Structure Formation through Cosmic Bose Einstein Condensation -Unified View of Dark Matter and Energy-

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Structure formation processes are discussed in the BEC(Bose-Einstein Condensation) cosmological model, in which, the boson dark matter (DM) gradually condensates into the uniform dark energy (DE) with negative pressure. This global condensate itself yields the accelerated expansion of the Universe. On the other hand, this component becomes unstable after its dominance and eventually collapses into black holes and other compact objects. Such components behave as highly localized cold dark matter. They become the seeds of galaxies and quasars. It is important that such rapid formation of highly compact object does not affect the uniformity in CMB and the power spectrum of density fluctuations in the linear stage. The possibility of the repeated sedimentation of condensate is proposed, which guarantees the autonomous self-regularization of the ratio DE/DM to become order one.

1. INTRODUCTION

The present model of the universe is hybrid in the sense that the unknown dark matter (DM) and dark energy (DE) independently dominate the cosmic energy density. [1 2] A complete cosmological model cannot be established without clarifying such unknown components in the universe.

There are several models which unifies the above two dark components introducing a scalar field [3] or the fluid with peculiar equation of state [4]. In this article, we would like to explore further such unified model introducing the Bose-Einstein condensation model for DE/DM. We propose a unified model of DE and DM in the context of a cosmic phase transition and, consequently, we are led to a new scenario for the early formation of highly non-linear objects. The key feature of the DE would be the volume-independent negative pressure, which guarantees the accelerated cosmic expansion through the Einstein equation for the scale factor $\ddot{a}(t) = -(4\pi G/3)(\rho + 3p)a(t)$.

We would like to reconsider the origin of the scalar field which is often used in the cosmological models. Moreover, we would like to consider not only the coherent field component but also the cold gas component as well. A natural consideration leads to the model of the Bose-Einstein condensation (BEC) of a boson field with attractive interaction [5]. A naïve expectation would be to identify the condensation as DE and the exited boson gas as DM. However such expectation needs some elaboration as we will see later in detail. The inevitable negative pressure of the condensate makes this component unstable, and they would eventually collapse into highly non-liner objects such as black holes very rapidly. They will behave as cold dark matter as well in the later stage. Thus we should consider the three different kinds of components in our BEC model; uniform excited boson gas, uniform condensate, and highly non-linear compact objects.

In this BEC model, we are faced upon the quantum structure of the Universe since the DE dominance means that the present universe is in almost the ground state described by a big wave function of the quantum condensate.

This nature is in accord with a general mechanism of the structures formation in cosmology. That is, all the basic structures are formed through phase transitions and the quantum mechanical principle yields the origin of detailed structures. This general mechanism especially applies to the inflationary stage in the early universe. The evolution of the classical inflaton filed corresponds to the development of the c-number order parameter in a big phase transition and the density fluctuation owes its origin in the quantum uncertainty principle.

There are plenty of examinations on the observational validity of the generalizations of or deviations from the standard cosmological model [6]. Especially the power spectrum and the uniformity in CMB data allow only very small deviations from the standard model. Therefore for example, the parameter rage of the generalized chaplygin gas model is strongly suppressed; actually no deviation is allowed from the standard equation of state.

On the other hand in our model, we do not seek for a small allowed deviation from the standard model, but we explore the different extreme limit. The homogeneous BEC is quite unstable and all the fluctuation modes develop. Among them, the instability in the smallest scale is the largest and the time scale of the collapse is the shortest. Therefore the smallest scale non-linearity naturally develops first. Moreover in this process the uniform original BEC component disappears in the course of the formation of compact objects. Therefore this process has no significant effect on the larger scale density fluctuations.

The above mechanism of BEC collapse is similar to the collapse of a huge soap bubble. The bubble surface rapidly fragments into many pieces when a tiny trigger is applied to the original soap bubble.

Basic model we construct would have the following general features:

1. *Bose gas* is introduced as DM which initially dominates the energy density and the *condensate* of the boson is identified as DE.
2. The condensate has negative pressure due to its attractive interaction. For the spatially uniform component of BEC, this negative pressure works as a cosmological constant and guarantees the *accelerated expansion*.

3. The *sedimentation* of the condensate slowly proceeds in the cosmic evolution.
4. When the energy density of BEC exceeds some critical value, it *collapses into compact boson stars or black holes*, which work as the standard cold dark matter and provides the seeds of galaxies. Simultaneously a new sedimentation process begins. This *cycle of slow sedimentation-and-rapid collapse repeats* many times. These rapid collapses take place well *after* the photon-decoupling stage, and therefore the large scale structure predicted by the Λ CDM model and actually observed pattern in CMB fluctuations would not strongly be violated.

2. COSMIC BEC MECHANISM

General BEC initiates when the quantum coherence of each constituent particle overlaps with each other and the coherence spreads over the whole system. More precisely the BEC takes place in the system when the thermal de Broglie length $\lambda_{dB} \equiv \left(2\pi\hbar^2/(mkT)\right)^{1/2}$ exceeds the mean separation length of particles $n^{-1/3}$. This condition turns out to be $T < T_c$, where the critical temperature becomes

$$T_c = \frac{2\pi\hbar^2 n^{2/3}}{m\zeta(3/2)} \qquad (1)$$

and $n = N/V$ is the mean number density of the boson of the mass m. The critical temperature is also characterized by the transition point that the chemical potential μ associated with the conservation of particle number N within the volume V shows singular behavior $\mu \to -0$. The condensation is possible only for non-relativistic stage $T < m$ and the particle number N is conserved. On this non relativistic stage, the cosmic energy density generally behaves as

$$n = n_0 \left(\frac{m}{2\pi\hbar^2}\frac{T}{T_0}\right)^{3/2}, \qquad (2)$$

where n_0, T_0 are the number density and the temperature at some moment in the non-relativistic stage. In deriving eq.(2), we have used the fact that the evolution is adiabatic in the sense that the entropy per particle

$$s/n = \ln\left(e^{5/2}(mT)^{3/2}/\left(2\pi\hbar^2\right)^{3/2}/n\right) \quad (3)$$

is conserved. The dominant component is the matter and is not the radiation on this stage; the temperature in Eq.(2) is the matter temperature. The number-density dependence of the temperature $T \propto n^{2/3}$ in Eq.(2) is the same as that in Eq.(3). (Figure 1.) Therefore the condition $T < T_C$ sets the upper limit of the boson mass; if we adopt the value $\rho_{now} = 9.44\ 10^{-30}\,\mathrm{g/cm^3}$, then $m < 2\ \mathrm{eV}$. If this condition holds, the BEC process starts when the boson becomes non-relativistic and it continues afterwards.

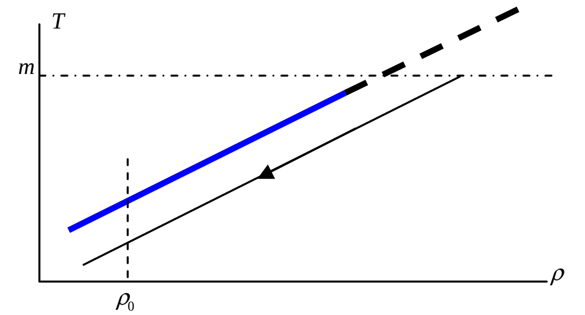

Figure 1. A schematic diagram of the temperature-density relation in the Universe. A thick line represents the critical temperature, Eq.(1), below which the BEC proceeds. The part of broken line is in the temperature higher than m and the matter is relativistic. Therefore BEC does not take place there. A thin solid line represents the evolution of the cosmic energy density Eq.(2). The arrow shows the direction of the cosmic evolution.

3. QUANTUM LIQUID MODEL OF COSMIC BEC

For describing the dynamics of BEC, it is usually adopted to use the mean-field analysis [7] based on the Gross-Pitaevskii equation. This equation has a form of non-linear Schrödinger equation,

$$i\hbar\frac{\partial\psi}{\partial t} = -\frac{\hbar^2}{2m}\Delta\psi + V\psi + g|\psi|^2\psi, \qquad (4)$$

where $\psi(\vec{x},t)$ is the condensate wave function, $V(\vec{x})$ is the potential, $g = 4\pi\hbar^2 a/m$, and a is the s-wave scattering length. If we polar decompose the wave function as $\psi = \sqrt{n}e^{iS}$, and define the velocity as $\vec{v} = \hbar\vec{\nabla}S/m$, then Eq.(4) reduces to a pair of the continuity equation and the hydrodynamic-like equation,

$$m\frac{\partial\vec{v}}{\partial t} + \vec{\nabla}\left(\frac{mv^2}{2} + V + gn - \frac{\hbar^2}{2m\sqrt{n}}\Delta\sqrt{n}\right) = 0, \quad (5)$$

If we disregard, in this equation, the last term in the parenthesis, which is quantum origin ($\propto \hbar^2$), it would be a complete hydrodynamic equation. This term can be actually neglected if the wave number of the mode k satisfies $k^2 < k_c^2 \equiv 2mgn/\hbar^2$; i.e. large scale mode. Further, if we choose the attractive interaction ($g < 0$), BEC can be described as fluid with negative pressure.

We introduce the following simple model for the BEC: Boson gas is identified as cold DM with the equation of state $p = 0$ and the condensate as DE with $p = -\rho$. The sedimentation of BEC in the uniformly expanding Universe slowly proceeds with the time scale Γ^{-1}. This setting is very similar to the chaplygin gas model [8] with the equation of state $p = -A/\rho$, except that in the latter, DE and DM properties are simultaneously included in this single equation of state of a single phase.

Since the Universe is initially extremely uniform, this condensation would also be uniform. The energy density of the excited boson gas is diluted by the cosmic expansion, however, that of condensate is not diluted. This is because the work supplied to expand the volume V to $V + dV$ is $-pdV = \rho dV$, which is exactly the necessary and sufficient amount of energy to produce the new condensate in the region dV with the same energy density. Therefore eventually the condensate would dominate the excited gas component and the expansion law of the Universe changes from the decelerated expansion to the accelerated expansion.

In the early stage when the boson gas density dominates that of condensate, density fluctuations are controlled by the dominant component, i.e. the boson gas, and their evolution is described by the standard Λ CDM model. However in the later stage when the condensate density dominates the boson gas density, the situation drastically changes. The linear perturbation equation for the gauge invariant quantity

$$\Delta = \delta + (1+w)\dot{a}(v-b)/k \qquad (6)$$

obeys the evolution equation

$$\Delta_k'' + \left(2 + \xi - 3\left(2w - c_s^2\right)\right)\Delta_k' = \left(\frac{3}{2}\left(1 - 6c_s^2 + 8w - 3w^2\right) - \left(\frac{kc_s^2}{aH}\right)^2\right)\Delta_k \quad ,(7)$$

where $(\cdots)' \equiv \frac{d(\cdots)}{d\ln a}$, and we have set zero spatial curvature[9]. For the equation of state $p = -\rho$,

$$w \equiv \frac{p}{\rho} = -1, c_s^2 \equiv \frac{\partial p}{\partial \rho} = -1, \xi \equiv \frac{(H^2)'}{2H^2} = 0, \quad (8)$$

and therefore

$$\Delta_k = \delta_k \equiv \delta\rho(k)/\rho\,. \qquad (9)$$

Then Eq.(7) reduces to

$$\delta_k'' + 5\delta_k' = -\left(6 - \left(\frac{k}{aH}\right)^2\right)\delta_k\,. \qquad (10)$$

According to this, a small scale mode $\tilde{k}^2 > 6H^2$ rapidly grows, and an almost cosmic horizon scale mode $\tilde{k}^2 < 6H^2$ slowly decays, where the comoving wave number $\tilde{k} \equiv k/a$ is defined. Moreover, the smaller the fluctuation scale, the faster the growing process:

$$\delta_k \propto \exp\left(t\tilde{k}/H\right). \qquad (11)$$

Note that this rapid collapse is related with the fact that in the gas of negative pressure, there is no sound wave ($c_S^2 < 0$). The situation now considering is not the (never growing) density fluctuations in the de Sitter space in which the pressure is a strict constant.

In the above, we have considered only the linear stage of the collapse of BEC. Eventually the dynamics terns into the non-linear collapsing phase on which we now study. We suppose a uniform spherically distributed over-density region of BEC of radius r and density ρ as an initial condition. Since the pressure gradient is non vanishing only on the surface of the sphere, the surface is isotropically compressed to form a dense skin. In this process, the skin region with the width dr acquires energy $4\pi r^2 dr(-p) = 4\pi r^2 dr\rho$ which is exactly the mass of the skin. Therefore the skin soon acquires the light velocity. The skin itself has large negative pressure in magnitude and therefore self-focuses. Since this skin is still located in the same pressure gradient, it is further compressed and eventually it wipes up the whole condensate toward the center. (Figure 2.) Disregarding the gravity which is of a secondly importance in the present, this collapsing process can be expressed in the evolution equation of the skin radius,

$$\frac{d(m_t\gamma\dot{r})}{dt} = -4\pi r_t^2\rho\,, \qquad (12)$$

where

$$m_t = \frac{4\pi}{3}\left(r_0^3 - r_t^3\right)\rho \qquad (13)$$

is the time dependent total mass of the skin, and the right hand side of Eq.(12) is the total force acting on the skin $4\pi r_t^2 p$. All solutions of Eq.(12) turn out to approach asymptotically to the constant velocity solution

$$\gamma \approx 1.62, \dot{r} \approx 0.79\,. \qquad (14)$$

Thus the collapse of the condensate is almost the light velocity.

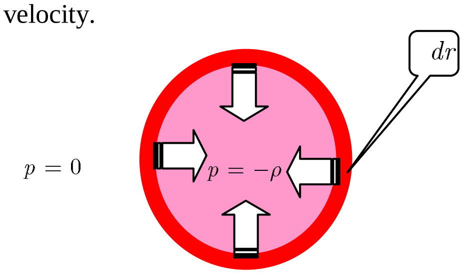

Figure 2. A schematic diagram of BEC collapse. A spherical symmetric condensate (marked in pink and red) region has a pressure gradient on the surface. The compression from outside yields the high density skin (marked in red), which eventually collapses toward the center with almost the speed of light.

Self gravity of the sphere would further accelerate the skin collapse especially in the later stage. The collapse would continue until the Heisenberg uncertainty principle begins to support the structure (the same effect due to the last term in the parenthesis in Eq.(5)), or a black hole is formed, or it bounces back outward if the condensate melts at the final stage of the collapse. Anyway the collapsed condensate forms highly localized compact objects classified as cold dark matter.

The above consideration is only on the unnatural spherically symmetric configuration. However the collapsing mechanism is robust and the similar argument holds, for example, also for one-dimensional collapse. Suppose a negative energy region is sandwiched by a low energy region, with parallel flat boundary surfaces. Then the width of the high energy region $r(t)$ follows the evolution equation, disregarding gravity,

$$\frac{d(m_t \gamma \dot{r})}{dt} = p = -\rho \qquad (15)$$

where the mass of the compressed skin of unit area is

$$m_t = (\rho/2)(r(0) - r(t)). \qquad (16)$$

All the solutions asymptotically approach toward the solution with constant velocity

$$\dot{r} \approx 0.91, \gamma \approx 2.41, \qquad (17)$$

as in the previous example. Thus the BEC collapse with almost the light speed is considered to be a general robust feature of the fate of BEC in the Universe.

4. GRADUAL SEDIMENTATION OF THE CONDENSATE -SELF ORGANIZED CRITICALITY-

We now turn our attention to the global evolution of DE/DM in the expanding Universe. The evolution of the various energy densities are phenomenologically given by

$$\begin{aligned} &\rho = \rho_c + \rho_g + \rho_l, \quad H \equiv \frac{\dot{a}}{a} = \sqrt{\frac{8\pi G \rho}{3}}, \\ &\dot{\rho}_c = \Gamma \rho_g, \quad \dot{\rho}_g = -3H\rho_g - \Gamma \rho_g, \quad \dot{\rho}_l = -3H\rho_l, \end{aligned} \qquad (18)$$

where ρ_c, ρ_g, and ρ_l are the energy densities of condensate, excited boson gas, and the localized energy density after the rapid collapse, respectively. The latter two are classified as the cold dark matter.

This set of evolution equations actually applies when the condensate does not dominate the energy density: $\rho_c < \rho_g + \rho_l$. Once it dominates $\rho_c > \rho_g + \rho_l$ after the time scale Γ^{-1} at around $z = z_c$, inhomogeneous components of the condensate would rapidly collapse, and some fraction of ρ_c is transformed into ρ_l. Then the condition $\rho_c < \rho_g + \rho_l$ is recovered and the gradual sedimentation of the condensate proceeds again following Eq.(18) during the time scale Γ^{-1}. This repeated "chase and collapse" process by DE(ρ_c) and DM($\rho_g + \rho_l$) self-regularizes the ratio of them to be fixed of order unity: $\rho_c \approx \rho_g + \rho_l$. (Figure 3.) This kind of autonomous dynamics designated as Self Organized Criticality (SOC) is widely known and observed in nature in various places[10].

In the late stage when the Universe is locked in this SOC phase, let us approximate $\rho_c = \rho_g + \rho_l$. Then the Einstein equation

$$\begin{aligned} &\ddot{a}(t) = -(4\pi G/3)(\rho + 3p)a(t), \\ &(\dot{a}/a)^2 = (8\pi G/3)\rho \end{aligned} \qquad (19)$$

has the solution

$$a(t) \propto t^{4/3}, \quad \rho(t) \propto a(t)^{-3/2} \qquad (20)$$

which corresponds to the deceleration parameter $q = -\ddot{a}a\dot{a}^{-2} = -1/4$.

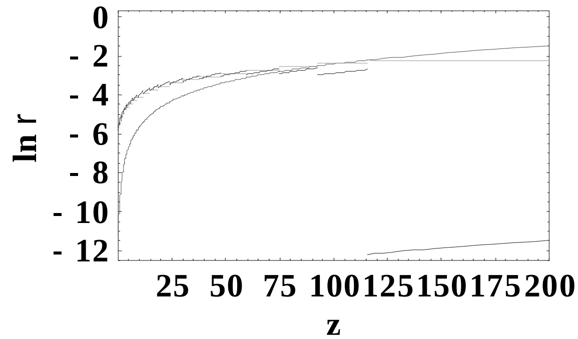

Figure 3. A simulation of the set of Eq.(18). The various energy density is plotted against the redshift. The green line represents the BEC condensation ρ_c, the red line represents the gas component ρ_g, and the blue line represents the compact collapsed component ρ_l. We set the parameter $\Gamma = 0.01$ in the unit of $8\pi G/3 = 1$, and the transition rate from ρ_c to ρ_l is set to be $1/3$ at each BEC collapse. The first BEC collapse occurs, in this simulation, at redshift about 120, which is well after the photon decoupling. After 22 cycles of BEC collapse and ρ_c dominance, DE(ρ_c) and DM($\rho_g + \rho_l$) autonomously self-regularize their ratio to be of order unity. These basic features are robust and are independent of the choice of parameters.

5. PREDICTIONS AND OBSERVATIONAL TESTS OF THE MODEL

The above unified model of DE/DM through BEC phase transition has various characteristic properties in practical applications. We now examine some of them and show how our model is tested.

(a) Power spectrum of the density fluctuations

The collapse of the condensate proceeds in the smallest scale. This is because the density fluctuation is stronger in the smaller scale, and the collapse proceeds with almost the speed of light. Consequently vary compact highly non-linear objects are coherently formed everywhere in the Universe. This feature may mislead the reader to believe that all the fluctuations are extraordinary enhanced in our model. We now see this is actually not a case. The essence is the fact that the unstable period is too short for the density fluctuations to evolve sufficiently.

Suppose the linear density fluctuation of the scale $l = (a/k) = 1/\tilde{k}$ with $\tilde{k}/H \gg 1$ which corresponds to the small physical scale, and we assume the power law cosmic expansion $a \propto t^{2/3} \propto H^{-2/3}$ for the moment. Then according to Eq.(10), the growing mode behaves as

$$\delta \approx a^{2\tilde{k}/H} \approx t^{4\tilde{k}/(3H)}. \qquad (21)$$

On the other hand the time duration for the region of scale l to collapse according to the previous mechanism would be $t_c = l/c$ which is proportional to $1/\tilde{k}$. Then within this time interval $t_c = l/c$, the density fluctuation evolves as

$$\frac{\delta_{t+t_c}}{\delta_t} = \left(1 + \frac{H}{\tilde{k}}\right)^{4\tilde{k}/(3H)} \approx e^{4/3} \approx 3.8 , \quad (22)$$

and after this short period, after the BEC collapse, the Universe recovers the ordinary evolution dynamics dominated by the cold dark matter. The above ratio is the largest enhancement; larger scale density fluctuation is less amplified. Actually, for example, the fluctuation mode of the scale ten times larger than l would have the enhancement factor about 1.1 . Even if the cosmic expansion is the power law, $a \propto e^{Ht}$, the enhancement in the density contrast becomes

$$\frac{\delta_{t+t_c}}{\delta_t} = e^{2\tilde{k}t_c} \approx e^2 \approx 7.4 \quad (23)$$

The reason of these comparatively small enhancements in the limited region of scale is the fact that, despite the quite large instability for δ , the evolution period is limited to be within a tiny time interval $[t, t+t_c]$. Therefore the overall enhancement factor is cancelled and stays to be a tiny value.

Of course the enhancement factor below the scale l is extremely large and well within the non-linear regime. Since such compact objects are uniformly distributed everywhere in the Universe, they cannot gravitationally affect the fate of the larger scale density fluctuations.

(b) Power spectrum of CMB

The collapse of the condensate must take place well after the photon decoupling time ($z_c < 1000$). Otherwise, the BEC collapse definitely leaves any imprints in CMB fluctuation pattern and the model severe conflicts with observations. In our previous simulations, the first collapse takes place at about redshift 120.

However the integrated Sacks-Wolfe effect, which originates from the non-uniform gravitational potential after the decoupling time, has a chance to modify the spectrum. Even this is the case, since the strongest non-linear structure forms in the smallest fluctuation size in our BEC model, such effect would actually never been detected.

(c) Compact objects

Our model has conspicuous features especially in the smaller scales. Once the condensate dominates the comic energy density, vary rapid collapses take place in the smallest scale of density fluctuations everywhere in the Universe. Associated with this process, the collapsing object can easily fragment into many pieces because the pressure is always negative. Therefore we can expect the large amount of collapsing objects.

For bosons, only the Heisenberg uncertainty principle can support the structure against the complete collapse. This is the quantum pressure expressed in the last term of the LHS in Eq.(5). This structure is known as the boson star [11] . The size R of the object is of order of the compton wave length: $\lambda_{comton} = \frac{2\pi\hbar}{mc} \approx 2R$. This size must be larger than the Schwarzschild radius $R > 2GM/c^2$ for this object not to collapse into a black hole. These equations yield the critical mass for the boson star,

$$M_{critical} \approx m_{pl}^2 / m \equiv M_{KAUP} , \quad (24)$$

only below which a structure can exist. For example, $m = 10^{-5}$ eV yields the critical mass abut the Earth mass: $M_{critical} = 10^{-5} M_{\odot} = M_{\oplus}$. These compact structures would have captured in the process of star formation. Moreover, such seed boson star and the condensate should be melted into boson gas in the high temperature stellar center, leaving no detectable relic in principle.

(d) A first galaxy harbors a giant black hole.

If the fluctuation mass is larger than the critical mass $M > M_{critical}$, then the collapse continues until a black hole is formed. In this process, since $p < 0, dV < 0$, no heating is expected thermodynamically. However the gravitational energy released in this collapse would be GM^2/R , and if this amount of energy is used to heat up the condensate, then the temperature would be, from $NT \approx GM^2/R$ and $R \approx GM$,

$$T \approx \frac{GM}{R} m \approx m . \quad (25)$$

Precisely at this point the boson becomes relativistic $T \approx m$ and the particle number no longer conserves. Then the chemical potential is not well defined and the condensation melts into the ordinary gas with positive pressure. Therefore the boson gas stops collapse and violently expands outward. In this process, some fraction of the condensate would form a central black hole and the rest of the condensate would melt into the ordinary gas and expand outward. The point of our model is the fact that the formation of giant black holes is quite easy and natural. Even more, there would be a variety in mass spectrum of black holes thus formed through BEC collapse.

The gravitational potential of this structure attracts baryon to form a cluster around the central black hole. If the size is appropriate there forms a galaxy, which harbors a black hole in the center and the boson gas and baryon in the outskirts. The size of the main part of the galaxy would be determined by the initial gravitational potential formed by the central black hole.

What would happen for the expanding boson gas? The expansion of once melted boson gas keeps the relation

$$T \approx \frac{GM}{R} m, \rho \approx \frac{M}{R^3} \quad (26)$$

and therefore $T \propto \rho^{1/3}$. Since this temperature is still below the critical temperature, the boson gas would eventually re-collapse. Then the condensation melts and the boson expands again. Apparently this bounce repeats multiple times with dissipation until the gas thermalizes completely.

Our model can provide a mechanism of biasing, which is inevitable in the ordinary scenario of structure formation only through the gravitational instability. The early formation of highly non-linear objects is due to the pressure of BEC and not due to the gravity. In the literature, there have been many such non-gravitational scenarios of the structure formation. Our model can be seen as one of those approaches. Our model is strongly bottom-up type.

(e) Re-ionization of the Universe

One of the conspicuous features of BEC model is the *sudden coherent formation of many huge black holes everywhere in the Universe in the very early stage*. The coherence is naturally expected since the BEC collapse is a phase transition. According to this scenario, the first astronomical object which emits light would be the system of a black hole and the accreting matter around it. Let us now call this system as quasar in general.

Quasars can re-ionize the Universe more efficiently that the ordinary stars. This is because the observation of the distant quasars, assuming they are the same type as the primordial quasars in our model, reveals that the luminosity of the quasars are about 100 to 1000 times larger than that of the ordinary galaxies as simple collection of shining stars. Moreover, all the features of BEC model such as "early sudden coherent formation of many huge black holes everywhere" naturally lead to the concept of "cosmic re-ionization phase transition". Quantitative investigations are at present under progress.

6. SUMMARY AND FURTHER DEVELOPMENTS

We have so far explored a cosmological model based on the BEC. The condensate of the boson is identified as the dark energy (DE) and the excited gas, as well as the collapsed component, is naturally identified as dark matter (DM). In order to guarantee the cosmic acceleration observed, BEC should have negative pressure which is, for example, induced by the attractive self-interaction of the boson field. This point is different from other approach to yield negative pressure based on the vacuum property.

Once gradually condensed DE dominates the cosmic energy density at $z = z_c$, rapid collapses of the condensate occur in the smallest scale to form compact objects such as boson stars and black holes. They provide the seeds of galaxies and quasars, the first object which emits light. This non-linear collapsing process should be studied more precisely.

After the depletion of BEC condensate, the sedimentation starts again until it dominates and rapidly collapses. This sedimentation-collapse cycle repeats many times and the Universe self-organizes the DE/DM ratio of order one.

In the context of BEC, there are many laboratory experiments using alkaline atom gas, in which almost ideal BEC is realized [12]. Especially the collapse dynamics of BEC with negative pressure [13, 14] may be useful to develop our cosmological BEC model. In this context, for the precise argument, we need to generalize the Gross-Pitaevskii equation Eq.(4) relativistic. The formal development is easy. Actually starting from the Kline-Gordon equation

$$\left(\Box + m^2\right)\phi = -\frac{\lambda}{3!}\left(\phi^\dagger\phi\right)\phi \tag{27}$$

where $\Box = \partial_\mu\partial^\mu = \partial_t^2 - \Delta$, and similar decomposition $\phi = Ae^{iS}$ and the definition of the velocity $u_\mu = \frac{\hbar}{m}\partial_\mu S$, $\vec{v} = \frac{\hbar}{m}\vec{\nabla}S$ yields the continuity equation and the relativistic fluid equation.

$$\varepsilon\frac{\partial\vec{v}}{\partial t} + \vec{\nabla}\left(\frac{m\vec{v}^2}{2} + \frac{1}{2}m^2 + \frac{\lambda}{12}A^2 + \frac{\hbar^2}{2mA}\Box A\right) = 0 \tag{28}$$

where $\varepsilon = m\gamma$. In the non-relativistic limit $\varepsilon \to m$ and $\ddot{A} \to 0$, this equation reduces to (5) with the correspondence $\lambda / \varepsilon \leftrightarrow 12g$. What we need is to extract the exact equation of state from this form. Further developments will be reported in some other places.

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