phenomenon. We analyze the case of a Kerr black hole rotating at arbitrary speed for some selected positions of a distant observer with respect to the equatorial plane of a Kerr black hole. Some time ago Falcke, Melia & Agol [3] suggested to search shadows at the Galactic Center. In this paper we present the boundaries for shadows calculated numerically. We also propose to use future radio interferometer RADIOASTRON facilities to measure shapes of mirages (glories) and to evaluate the black hole spin as a function of the position angle of a distant observer.

Recently Holz & Wheeler [1] have suggested that a Schwarzschild black hole may form retro-images (called retro-MACHOs) if it is illuminated by the Sun. We analyze a rapidly rotating Kerr black hole case for some selected positions of a distant observer with respect to the equatorial plane of the Kerr black hole. We discuss glories (mirages) formed near a rapidly rotating Kerr black hole horizon and propose a procedure to measure the mass and the black hole spin analyzing the mirage shapes. Since a source illuminating the black hole surroundings may be located in an arbitrary direction with respect to the observer line of sight, a generalization of the retro-gravitational lens idea suggested by Holz & Wheeler [1] is needed. A strong gravitational field approximation for a gravitational lens model was considered recently in several papers [4–16].

Here we will consider images formed by retro-photons, but in contrast to Holz & Wheeler [1] we will analyze forms of images near black holes but not a light curve of an image formed near black hole as Holz & Wheeler [1] did. In our consideration a location of source could be arbitrary in great part (in accordance with a geometry different parts of images could be formed),[1] for example, accretion flows (disks) could be sources forming such images. Since in such cases images formed by retro-photons are considered, we call it like retro gravitational lensing even if a source is located near a gravitational lens (a black hole) in contrast to a standard gravitational lens model.

[1]However, if a source is located between black hole and an observer, images formed by retro-photons and located near black holes could be non-detectable.

As usual, we use geometrical units with $G = c = 1$. It is convenient also to measure all distances in black hole masses, so we may set $M = 1$ (M is a black hole mass). Calculations of mirage forms are based on qualitative analysis of different types of photon geodesics in a Kerr metric (for references see [17–20]). In fact, we know that impact parameters of photons are very close to the critical ones (which correspond to parabolic orbits). One can find some samples of photon trajectories in [18, 21]. This set (critical curve) of impact parameters separates escape and plunge orbits (see [17–20] for details) or otherwise the critical curve separates scatter and capture regions for unbounded photon trajectories. Therefore the mirage shapes almost look like to critical curves but are just reflected with respect to z-axis. We assume that mirages of all orders almost coincide and form only one quasi-ring from the point of view of the observer. We know that the impact parameter corresponding to the π deflection is close to that corresponding to a $n\pi$ deflections (n is an odd number). For more details see [1] (astronomical applications of this idea was discussed by De Paolis et al. [22] and its generalizations for Kerr black hole are considered by De Paolis et al. [23]). We use prefix "quasi" since we consider a Kerr black hole case, so that mirage shapes are not circular rings but Kerr ones. Moreover, the side which is formed by co-moving (or co-rotating) photons is much brighter than the opposite side since rotation of a black hole squeeze deviations between geodesics because of Lense - Thirring effect. Otherwise, rotation stretches deviations between geodesics for counter-moving photons.

The full classification of geodesic types for Kerr metric is given in [19]. As it was shown in this paper, there are three photon geodesic types: capture, scattering and critical curve which separates the first two sets. This classification fully depends only on two parameters $\xi = L_z/E$ and $\eta = Q/E^2$, which are known as Chandrasekhar's constants [18]. Here the Carter constant Q is given by Carter [24]

$$Q = p_\theta^2 + \cos^2\theta \left[a^2\left(m^2 - E^2\right) + L_z^2/\sin^2\theta\right], \quad (1)$$

where $E = p_t$ is the particle energy at infinity, $L_z = p_\phi$ is z-component of its angular momentum, $m = p_i p^i$ is the particle mass. Therefore, since photons have $m = 0$

$$\eta = p_\theta^2/E^2 + \cos^2\theta \left[-a^2 + \xi^2/\sin^2\theta\right]. \quad (2)$$

The first integral for the equation of photon motion (isotropic geodesics) for a radial coordinate in the Kerr metric is described by the following equation [18, 19, 24, 25]

$$\rho^4 (dr/d\lambda)^2 = R(r), \quad (3)$$

where

$$R(r) = r^4 + (a^2 - \xi^2 - \eta)r^2 + 2[\eta + (\xi - a)^2]r - a^2\eta,$$

and $\rho^2 = r^2 + a^2\cos^2\theta, \Delta = r^2 - 2r + a^2, a = S/M^2$. The constants M and S are the black hole mass and angular momentum, respectively. Eq. (3) is written in dimensionless variables (all lengths are expressed in black hole mass units M).

We will consider different types of geodesics on r - coordinate in spite of the fact that these type of geodesics were discussed in a number of papers and books, in particular in a classical monograph by Chandrasekhar [18] (where the most suited analysis for our goals was given). However, our consideration is differed even from Chandrasekhar's analysis in the following items.

i) Chandrasekhar [18] considered the set of critical geodesics separating capture and scatter regions as parametric functions $\eta(r), \eta(r)$, but not as the function $\eta(\xi)$ (as we do). However, we believe that a direct presentation of function $\eta(\xi)$ is much more clear and give a vivid illustration of different types of motion. Moreover, one could obtain directly form of mirages from the function $\eta(\xi)$ (as it will be explained below).

ii) Chandrasekhar [18] considered the function $\eta(r)$ also for $\eta < 0$ and that is not quit correct, because for $\eta < 0$ allowed constants of motion correspond only to capture (as it was mentioned in the book [18]). This point will be briefly discussed below.

If we fix a black hole spin parameter a and consider a plane (ξ, η) and different types of photon trajectories corresponding to (ξ, η), namely, a capture region, a scatter region and the critical curve $\eta_{\rm crit}(\xi)$ separating the scatter and capture regions. The critical curve is a set of (ξ, η) where the polynomial $R(r)$ has a multiple root (a double root for this case). Thus, the critical curve $\eta_{\rm crit}(\xi)$ could be determined from the system [19, 25]

$$R(r) = 0,$$
$$\frac{\partial R}{\partial r}(r) = 0, \quad (4)$$

for $\eta \geq 0, r \geq r_+ = 1 + \sqrt{1 - a^2}$, because by analyzing of trajectories along the θ coordinate we know that for $\eta < 0$ we have $M = \{(\xi, \eta) | \eta \geq -a^2 + 2a|\xi| - \xi^2, -a \leq \xi \leq a\}$ and for each point $(\xi, \eta) \in M$ photons will be captured. If instead $\eta < 0$ and $(\xi, \eta) \bar{\in} M$, photons cannot have such constants of motion, corresponding to the forbidden region (see, [18, 19] for details).

One can therefore calculate the critical curve $\eta(\xi)$ which separates the capture and the scattering regions [19, 25]. We remind that the maximal value for $\eta_{\rm crit}(\xi)$ is equal to 27 and is reached at $\xi = -2a$. Obviously, if $a \to 0$, the well-known critical value for Schwarzschild black hole (with $a = 0$) is obtained.

Thus, at first, we calculate the critical curves for chosen spin parameters a which are shown in Fig. 1 (the critical curves for other spin parameters are presented in [2]). The shape of the critical curve for $a = 0$ (Schwarzschild black hole) is well-known because for this case we have $\eta_{\rm crit}(\xi) = 27 - \xi^2$ for $|\xi| \leqslant 3\sqrt{3}$, but we show the critical curve to compare with the other cases.

By following this approach we can find the set of critical impact parameters (α, β), for the image (mirage or glory) around a rotating black hole. The sets of critical parameters form caustics around black holes and it is well-known that caustics are the brightest part of each image (numerical simulations of caustic formations were done by Rauch & Blandford [26]). We remind that (α, β) parameters could be evaluated in terms of $(\xi, \eta_{\rm crit})$ by the following way [18]

$$\alpha(\xi) = \xi/\sin\theta_0, \quad (5)$$

$$\begin{aligned}\beta(\xi) &= (\eta_{\rm crit}(\xi) + a^2\cos^2\theta_0 - \xi^2\cot^2\theta_0)^{1/2} \\ &= (\eta_{\rm crit}(\xi) + (a^2 - \alpha^2(\xi))\cos^2\theta_0)^{1/2}. \quad (6)\end{aligned}$$

Actually, the mirage shapes are boundaries for shadows considered by Falcke, Melia & Agol [3] (see also [27]).

We note that the precision we obtain by considering critical impact parameters instead of their exact values for photon trajectories reaching the observer is good enough. In particular, co-rotating photons form much brighter part of images with respect to retrograde photons. Of course, the larger is the black hole spin parameter the larger is this effect (i.e. the co-rotating part of the images become closest to the black hole horizon and brighter).

This approximation is based not only on numerical simulation results of photon propagation [28–44]

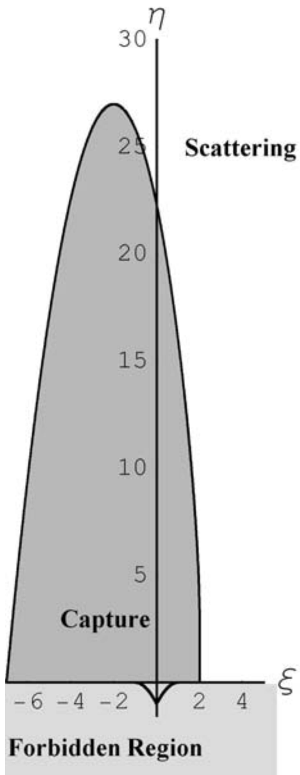

Figure 1: Different types for photon trajectories and an extreme spin parameter ($a = 1.,$). Critical curves separate capture and scatter regions. Here we show also the forbidden region corresponding to constants of motion $\eta < 0$ and $(\xi, \eta) \bar{\in} M$ as it was discussed in the text.

(about 10^9 photon trajectories were analyzed) but also on analytical results (see, for example [18, 19]).

Let us assume that the observer is located in the equatorial plane ($\theta = \pi/2.$). For this case we have from Eqs. (5) and (6)

$$\alpha(\xi) = \xi, \qquad (7)$$

$$\beta(\xi) = \sqrt{\eta_{\rm crit}(\xi)}. \qquad (8)$$

As mentioned earlier, the maximum impact value $\beta = 3\sqrt{3}$ corresponds to $\alpha = -2a$ and if we consider the extreme spin parameter $a = 1$ a segment of straight line $\alpha = 2, 0 < |\beta| < \sqrt{3}$ belongs to the mirage (see images in Fig. 2 for different spin parameters). It is clear that for this case one could easy evaluate the black hole spin parameter after the mirage shape reconstruction since we have a rather strong dependence of the shapes on spins. As it was explained earlier, the maximum absolute value for $|\beta| = \sqrt{27} \approx 5.196$ corresponds to $\alpha = -2a$ since the maximum value for $\eta(\xi)$ corresponds to $\eta(-2a) = 27$ as it was found by

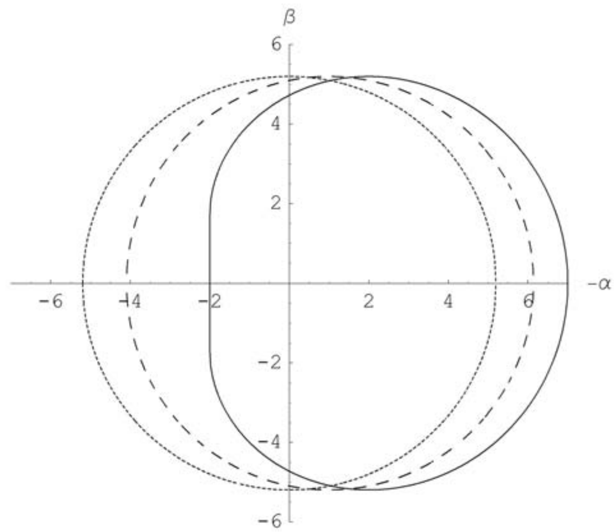

Figure 2: Mirages around black hole for equatorial position of distant observer and different spin parameters. The solid line, the dashed line and the dotted line correspond to $a = 1, a = 0.5, a = 0$ correspondingly.

Zakharov [19]. Therefore, in principle it is possible to estimate the black hole spin parameter by measuring the position of the maximum value for β, but probably that part of the mirage could be too faint to be detected.

Let us consider different angular positions of a distant observer $\theta = \pi/2, \pi/3, \pi/4$ and $\pi/6$ for the extreme spin parameter (Fig. 3). From Fig. 3 (and other Figs. presented in [2]) one can see that angular positions of a distant observer could be evaluated from the mirage shapes only for rapidly rotating black holes ($a \sim 1$), but there are no chances to evaluate the angles for slowly rotating black holes, because even for $a = 0.5$ the mirage shape differences are too small to be distinguishable by observations. Indeed, mirage shapes weakly depend on the observer angle position for moderate values of a black hole spin.

According to the schedule the space radio telescope RADIOASTRON will be launched in 2006 or 2007. This space based 10-meter radio telescope will be used for space – ground VLBI measurements. The measurements will have extraordinary high angular resolutions, namely about 1 – 10 microarcseconds (in particular about 8 microarcseconds at the shortest wavelength 1.35 cm and a standard orbit and could be about 0.9 microarcseconds for the high orbit at the same wavelength. For observations four wave bands will be used corresponding to $\lambda = 1.35$ cm, $\lambda = 6.2$ cm, $\lambda = 18$ cm, $\lambda = 92$ cm.

The fringe sizes (in micro arc seconds) for the apogee of the above-mentioned orbit and for all RADIOASTRON bands are given in Table 1.

Thus, there are non-negligible chances to observe such mirages around the black hole at the Galactic

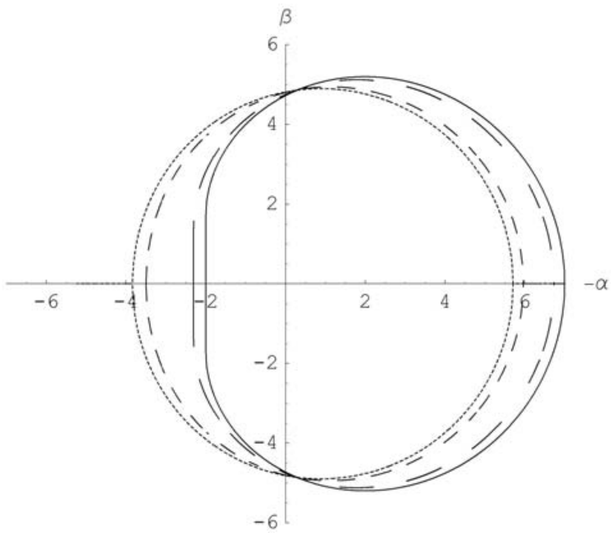

Figure 3: Mirages around black hole for different angular positions of a distant observer and the spin $a = 1$. Solid, long dashed, short dashed and dotted lines correspond to $\theta_0 = \pi/2, \pi/3, \pi/6$ and $\pi/8$, respectively.

Table I The fringe sizes (in micro arc seconds) for the standard and advanced apogees B_{max} (350 000 and 3 200 000 km correspondingly).

B_{max}(km)\\λ(cm)	92	18	6.2	1.35
3.5×10^5	540	106	37	8
3.2×10^6	59	12	4	0.9

Center and in nearby AGNs and microquasars in the radio-band using RADIOASTRON facilities.

Observations of Sgr A* in radio, near-infrared and X-ray spectral bands develop very rapidly [45–53][2] also because it harbours the closest massive black hole. The mass of this black holes is estimated to be $4 \times 10^6\ M_\odot$ [52, 55–57] and its intrinsic size from VLBA observations at wavelengths $\lambda = 2$ cm, 1.3 cm, 0.6 cm and 0.3 cm [52].

Similarly to Falcke, Melia & Agol [3] we propose to use VLBI technique to observe the discussed mirages around black holes. They used ray-tracing calculations to evaluate the shapes of shadows. The boundaries of the shadows are black hole mirages (glories or "faces") analyzed earlier. We use the length parameter $r_g = \dfrac{GM}{c^2} = 6 \times 10^{11}$ cm to calculate all values in these units as it was explained in the text. If we take into account the distance towards the Galactic Center $D_{\rm GC} = 8$ kpc then the length r_g corresponds to angular sizes $\sim 5\mu$as. Since the minimum arc size for the considered mirages are about $2r_g$, the standard RADIOASTRON resolution of about 8 μas is comparable with the required precision. The resolution in the case of the higher orbit and shortest wavelength is $\sim 1\mu$as (Table 2) good enough to reconstruct the shapes. Therefore, in principle it will be possible to evaluate a and θ parameters after mirage shape reconstructions from observational data even if we will observe only the bright part of the image (the bright arc) corresponding to positive parameters α. However, Gammie, Shapiro & McKinney [58] showed that black hole spin is usually not very small and could reach $0.7 - 0.9$ (numerical simulations of relativistic magnetohydrodynamic flows give $a \sim 0.9$). Taking into account detections of 106 day cycle in Sgr A* radio variability seen at 1.3 cm and 2.0 cm by Zhao, Bower & Goss [59] at Very Large Array (VLA), Liu & Melia [60] suggested a procedure to evaluate the black hole spin assuming that the variability could be caused by spin induced disk precession. Moreover, the recent analysis by Aschenbach et al. [61] of periodicity of X-ray flares from the Galactic Center black hole gives an estimate for the spin as high as $a = 0.9939^{+0.0026}_{-0.0074}$. Actually, the authors used generalizations of the idea proposed by Melia et al. [62] that the minimum rotation period for Schwarzschild black hole (for an assumed black hole mass of $2.6 \times 10^6 M_\odot$) is about $P_0 \approx 20$ minutes and could be in the range $P_0 \in [2.6, 36]$ minutes depending on the black hole spin and prograde and retrograde accretion flows generating the quasi-periodic oscillations. Using this idea and analyzing quasi-periodic variabilities in a infrared band Genzel et al. [47] concluded that the black hole spin should be $a \sim 0.5$. However, this conclusion is based on the assumption that the emitting region is located at the marginally stable orbit, therefore if the periodicity is related to the emitting gas motion around the black hole, we should conclude that the black hole spin is $a \gtrsim 0.5$. One could also mention that such a determination of the black hole spin is indirect and actual typical frequencies for real accretion flows could be rather different from frequencies considered by the authors. We may summarize by saying that there are indications that the spin of the Galactic Center black hole can be very high, although this problem is not completely solved up to date.

As stated earlier, the part of Kerr quasi-rings formed by co-rotating photons is much brighter with respect to the opposite side (i.e. the part of the image formed by counter-rotating photons) and in principle can be detected much more easily. However, even the bright part of the quasi-ring can give information about mass, rotation parameter and inclination angle of the black hole. Of course, if the black hole - observer distance is unknown, the black hole mass can be evaluated in units of the distance. Even if the faint

[2] An interesting idea to use radio pulsars to test a region near black hole horizon was proposed in [54].

part of image (which is formed by counter-rotating photons) is not detectable, one can try to reconstruct the shape of the total image searching for the best fit of the full image using only the bright part of the image.

We could summarize that angular resolution of the space RADIOASTRON interferometer will be high enough to resolve radio images around black holes therefore analyzing the shapes of the images one could evaluate the mass and the spin a for the Kerr black hole inside the Galactic Center and a position angle θ_0 for a distant observer and as it is clear a position angle could be determined by more simple way for rapidly rotating black holes $a \sim 1$ (in principle, measuring the mirage shapes we could evaluate mass, inclination angle and spin parameter if we know the distance toward the observed black hole. Otherwise one can only evaluate the spin parameter in units of the black hole mass since even for not very small spin $a = 0.5$ we have very weak dependence on θ_0 angle for mirage shapes and hardly ever one could determine θ_0 angle from the mirage shape analysis. Moreover, we have a chance to evaluate parameters a and θ (for rapidly rotating black holes) if we reconstruct only bright part of the mirages (bright arcs) corresponding to co-moving photons ($\alpha > 0$). However, for slow rotating black holes $\alpha \lesssim 0.5$ it would be difficult to evaluate parameters a and θ because we have very slow dependence of mirage shapes on these parameters.

However, there are two kind of difficulties to measure mirage shapes around black holes. First, the luminosity of these images or their parts (arcs) may not be sufficient to being detectable by RADIOASTRON. However, numerical simulations by Falcke, Melia & Agol [3], Melia & Falcke [55] give hope that the luminosity could be not too small at least for arcs of images formed by co-rotating photons ($\alpha > 0$).

Recent observations of simultaneous X-ray and radio flares at 3 mm, 7 mm, 1.3 cm and 2 cm with the few-hundred second rise/fall timescales gave indirect evidences that X-ray and radio radiation from the close vicinity of Sgr A* was detected because of that is the most natural interpretation of these flares. However, another interpretations of these flares could not be ruled out and in this case an optical depth for radio waves at 1.3 cm wavelength toward Sgr A* may be not very small.

Few years ago a possibility to get images of nearby black holes in X-ray band was discussed by White [64], Cash et al. [65], moreover Cash et al. [65] presented a laboratory demonstration of the X-ray interferometer. If the project will be realized, one could get X-ray images of black holes with 0.1×10^{-6} arcsec resolution, thus using this tool one could detect X-ray images around the Galactic Centre and around the black hole in M87 Galaxy.

One could mention also that if the emitting region has a degenerate position with respect to the line of sight (for example, the inclination angle of an accretion disk is $\gtrsim 85^0$) strong bending effects found by Matt, Perolla & Stella [66] and analyzed later by Zakharov & Repin [38] do appear.

In spite of the difficulties of measuring the shapes of images near black holes is so attractive challenge to look at the "faces" of black holes because namely the mirages outline the "faces" and correspond to fully general relativistic description of a region near black hole horizon without any assumption about a specific model for astrophysical processes around black holes (of course we assume that there are sources illuminating black hole surroundings). No doubt that the rapid growth of observational facilities will give a chance to measure the mirage shapes using not only RADIOASTRON facilities but using also other instruments and spectral bands (for example, X-ray interferometer [64, 65] or sub-mm VLBI array [67]).

A detailed description of the problem and additional figures illustrating the consideration could be found in [2].

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