Dark Matter Abundance in Brane Cosmology

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We investigate the thermal relic density of a cold dark matter in the brane world cosmology. Since the expansion law in a high energy regime is modified from the one in the standard cosmology, if the dark matter decouples in such a high energy regime its relic number density is affected by this modified expansion law. We derive analytic formulas for the number density of the dark matter. It is found that the resultant relic density is characterized by the "transition temperature" at which the modified expansion law in the brane world cosmology is connecting with the standard one, and can be considerably enhanced compared to that in the standard cosmology, if the transition temperature is low enough. As an application, the thermal relic density of the neutralino dark matter in the brane world cosmology is studied. For the neutralino, if the five dimensional Planck mass M_5 is lower than 10^4 TeV, the brane world cosmological effect is significant at the decoupling time and the resultant relic density is enhanced. We calculate the neutralino relic density in the Constrained Minimal Supersymmetric Standard Model (CMSSM) and show that the allowed region is dramatically modified from the one in the standard cosmology and eventually disappears as M_5 is decreasing. Thus, we find a new lower bound on $M_5 \gtrsim 600$ TeV based on the neutralino dark matter hypothesis, namely the lower bound in order for the allowed region of the neutralino dark matter to exist.

1. INTRODUCTION

Recent cosmological observations, especially the Wilkinson Microwave Anisotropy Probe (WMAP) satellite [1], have established the ΛCDM cosmological model with a great accuracy and the relic abundance of the cold dark matter is estimated as (in 2 σ range)

$$\Omega_{CDM} h^2 = 0.1126^{+0.0161}_{-0.0181}. \tag{1}$$

On the other hand, theoretically, if the dark matter is the thermal relic, the present number density is estimated as

$$Y|_0 \equiv \left.\frac{n_{CDM}}{s}\right|_0 \simeq \begin{cases} \dfrac{x_d}{\lambda\sigma_0} & \text{for} \quad n=0, \\ \dfrac{2x_d^2}{\lambda\sigma_1} & \text{for} \quad n=1, \end{cases} \tag{2}$$

with a constant

$$\lambda = 0.26 \left(\frac{g_{*S}}{g_*^{1/2}}\right) \frac{m}{\sqrt{G}} \tag{3}$$

for models in which the thermal averaged product of the annihilation cross section σ and the relative velocity v, $\langle\sigma v\rangle$, is approximately parametrized as $\langle\sigma v\rangle = \sigma_n x^{-n}$ with $x = m/T$, where g_* is the effective total number of relativistic degrees of freedom, $x_d = m/T_d$, T_d is the decoupling temperature and m is the mass of the dark matter particle and G is the Newton's gravitational constant [2].

Note that the thermal relic density of the dark matter depends on the underlying cosmological model as well as its annihilation cross section. A brane world cosmological model which has been intensively investigated [3] is a well-known example as such a non-standard cosmological model. The model is a cosmological version of the so-called "RS II" model first proposed by Randall and Sundrum [4], where our 4-dimensional universe is realized on the "3-brane" located at the ultra-violet boundary of a five dimensional Anti de-Sitter spacetime. In this setup, the Friedmann equation for a spatially flat spacetime is found to be

$$H^2 = \frac{8\pi G}{3}\rho\left(1+\frac{\rho}{\rho_0}\right), \tag{4}$$

where

$$\rho_0 = 96\pi G M_5^6, \tag{5}$$

H is the Hubble parameter, ρ is the energy density of matters, M_5 is the five dimensional Planck mass, and we have omitted the four dimensional cosmological constant and the so-called dark radiation term. The second term proportional to ρ^2 is a new ingredient in the brane world cosmology and lead to a non-standard expansion law. Since at a high energy regime this term dominates and the universe obeys a non-standard expansion law, some results previously obtained in the standard cosmology can be altered. In fact, some interesting consequences in the brane world cosmology have been recently pointed out. For example, the novel possible solution to the "gravitino problem" by the modified expansion law is pointed out [5].

Here, we investigate the brane cosmological effect for the relic density of the dark matter due to the non-standard expansion law. If the new term in Eq. (4)

dominates over the term in the standard cosmology at the freeze out time of the dark matter, it can cause a considerable modification for the relic abundance of the dark matter [6].

After deriving an analytic formula for the relic density of the dark matter [6], we apply this result to a concrete candidate, neutralino. The lightest supersymmetric particle (LSP) is suitable for a cold dark matter, because they are stable owing to the conservation of R-parity. In the minimal supersymmetric standard model (MSSM), the lightest neutralino is typically the LSP and the promising candidate for the cold dark matter. In the light of the WMAP data, the parameter space in the Constrained MSSM (CMSSM) which allows the neutralino relic density suitable for the cold dark matter has been recently re-analyzed [7]. It has been shown that the resultant allowed region is dramatically reduced due to the great accuracy of the WMAP data. By taking account of the modified expansion law in brane world cosmology, we estimate numerically the neutralino relic density in the brane world cosmology and show that the allowed region for the neutralino dark matter in the CMSSM is dramatically modified in the brane world cosmology [8].

2. DARK MATTER RELIC DENSITY IN BRANE WORLD COSMOLOGY

In this section, we derive the general analytic formula for the relic density of the dark matter in the brane world cosmology with a low five dimensional Planck mass [6].

In the context of the brane world cosmology, the thermal relic density of a dark matter particle is evaluated by solving the Boltzmann equation

$$\frac{dn}{dt} + 3Hn = -\langle \sigma v \rangle (n^2 - n_{EQ}^2), \tag{6}$$

with the modified Friedmann equation Eq. (4), where n is the actual number density of the dark matter particle, n_{EQ} is the equilibrium number density. It is useful to rewrite Eq. (6) into the form,

$$\frac{dY}{dx} = \lambda \frac{x^{-2}}{\sqrt{1+\left(\frac{x_t}{x}\right)^4}} \langle \sigma v \rangle (Y^2 - Y_{EQ}^2), \tag{7}$$

where x_t is defined as

$$x_t^4 \equiv \left. \frac{\rho}{\rho_0} \right|_{T=m}. \tag{8}$$

At the era $x \ll x_t$ the ρ^2 term dominates in Eq. (4), while the ρ^2 term becomes negligible after $x \gg x_t$ and the expansion law in the standard cosmology is realized. Hereafter we call the temperature defined as $T_t = m x_t^{-1}$ (or x_t itself) "transition temperature" at which the expansion law of the early universe changes from the non-standard one to the standard one. Since we are interested in the effect of the ρ^2 term for the dark matter relic density, we consider the case that the decoupling temperature of the dark matter T_d is higher than the transition temperature T_t, namely $x_t > x_d = m/T_d$.

At the early time, the dark matter particles are in the thermal equilibrium and $Y = Y_{EQ} + \Delta$ tracks Y_{EQ} closely. After the temperature decreases, the decoupling occurs at x_d roughly evaluated as $\Delta(x_d) \simeq Y(x_d) \simeq Y_{EQ}(x_d)$. The solutions of the Boltzmann equation during the $x_t > x > x_d$ epoch are given as

$$\begin{aligned} \frac{1}{\Delta(x)} - \frac{1}{\Delta(x_d)} &= \frac{\lambda \sigma_0}{x_t^2}(x - x_d) \quad \text{for} \quad n=0, \\ & \quad \frac{\lambda \sigma_1}{x_t^2} \ln\left(\frac{x}{x_d}\right) \quad \text{for} \quad n=1. \end{aligned} \tag{9}$$

Note that $\Delta(x)^{-1}$ is continuously growing without saturation for $n \le 1$. This is a very characteristic behavior of the brane world cosmology, comparing the case in the standard cosmology where $\Delta(x)$ saturates after decoupling and the resultant relic density is roughly given by $Y(\infty) \simeq Y(x_d)$. For a large $x \gg x_d$ in Eq. (9), $\Delta(x_d)$ and x_d can be neglected. When x becomes large further and reaches x_t, the expansion law changes into the standard one, and then Y obeys the Boltzmann equation with the standard expansion law for $x \ge x_t$. Since the transition temperature is smaller than the decoupling temperature in the standard cosmology (which case we are interested in), the number density freezes out as soon as the expansion law changes into the standard one and can be roughly evaluated as $Y(\infty) \sim \Delta(x_t)$ in Eq. (9).

By adopting the approximate parametrization for $\langle \sigma v \rangle$, we can obtain simple analytic formulas for the relic number density of the dark matter as

$$\begin{aligned} Y(x \to \infty) &\simeq 0.54 \frac{x_t}{\lambda \sigma_0} \quad \text{for} \quad n=0, \\ & \quad \frac{x_t^2}{\lambda \sigma_1 \ln x_t} \quad \text{for} \quad n=1, \end{aligned} \tag{10}$$

in the limit $x_d \ll x_t$, where x_d is the decoupling temperature. Note that the results are characterized by the transition temperature rather than the decoupling temperature. It is interesting to compare these results to that in the standard cosmology. Using the well-known approximate formulas in the standard cosmology, Eq. (2), we obtain the ratio of the relic energy density of the dark matter in the brane world cosmology ($\Omega_{(b)}$) to the one in the standard cosmology ($\Omega_{(s)}$) such that

$$\begin{aligned} \frac{\Omega_{(b)}}{\Omega_{(s)}} &\simeq 0.54 \left(\frac{x_t}{x_{d(s)}}\right) \quad \text{for} \quad n=0, \\ & \quad \frac{1}{2\ln x_t}\left(\frac{x_t}{x_{d(s)}}\right)^2 \quad \text{for} \quad n=1, \end{aligned} \tag{11}$$

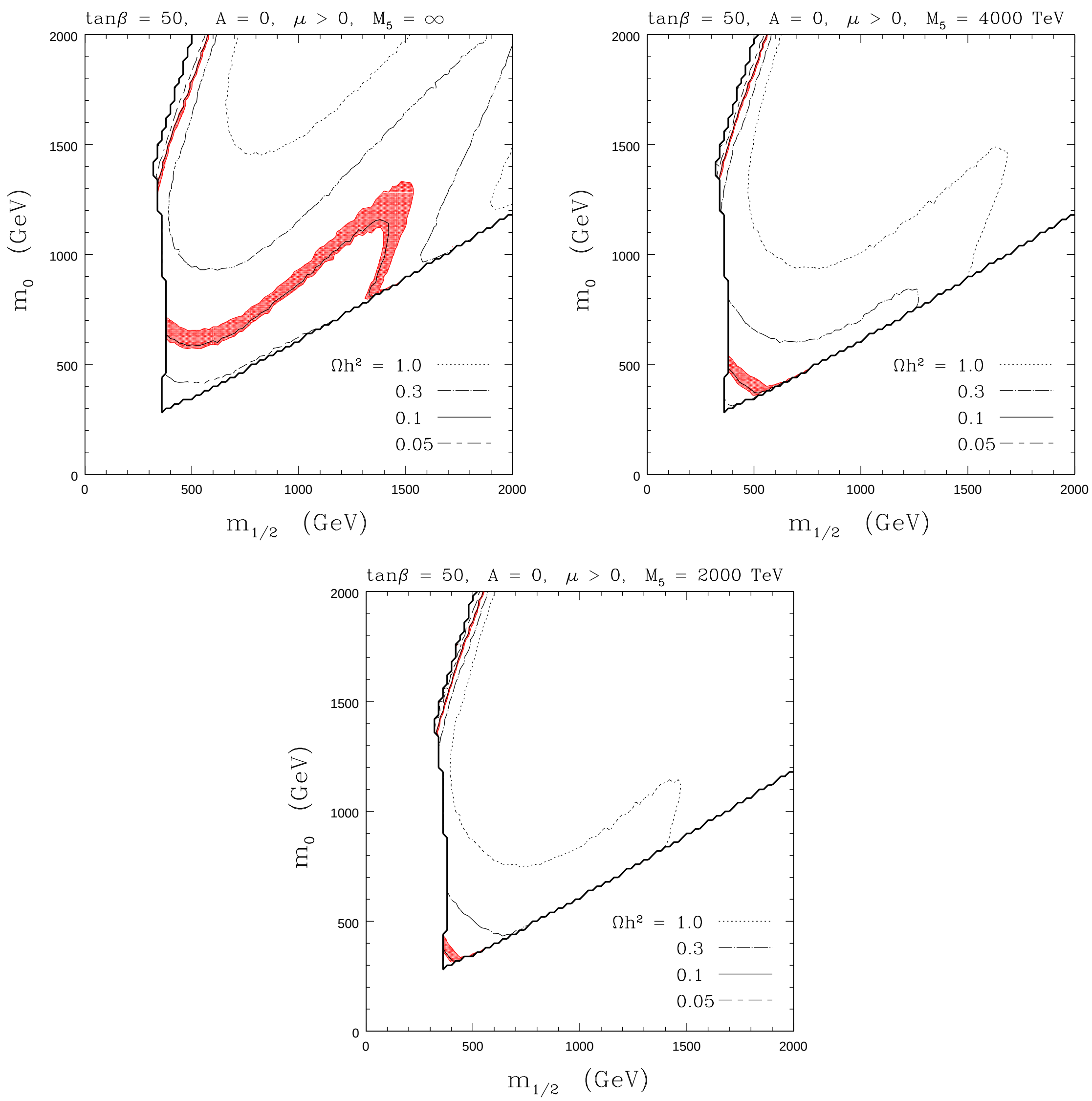

Figure 1: Contours of the neutralino relic density $\Omega_\chi h^2$ in the $(m_{1/2}, m_0)$ plane for $M_5 = \infty$ (upper left window), 4000 TeV (upper right window), and 2000 TeV (lower window) in the case of $\tan\beta = 50$, $A = 0$ and $\mu > 0$. The dotted, dashed, solid and short-dash–long-dash lines correspond to $\Omega_\chi h^2 = 1.0$, 0.3, 0.1 and 0.05, respectively. The shaded regions (in red) are allowed by the WMAP constraint. The region outside the bold line, including the two axes, are excluded by experimental constraints or the condition for successful electroweak symmetry breaking.

where $x_{d(s)}$ denotes the decoupling temperature in the standard cosmology.

Thus, the consequence for the dark matter in brane world cosmology is that the relic energy density can be enhanced from the one in the standard cosmology if the transition temperature is low enough.

If the above discussion is applied to detailed analysis of the relic abundance of the neutralino dark matter, we can expect a dramatic modification of the allowed region in the CMSSM. In the next section, we present the results.

3. THE APPLICATION TO THE NEUTRALINO DARK MATTER

In this section, we study the enhancement effect on the parameter space in a neutralino dark matter model [8]. We calculate the relic density of the neutralino, $\Omega_\chi h^2$, in the CMSSM with the modified Friedmann equation Eq. (4). For this purpose, we have modified the code DarkSUSY [9] so that the modified Friedmann equation is implemented.

The mass spectra in the CMSSM are determined by the following input parameters

$$m_0, \quad m_{1/2}, \quad A, \quad \tan\beta, \quad \mathrm{sgn}(\mu), \tag{12}$$

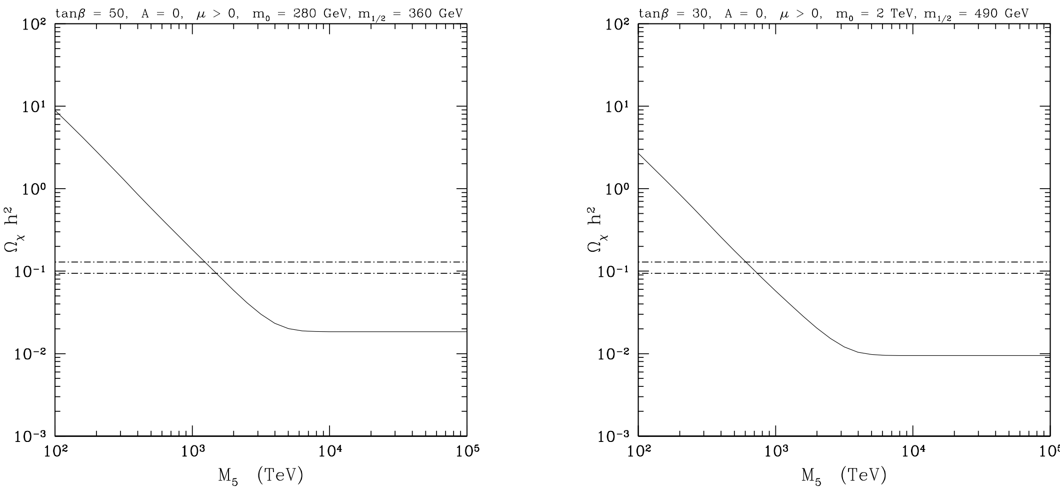

Figure 2: $\Omega_\chi h^2$ *vs.* M_5 for $\tan\beta = 50$, $m_0 = 280$ GeV, $m_{1/2} = 360$ GeV (left window) and $\tan\beta = 30$, $m_0 = 2$ TeV, $m_{1/2} = 490$ GeV (right window) with $A = 0$ and $\mu > 0$. The range of $\Omega_\chi h^2$ between the two dash–dotted lines satisfies the WMAP constraint.

where m_0 is the universal scalar mass, $m_{1/2}$ is the universal gaugino mass, A is the universal coefficient of scalar trilinear couplings, $\tan\beta$ is the ratio of the vacuum expectation values of the two neutral Higgs fields, and $\mathrm{sgn}(\mu)$ is the sign of the higgsino mass parameter μ. With these input parameters, renormalization group equations for the CMSSM parameters are solved using the code ISASUGRA [10] to obtain the mass spectra at the weak scale. In the present analysis, we take $A = 0$ and $\mu > 0$.

In Fig. 1, we show the allowed region in the $(m_{1/2}, m_0)$ plane consistent with the WMAP 2σ allowed range $0.094 < \Omega_\chi h^2 < 0.129$ for $\tan\beta = 50$, $A = 0$ and $\mu > 0$. The shaded regions (in red) are allowed by the WMAP constraint. These figures contains the contour plots of $\Omega_\chi h^2$. The dotted, dashed, solid and short-dash–long-dashed lines correspond to $\Omega_\chi h^2 = 1.0$, 0.3, 0.1 and 0.05, respectively. The region among the bold line and the two coordinate axes is excluded by various experimental constraints (the lightest Higgs mass bound, $b \to s\gamma$ constraint, the lightest chargino mass bound etc.) [7, 11] or the condition for the successful electroweak symmetry breaking. The upper left window corresponds to the usual result in the standard cosmology ($M_5 = \infty$). The upper right window and the lower window are the corresponding results for $M_5 = 4000$ TeV and 2000 TeV, respectively. These figures clearly indicate that, as M_5 decreases, the allowed regions shrink significantly. The allowed region eventually disappears as M_5 decreases further.

Next, we present sensitivity of the relic density to M_5 in Fig. 2, fixing m_0 and $m_{1/2}$ as well as $\tan\beta$, A and $\mathrm{sgn}(\mu)$. The left window, where we take $\tan\beta = 50$, $m_0 = 280$ GeV and $m_{1/2} = 360$ GeV, corresponds to the point giving the smallest value of $\Omega_\chi h^2$ in the small mass region (m_0, $m_{1/2} < 1$ TeV) for $\tan\beta = 50$. The range of $\Omega_\chi h^2$ between the two dash–dotted lines satisfies the WMAP constraint. For large $M_5 \gtrsim 10^4$ TeV, the (too small) relic density ($\Omega_\chi h^2 \approx 0.02$) in the standard case is reproduced independently of M_5. This is because $x_t \lesssim x_{d(s)}$ is obtained for a such large M_5. As M_5 decreases, however, the relic density $\Omega_\chi h^2$ starts to be enhanced significantly, and it amounts to ≈ 10 for $M_5 \approx 100$ TeV. It is found that, through the enhancement, an allowed region comes out for M_5 in the range of 1000 TeV $\lesssim M_5 \lesssim$ 1500 TeV.

On the other hand, the right window, where we take $\tan\beta = 30$, $m_0 = 2$ TeV and $m_{1/2} = 490$ GeV, represents the point giving the smallest value of $\Omega_\chi h^2$ in the higgsino-like region for $\tan\beta = 30$. The M_5 dependence is quite similar to the case in the left window. However, since the relic density $\Omega_\chi h^2 \approx 0.01$ in the standard case is smaller than that in the left window, the WMAP allowed region is shifted to smaller M_5 region as 600 TeV $\lesssim M_5 \lesssim$ 800 TeV.

We cannot find the WMAP allowed region for $M_5 \lesssim$ 600 TeV. Hence,

$$M_5 \gtrsim 600\,\mathrm{TeV}, \tag{13}$$

is the lower bound on M_5 in the brane world cosmology based on the neutralino dark matter hypothesis, namely the lower bound in order for the allowed region of the neutralino dark matter to exist.

4. SUMMARY

We have investigated the thermal relic density of the cold dark matter in the brane world cosmology. If the five dimensional Planck mass is small enough, the ρ^2 term in the modified Friedmann equation can be effective when the dark matter is decoupling. We have derived the analytic formulas for the relic density and found that the resultant relic density can be enhanced. The enhancement factor is characterized by the transition temperature x_t, at which the expansion law changes from the non-standard law to the standard one.

In addition, we have studied the neutralino relic density in the CMSSM in the brane world cosmology. If the five dimensional Planck mass is low enough, $M_5 \lesssim 10^4$ TeV, the ρ^2 term in the modified Friedmann equation can be effective at the decoupling time. We have presented our numerical results and shown that the allowed region shrinks and eventually disappears as M_5 decreases. Through the numerical analysis, we have found a lower bound on $M_5 \gtrsim 600$ TeV in the brane world cosmology based on the neutralino dark matter within the CMSSM.

Acknowledgments

The works of T.N. and N.O. are supported in part by the Grant-in-Aid for Scientific Research (#16740150 and #15740164) from the Ministry of Education, Culture, Sports, Science and Technology of Japan. The work of O.S. is supported by PPARC.

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