Gravitational Radiation from Rotating White Dwarfs and Neutron Stars

D.M. Sedrakian[1], M. Benacquista[2], K. Shahabasyan[1], M. Hayrapetian[1], A.A.Sadoyan[1]

[1]Yerevan State University, Alex Manoogian 1,375025, Armenia
[2]Montana State University-Billings, USA

Gravitational waves from isolated sources like single neutron stars or white dwarfs aren't as much intense as comparably very rare events in massive double systems, but the quantity of single stars and comparably small distances to Earth make these sources interesting from viewpoint of gravitational wave physics. We present our results of research on gravitational radiation from magnetized white dwarfs and neutron stars. We compare different energy sources for oscillation:

1) differential rotation energy for White dwarfs,
2) deformation energy of configuration for White Dwarfs and Neutron Stars
3) energy released during glitches for Neutron stars.

According to our estimations the most promising sources of gravitational waves should be Crab and Vela pulsars and may be G99-47 white dwarf. Registration of gravitational radiation from a given single source will open a new window for investigation of neutron stars and white dwarfs.

1. INTRODUCTION

A recent trend in modern astrophysics is the study of potential sources of gravitational radiation. Because gravitational waves are not absorbed by intervening matter, the observation of gravitational radiation from compact objects (neutron stars and white dwarfs), can provide additional information about the state of the matter and internal processes in their core.

2. GRAVITATIONAL WAVES FROM WHITE DWARFS

2.1. Magnetized White Dwarfs

There are a number of gravitational radiation detectors, planned, under construction, and operational covering a wide frequency spectrum from ~10^{-9}Hz all the way up to~10^4Hz. The coverage of the spectrum is not complete and the gap between space-based interferometer and ground-based interferometers has been proposed as a possible "window", devoid of continuous foreground sources, through which the cosmological background of gravitational radiation could be seen [1]. Magnetized rotating white dwarfs undergoing self-similar quasi-radial oscillations will emit gravitational radiation in the frequency range in 0.1-0.3 Hz [2]. Quasi-radial oscillations of rotating white dwarfs were investigated in the early 1970's [3,4] where the frequency spectrum of the fundamental oscillation mode for maximally rotating white dwarfs was determined. These stars are oblate due to their rotation and consequently they have a non-zero quadrupole moment. Oscillations add a time dependence to the quadrupole moment [5]. The oscillation is described by assigning each mass element a time dependent coordinate given by $x_\alpha = x_\alpha^0(1+\eta\sin\omega t)$ where $\eta << 1$ and is a constant. Thus, the reduced quadrupole moment is given by:

$$Q_{\alpha\beta} = \int \rho\left(x_\alpha x_\beta - \frac{1}{3}\delta_{\alpha\beta}x^2\right)d^3x \approx Q^0_{\alpha\beta}(1+2\eta\sin\omega t)$$

where $Q^0_{\alpha\beta}$ are the components of the quadrupole moment of the rotating oblate white dwarf in equilibrium and we have neglected terms of order η^2.The radiation power is

$$J = \frac{6G}{5c^5}\eta^2\omega^6\left|Q^0\right|^2\cos^2\omega t' = J_0\cos^2\omega t'$$

where the retarded time is $t' = t - r/c$ for a source at distance r. We can express the strain amplitude as [2]:

$$h_+ = \sqrt{\frac{15GJ_0}{2c^3}}\frac{1}{r\omega}sin^2\theta\, sin\,\omega t' = h_0\, sin^2\theta\, sin\,\omega t'$$

We assume that permanent source of energy to feed the gravitational radiation is deformation energy released during spin-down [2]. This energy is calculated in [6]. We relate the power of gravitational radiation to the decrease in deformation energy by: $J_0 = \beta\frac{W_g}{\tau}$, where β is "branching ratio" that quantifies the fraction of deformation energy that goes into gravitational radiation from fundamental mode, τ is a spin-down time.

Now let's turn to determination of spin-down time τ. We assume that white dwarf spins-down due to magneto-dipole radiation torque, which occurs if the magnetic field is oblique [8]. Observational data for 65 isolated white dwarfs indicates the magnetic field strength on the surface of these stars lies in the range ~$3{\cdot}10^4$ to ~10^9G [9]. If we define α to be the angle between the magnetic and rotation axes, the spin-down rate of the white dwarf is given by:

$$\dot{\Omega} = -\frac{2\mu^2\Omega^3}{3Ic^3}\sin^2\alpha$$

where $\mu = BR^3$ is the magnetic moment, B is the magnetic field strength. Then the characteristic time scale will be:

$$\tau = \frac{\Omega}{2|\dot{\Omega}|}$$

We calculate thermal energy losses due to magneto-hydrodynamic mechanism [7].

The expected gravitational energy flux F on earth (in erg.s^{-1}.cm^{-2}) for a population made entirely of each type of white dwarf is is calculated using formula

$$F = \frac{4\pi\rho_s hf(h) J_0}{4\pi\left(3\cdot10^{18}\right)^2},$$

where $h = 200pc$ and $f(h) = 6.15$ is calculated using Appendix A of [20]. Finally, we note that a simple average of the strain amplitudes in Table 3 gives $h_0 = 5.7\cdot10^{-26}$ and an average flux of $F = 8.3\cdot10^{-12} erg.s^{-1}.cm^{-2}$. The flux is spread out over a frequency band of $\nu_1 = 0.12$ to $\nu_2 = 0.32Hz$, and we can estimate an average strain amplitude for the galactic population of pulsating white dwarfs by using the angle and polarization averaged expression of and averaging over the frequency range $\Delta\nu = \nu_2 - \nu_1$ to obtain:

$$h_{0ave} = \frac{\ln \nu_2/\nu_1}{\Delta\nu}\sqrt{\frac{4GF}{\pi c^3}}$$

which gives $h_{0ave} = 7.94\cdot10^{-25}$.

Table 1: Gravitational waves from Differentially Rotating White Dwarfs(WD)

Name of WD	β	h_0	F
PG 1031+234	0.00184	$2.58\cdot10^{-28}$	$1.13\cdot10^{-15}$
EUVE J0317-855	0.90727	$9.69\cdot10^{-26}$	$6.04\cdot10^{-11}$
PG 1015+015	0.16853	$3.81\cdot10^{-28}$	$1.93\cdot10^{-15}$
Feige 7	0.80759	$1.47\cdot10^{-26}$	$3.96\cdot10^{-13}$
G99-47	0.99484	$3.45\cdot10^{-25}$	$5.84\cdot10^{-12}$
KPD 0253+5052	0.50833	$2.06\cdot10^{-28}$	$2.33\cdot10^{-16}$
PG 1312+098	0.41490	$9.38\cdot10^{-29}$	$1.56\cdot10^{-17}$
G217-037	0.99999	$8.97\cdot10^{-29}$	$8.19\cdot10^{-19}$

2.2. Differentially Rotating White Dwarfs

Now lets consider the differential rotation energy of the white dwarf as providing the energy to drive the oscillations. We presume that at birth the white dwarf was rotating differentially, with the angular velocity a function of the radius. Due to friction, the configuration will relax to uniform rotation after some time. The difference between the energies of the star in these two states we call the differential rotation energy. To describe a differentially rotating stellar configuration, one should give both the density distribution and the angular momentum distribution inside the star. Following Ostriker and Mark [10], we change to cylindrical coordinates where, according to the Poincare theorem, the angular momentum distribution depends only on the Lagrangian cylindrical mass u. This is the fraction of the total mass M for the configuration accumulated within a cylinder of radius $r_\perp$. For a star with radius R we have in cylindrical coordinates,

$$u(r_\perp) = \frac{2}{M}\int_0^R \int_0^{\sqrt{R^2 - r_\perp^2}} 2\pi r_\perp \rho(\sqrt{r_\perp^2 + z^2}) dr_\perp dz$$

where $\rho(r)$ is density distribution in spherical coordinate system.

Our calculations are made for the following two distributions of angular momentum [10]

$$l(u) = 5(1-(1-u(r_\perp)^{2/3})/2 \quad (1)$$

$$l(u) = 4.8239 + 1.8744(1-u)^{0.5622} - 6.6983(1-u)^{1/3} \quad (2)$$

The first distribution corresponds to rigidly rotating MacLoren spheroid, while the second interpolating formula corresponds with accuracy 1% to rigidly rotating polytrope with index n=1.5. If these distributions are applied for white dwarfs consisting of ultrarelativistic electronic gas where the density distribution is described by Line-Emden function of index 3, then they may correspond to strong differential rotation [10].

One can easily check that $\int_0^1 l(u)du = 1$ for both cases.

The available energy of differential rotation is equal to difference of rotating energy of differentially rotating star (E_{dr}) and rotating energy of uniformly rotating star($E_{ur} = I\Omega_0^2/2$). I is moment of inertia of the star, Ω_0 will be the angular velocity of uniformly rotating star if friction is zero. $\Omega_0 = const$ throughout the star as differential rotation had already dumped.

$$E_{diff} = 2\pi\int_0^R \int_0^{\sqrt{R^2 - r_\perp^2}} r_\perp \rho(\sqrt{r_\perp^2 + z^2})\left(\Omega^2(r_\perp) - \Omega_0^2\right) r_\perp^2 dr_\perp dz$$

where $\Omega(r_\perp)$ is angular velocity versus cylindrical radius. Angular velocity for each infinitesimally thin cylinder of radius $r_\perp$, mass $M\cdot du$ and angular momentum $L\cdot l(u)\cdot du$ equals to

Angular velocities of differential rotation is given by

$$\Omega(r_\perp) = 5I\Omega_0(1-(1-u(r_\perp)^{2/3})/2Mr_\perp^2$$

$$\Omega(r_\perp) = I\Omega_0(4.82 + 1.87(1-u(r_\perp))^{0.56} - 6.70(1-u(r_\perp))^{1/3})/Mr_\perp^2$$

where Ω_0 is the final angular velocity of white dwarf that is rotating uniformly. We consider Ω_0 to be equal Ω_k Keplerian angular velocity when there is an outflow of matter from equator of the star. For the same white dwarfs that were considered in 2.1, if driving energy of oscillations will be differential rotation we shall have gravitational waves described in Table 2

Table 2: Gravitational waves from Differentially Rotating White Dwarfs

White Dwarf	τ **(Gyr)**	**ho for dist. 1**	**ho for dist.2**
PG 1031+234	2,2	1.39E-27	1.14E-27
EUVE J0317-855	0,1	5.86E-26	4.72E-26
PG 1015+015	0,5	4.39E-27	3.56E-27
Feige 7	11,8	3.63E-27	3.00E-27
G99-47	11,8	4.89E-26	4.04E-26
KPD 0253+5052	2,2	2.18E-27	1.79E-27
PG 1312+098	2,2	2.68E-27	2.20E-27
G217-037	2,2	3.05E-26	2.50E-26
Averege		1.9E-26	1.6E-26

The average from the whole population of white dwarfs, if all were rotating differentially will be h_{oave}=2.0116E^{-25}

2.3. White Dwarfs with rough surfaces

A white dwarf rotating at a maximal angular velocity can take a form of a triaxial ellipsoid due to the rotation and the presence of mountains on its surface. Such object emits gravitational waves at a frequency of 2Ω, where Ω is the angular velocity of rotation. The gravitation radiation power of the WD with rough surface is following:

$$J_0 = \frac{32}{5}\frac{G}{c^5} I_3^2 \varepsilon^2 \Omega^6 \qquad (3)$$

where I_3 is the moment of inertia around z axes, ε – ellipticity [8,9]. We find also the expression for gravitational wave amplitude for a terrestrial observer at the distance r from the WD:

$$h_0 = \frac{1}{\Omega r}\sqrt{\frac{2.5 J_0 G}{c^3}} \qquad (4)$$

We suppose that the source of gravitational radiation is the kinetic energy of rotation of the star, i.e. the WD spins-down due to reaction of radiation. In this case we find characteristic spin-down time for the WD by gravitational radiation, which is

$$\tau_0 = \frac{\Omega}{2|\dot{\Omega}|} = \frac{5c^5}{64 G I_3 \varepsilon^2 \Omega^4} \qquad (5)$$

If τ_0 turns out to be on the order of lifetime of the WD (the age of Universe), then a WD with a rough surface will emit gravitational waves until now.

As we see from expressions (3) and (4), the basic characteristics of the gravitational radiation, such as the intensity and wave amplitude, depend on the ellipticity ε. In order to estimate this quantity, we have to know the characteristic scales of the mountains on the surface of a WD. The maximum height of a mountain that can support its own weight in the gravitational field of a white dwarf can be estimated to be [12,13]

$$H = \frac{10^{12}}{g}\rho^{1/3} cm \qquad (6)$$

After this, we obtain an expression for the ellipticity of the form $\varepsilon = H/a$, a is the equatorial radius of the star, which we shall use below for estimating the gravitational radiation intensity and gravitational wave amplitude. For example, for a WD with a central density $\rho_c = 2\cdot 10^7$g/cm^3 from (6) we find integral parameters of the star: mass $M = M_\Theta$, radius $a = 7.3\cdot 10^3$km, moment of inertia $I_3 = 8.9\cdot 10^{49}$ g.cm^2, and maximal rotation speed $\Omega_{max} = 0.48 s^{-1}$. Using these data, from (6) we get that H=0.19 km and as follows from (3) and (4) $J_0 = 10^{30}$ erg/s, and $h_0 = 10^{-24}$. In this way for extremely dense WD with $\rho_c = 2.6\cdot 10^9$g/cm^3 we find that $J_0 = 3.7\cdot 10^{31}$ erg/s, and $h_0 = 10^{-24}$ [12]. These calculations show that gravitational waves from rapidly rotating white dwarfs have quite high amplitudes and can be distinguished from the cosmic background by the new generations of detectors. Rapidly rotating white dwarfs are entirely possible, since the characteristic spin-down times due to gravitational radiation, as it follows from (5), are on the order of 10^{11} years, which is on the order of the age of the universe. Thus, if a star had an angular velocity close to maximal when it was born, it would have a fairly high angular velocity in our time and our calculations would remain valid.

3. GRAVITATIONAL WAVES FROM NEUTRON STARS

We consider undamped, self-similar quasiradial pulsations of a rotating neutron star consisting of a real baryon gas as a source of gravitational radiation.

Another mechanism for generating gravitational radiation can be the precession of the neutron star [14,15] in which the symmetry axis rotates about the angular momentum vector. This kind of precession was used to explain the fluctuations of the angular velocity of the Crab and Vela pulsars. Observational data from pulsar PSRB 1828-11 supports the existence of precession [16]. It has also been proposed that the recently discovered pulsar in the remnant of supernova 1987a is spinning down due to the emission of gravitational radiation caused by the free precession of the star [17]. However, as it was shown in [18] gravitational waves from precession of isolated neutron stars should be smaller then $h_{omax} = 10^{-30}$.

3.1. Oscillating Neutron Stars

One possible source of energy is the deformation energy of the neutron star. For rotating neutron stars, the surfaces

of constant density are rotating elipsoids. During the spin-down, these surfaces tend towards sphericity. Because the crust is a crystalline solid, the process of spin-down will be accompanied by starquakes which will relieve the stress built up in the core and drive quasi-radial oscillations. We propose that part of the deformation energy is converted to gravitational radiation in this process.
Let us take a model of a neutron star with a central density $\rho_c = 1.14 \cdot 10^{15}$ g/cm^3 and a mass $M = 1.4 M_\odot$. This model [3] has the following characteristics:
$\Omega_k \approx 8.4 \cdot 10^3 s^{-1}$ and $W_{def}(\Omega) \approx 1.8 \cdot 10^{53}$ erg. We find the gravitational radiation intensity to be $J_0 \approx 3.9 \cdot 10^{37}$ erg/s. We bring maximally possible values for parameters of gravitational waves from Crab and Vela pulsars in Table 3. For the neutron star model we have chosen, the oscillation frequency is $\omega \approx 5 \cdot 10$ khz [3,4]. In reality branching ratio will be about 0.01 or less that will result in coefficient $\sqrt{\beta}$ for strain amplitudes.

Table 3: Gravitational wave Maximal parameters from oscillating Neutron Stars(branching ratio β=1)

Neutron Star	r distance	τ (Gyr)	ho	F
Crab	2.5kpc	2.27E-6	9.20E-25	4.11E-4
Vella	0.3kpc	2.22E-5	9.04E-25	3.97E-4

3.2. Gravitational Waves from Glitches

Glitches (jumps) and fluctuations on the order of $\Delta\Omega / \Omega \approx 10^{-6} \div 10^{-9}$ are superimposed on the irregular variation in the angular velocity. The derivative of the angular velocity also experiences relative changes on the order of $\Delta\dot{\Omega}/|\dot{\Omega}| \approx 10^{-2} \div 10^{-4}$. In the following we assume that one of the possible sources for generating and maintaining the quasiradial oscillations may be the glitches and fluctuations in the star's angular velocity. It can be assumed that part of the rotational energy is transferred to the crust of a neutron star during these irregular changes in the angular velocity through the excitation of harmonic oscillations. The energy transferred to the crust is subsequently removed by gravitational radiation. The energy involved in the acceleration of a neutron star is

$$\Delta W = I\Omega \Delta\Omega$$

where Ω is the angular rotation velocity, $\Delta\Omega$ is the change in it, and I is the moment of inertia of the star. The power transferred to the star's crust is given by

$$\Delta\dot{W} = I\Omega\Delta\dot{\Omega} = I\Omega\dot{\Omega}\frac{\Delta\dot{\Omega}}{\dot{\Omega}}$$

where W& is the steady state loss of rotational energy by the neutron star during its secular deceleration.
If we assume that the energy of the quasiradial oscillations is entirely radiated in the form of gravitational waves, then we have to set

$$J_0 = \Delta\dot{W}$$

Substituting Glitch parameters from [19]and [20] for Crab and Vela pulsars we come to the following results

Table 4: Gravitational waves from Neutron Star Glitches

Neutron Star	$\frac{\Delta\dot{\Omega}}{\lvert\dot{\Omega}\rvert}$	ho	F
Crab	5E-3	1.0E-26	3.2E-7
Vella	1E-4	6.8E-27	7.78E-7

Registration of Gravitational Waves from Glitches will be very important, because it will give a chance to measure experimentally the velocity of propagation of gravitational wave.

4. CONCLUSIONS

The galactic population of white dwarfs is a large collection of potential sources of gravitational radiation. Although most oscillation modes of white dwarfs lie in the millihertz range, quasiradial self-similar oscillations lie in the decihertz range. We have investigated the possible strength of the gravitational radiation foreground due to these oscillations in the galactic population. We have identified a possible energy source in the differential rotation energy which can support the oscillations for roughly the lifetime of the galaxy, so that we can consider these oscillations to be long-lived, essentially monochromatic sources of gravitational radiation. Although we have not identified a mechanism by which the gradual relaxation of differential rotation can actually drive the oscillations, we have shown that if such a mechanism can sustain quasi-radial oscillations with an amplitude comparable to observed low frequency oscillation, then the galactic population of white dwarfs can produce a stochastic foreground of comparable strength to the expected cosmological background in this frequency band.

Pulsars with irregular variations in their angular velocity are good candidates for gravitational wave observations. Note that the best candidate for this purpose is the Vela pulsar, which is being continuously monitored.
This pulsar also differs from other pulsars in having large glitches in its angular momentum that occur at a high rate.

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References

[1] N. Seto,S. Kawamura,T. Nakamura,Phys.Rev.Lett., 87, 221103,2001.

[2] M. Benacqusta,D.M. Sedrakian,M.V. Hayrapetyan, K.M. Shahabasyan,A.A. Sadoyan, Astrophys. J., 596, L223-226, 2003.
[3] V.V.Papoyan,D.M.Sedrakian, E.V.Chubaryan, Astron. Zh., 49, 750 ,1972.
[4] G.G.Haroutyunyan, D.M.Sedrakian, E.V.Chubaryan, Astron. Zh., 49, 1216, 1972.
[5] Yu.L.Vartanian,G.S Hajian,Astron. Zh., 54, 1047, 1977.
[6] G.S.Sahakian, D.M.Sedrakian, E.V.Chubarian, Astrofizika, 8, 541, 1972.
[7] Benacquista, M., D.M. Sedrakian, M.V. Hairapetyan, K.M. Shahabasyan, A.A. Sadoyan Astrofizika 47, 381, 2004.
[8] G.D.Schmidt, S.Vennes, D.T.Wickramasinghe, L.Ferrario, Mon. Not. R. Astron. Soc., 328, 203, 2001.
[9] D.T.Wickgramasinghe, L.Ferrario, Proc. Astron. Soc. Pacific, 112, 873, 2000.
[10] Ostriker J P and Mark J W-K , Astrophys. J. 151 1075, 1968
[11] S. Shapiro and S.Tyukolski, Black Holes, White Dwarfs and Neutron Stars, vol.2, 1983.
[12] D.M.Sedrakian, M.V.Hayrapetyan, A.A.Sadoyan, Astrophysics, v.48, No. 1, 53, 2005.
[13] F.Dyson and D. ter Haar, Neutron Stars and Pulsars, 1971.
[14]. D. I. Jones and N. Andersson, Mon. Notic. Roy. Astron. Soc. 324, 811 2001.
[15]. M. Zimmerman and Jr. E. Szedenits, Phys. Rev. D 20, 351 (1979).
[16]. I. H. Stairs, A. G. Lyne, and S. L. Shemar, Nature 406, 484 (2000).
[17]. J. Middleditch et al. , astro-ph/0010044 2000.
[18]. ASedrakyan, I.Wasserman, & J.Cortes, ApJ 524,341,1999.
[19]. J. M. Cordes, G. S. Downs, and J. Krause-Polstorff, Astrophys. J. 330, 841 (1988).
[20]. N. Wand, R. N. Manchester, J. Zhang, X. -Y. Wu, A. Yusup, A. G. Lyne, K. S. Cheng, and M. Z. Chen, astro-ph/0111006 2001.