A Possible Solution to the Smallness Problem of Dark Energy

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The smallness of the dark energy density has been recognized as the most crucial difficulty in understanding dark energy and also one of the most important questions in the new century. In a recent paper[1], we proposed a new dark energy model in which the smallness of the cosmological constant is naturally achieved by invoking the Casimir energy in a supersymmetry-breaking brane-world. In this paper we review the basic notions of this model. Various implications, perspectives, and subtleties of this model are briefly discussed.

1. Introduction

The accelerating expansion of the present universe was first indicated by the type Ia supernova (SN Ia) data in 1998 [2, 3]. It is recently reinforced by new SN Ia data [4, 5] obtained using the Hubble Space Telescope. To explain this mysterious phenomena, cosmologists first resort to energy sources which would entail strongly negative pressure and thereby provide anti-gravity (i.e. repulsive gravitational) force, such as positive cosmological constant [6] and quintessence [7, 8]. It is in this viewpoint the term "dark energy" was introduced for the origin of the accelerating expansion.

In addition to the dark energy approach, which can be regarded as a modification of the right-hand side (i.e. the energy part) of the Einstein equations, there have also been attempts to modify its left-hand side (i.e. the geometry part) [9–16]. One may regard this latter category the "dark geometry" [11, 12, 16].

Among these proposals for the accelerating expansion, the positive cosmological constant approach appears conceptually the simplest. Nevertheless, while the notion of cosmological constant is hitherto consistent with all the observational results, it suffers the well-known fine-tuning problem, especially when facing the possible huge contribution from the quantum vacuum energy. The non-zero but extremely small cosmological constant, as implied by observations, remains a fundamental challenge.

Recently we proposed[1] a new dark energy model in which the smallness of the cosmological constant is naturally achievable by invoking the Casimir energy in a supersymmetry-breaking brane-world. In the following section we will give an overview of the existing approaches to the pre- and post-dark-energy cosmological constant problems. In Sec. 3 we review the basic notions of our model. We then investigate its implications and subtleties in Sec. 4, and briefly discuss future perspectives in Sec. 5.

2. Pre- and Post-Dark-Energy Cosmological Constant Problems

Before the discovery of the present accelerating expansion of the universe in 1998, the cosmological constant problem was how to make it vanish (pre-dark-energy). After 1998, there came another problem — how to make the cosmological constant slightly deviate from zero (post-dark-energy). The pre-dark-energy problem stems from our lack of an ultimate understanding of accommodating quantum vacuum in gravity. While it still awaits a profound answer from a future theory that would successfully combine general relativity and quantum theory, as we shall review below, some interesting proposals are already in sight. The post-dark-energy problem, on the other hand, appears soluble based on our current knowledge.

One interesting early idea for ameliorating the pre-dark-energy cosmological constant problem is, instead of making it small, to invoke extra dimensions such that the expansion rate of the imbedded (3+1) spacetime is independent of the vacuum energy [26]. This idea has received reviving interests in the post-dark-energy era, where physical models have been proposed based on the brane-world scenario. One such approach, for example, involves a codimension-two brane whose brane tension induces a deficit angle in the bulk, which in turn cancels the brane tension exactly [27–29]. Another approach [30–32] is to construct a modified Friedmann equation with the form $H^2 \propto (\rho + p)$, so that the vacuum energy (with the equation of state $\rho = -p$) would not contribute to the Hubble expansion. We shall refer to this generic idea, including the two types of models described above, as the self-tuning mechanism.

Even if the pre-dark-energy cosmological constant problem may eventually be solved by either the self-tuning mechanism or some as-yet undiscovered novel concept, the post-dark-energy problem would still remain. The tremendous hierarchy between the vacuum energy implied by the observations and the known particle physics energy scales remains a severe theoretical challenge by itself. What underlying physics could be responsible for such a tiny vacuum energy?

In Ref.[1] we focus on this latter issue, assuming that the pre-dark-energy problem can eventually be addressed. We point out that the same ingredients invoked in the self-tuning mechanism, that is, the extra dimensions and the brane-world scenario, when augmented with supersymmetry (SUSY), can in principle solve the post-dark-energy problem with very relaxed constraints.

SUSY guarantees the perfect cancellation of the vacuum energy. However, we know that SUSY has to be broken, at least in our (3+1)-dimensional world, with the symmetry-breaking scale $\sim$ TeV. Conventionally this would entail a vacuum energy that is much too large for dark energy. But we assume the brane tension so induced should be perfectly cancelled due to the self-tuning mechanism or its variant. If we further assume that SUSY is broken on the brane through, for example, a gauge-mediated SUSY-breaking mechanism (for a review, see [21]) while preserved in the bulk, then in this configuration we find that the leading contribution to the vacuum energy on the imbedded 3-brane a la Casimir effect can be dramatically suppressed relative to the SUSY-breaking scale.

We emphasize that there is a fundamental difference between such a Casimir energy and the conventional vacuum energy. The Casimir energy in our SUSY configuration is nontrivial only around the 3-brane, and, in particular, it entails the equations of state: $p_a = -\rho$ and $p_b > 0$, where p_a and p_b are its pressures along the 3-brane and the extra dimensions, respectively. In contrast, the brane tension from the conventional vacuum energy obeys the following equations of state: $p_a = -\rho$ and $p_b = 0$, which are a necessary condition for its cancellation via the self-tuning mechanism. Thus the Casimir energy cannot be removed by the same self-tuning procedure and should survive as the leading contribution to vacuum energy on the brane.

A similar concept of SUSY-breaking in a brane-world, called supersymmetric large extra dimensions (SLED), has been invoked to address the vacuum energy and the cosmological constant problem [28]. In this proposal SUSY in the bulk is slightly broken by the presence of non-supersymmetric 3-brane. By incorporating the self-tuning mechanism for two extra dimensions, the brane tension is exactly cancelled with the curvature [27], and as a result the leading contribution to vacuum energy is induced by the weak SUSY-breaking in the bulk. The smallness of dark energy in this proposal relies on the requirement of two large extra dimensions of the size around 0.1 mm, which is on the edge of the current experimental constraint [19].

3. Casimir Dark Energy in a Supersymmetry-Breaking Brane-World

The possibility that the Casimir energy in extra dimensions can play the role of dark energy has been explored by Milton [17]. In general, a positive Casimir energy in ordinary (3+1) dimensions cannot entail a negative pressure, while in extra dimensions, on the contrary, it can in principle induce a cosmological constant with the desired attribute. Its sign depends on the geometry, the boundary condition, and the field contents in the extra space. If all extra dimensions have the same size a, the value of the Casimir energy density measured on an imbedded 3-dimensional space is in general proportional to a^{-4}. Imposing the observed dark energy density of the universe, which is very close to the critical density ρ_c as indicated by CMB data [18], one can arrive at a lower limit on the extra-dimension size. From this requirement Milton obtained a stringent constraint. That is, the number of extra dimensions cannot exceed two and the extra-dimension size should be larger than microns [17]. This constraint on the extra-dimension size is not far from the current experimental upper limit around 100 microns [19].

Stringent constraints and fine-tunings are common features in many dark energy/dark geometry models. Generally speaking, it is very difficult to retain the smallness of the dark energy density if the energy scale invoked in the dark energy/dark geometry model is not a natural scale required by some symmetry principle. In our recently proposed dark energy model, which invokes the Casimir effect in a supersymmetry-breaking brane-world, this difficulty is apparently resolved [20].

In our model we consider a (3+n+1)-dimensional space-time with n compact extra dimensions, in which the standard model fields and their superpartners are confined on a imbedded 3-brane while the gravity (graviton-gravitino) sector resides in the (higher-dimensional) bulk. We assume that supersymmetry is broken, for example, through a gauge-mediated supersymmetry-breaking mechanism (for a review, see [21]), only around the brane while preserved in the bulk. In this configuration we find that the leading contribution to the vacuum energy a la Casimir effect can be dramatically suppressed relative to the supersymmetry-breaking scale.

As an example for demonstration, here we present the result of the case in which extra dimensions geometry is toroidal and only graviton and gravitino reside in the bulk. As a result of our supersymmetry-breaking assumption, gravitino acquires a mass m only around the brane and graviton remains massless everywhere. The resultant net Casimir energy density on the imbedded 3-brane can be shown to be

$$\delta\rho^{(4)}_{\not{S}} \cong \alpha_n \cdot m^2\, a^{-2} \cdot \left(\frac{V_\delta}{V_a}\right), \tag{1}$$

where V_δ, $\propto \delta^n$, is the extra-dimensional volume of the supersymmetry-breaking region, V_a, $\propto a^n$, is the total extra-dimensional volume, and α_n is a constant factor depending on the number of extra dimensions n and the boundary conditions in extra dimensions. It is natural to have the situation that $V_\delta \ll V_a$. Consequently the ratio V_δ/V_a provides a strong suppression factor in the above formula of the Casimir energy density. This suppression is a manifestation of the sharp contrast between the volume of the supersymmetry-breaking region around the brane and that of the supersymmetry-preserving bulk. This is the key for achieving the smallness of the cosmological constant/dark energy in our model.

The quantities a, δ, and m involved in the above expression of the Casimir energy density are related to several fundamental energy scales: the fundamental gravity scale $M_{\rm G}$, the string scale $M_{\rm S}$, the supersymmetry-breaking scale $M_{\rm SUSY}$, and the Planck scale $M_{\rm pl}$, as follows.

$$a \sim \left(M_{\rm pl}/M_{\rm G}\right)^{2/n} M_{\rm G}^{-1}, \tag{2}$$

$$\delta \sim l_{\rm S}\ (\text{string length}) = M_{\rm S}^{-1}, \tag{3}$$

$$m \sim M_{\rm SUSY}^2/M_{\rm pl}\,. \tag{4}$$

The Casimir energy expression can then be rewritten in terms of these fundamental scales as:

$$\delta\rho^{(4)}_{\not{S}} \sim \alpha_n\, M_{\rm G}^{(n+2)^2/n} M_{\rm S}^{-n} M_{\rm SUSY}^4 M_{\rm pl}^{-4-4/n}\,. \tag{5}$$

Identifying $\rho^{(4)}_{\not{S}}$ as the dark energy density $\sim 3\times 10^{-11}\,{\rm eV}^4$, we arrive at the following constraint among these fundamental scales:

$$\left(\frac{M_{\rm G}}{M_{\rm pl}}\right)^{(n+2)^2/n}\left(\frac{M_{\rm S}}{M_{\rm pl}}\right)^{-n}\left(\frac{M_{\rm SUSY}}{M_{\rm pl}}\right)^{4} \sim 10^{123}\,\alpha_n^{-1}\,. \tag{6}$$

This constraint is quite loose. Namely, it can be satisfied by a wide range of $M_{\rm G}$, $M_{\rm S}$ and $M_{\rm SUSY}$ values. Its looseness indicates that the smallness of the dark energy is easily attainable in our model. In the following we will see that this constraint remains flexible even after additional conditions are imposed.

Although there is no a priori reason why these scales should be related, it is desirable to reduce the large hierarchy among various energy scales. Motivated by this, we further impose two separate conditions in Eq. (6): (a) $M_{\rm SUSY} \sim M_{\rm G}$, i.e. to bridge the hierarchy between the SUSY-breaking scale and the fundamental gravity scale; (b) $M_{\rm S} \sim M_{\rm G} \sim M_{\rm SUSY}$, i.e. to insist that there is only one energy scale in our physics.

First we focus on the scenario where the mass shift of the gravitino is suppressed by the Planck scale: $\mu \sim M_{\rm SUSY}^2/M_{\rm pl}$ (i.e. $\eta \sim M_{\rm SUSY}/M_{\rm pl}$). Let us further assume that the values of α_n do not vary too drastically. Then in case (a) our general constraint, Eq. (6), is reduced to a more specific constraint on $M_{\rm S}$ and $M_{\rm G}$ under different choices of the extra-dimensionality, as represented by solid curves in Fig. 1. If one further insists on condition (b), then the solutions further reduce to the intersects between the dashed line for $M_{\rm S} = M_{\rm G}$ and the solid curves. Concentrating on the energies between TeV scale and

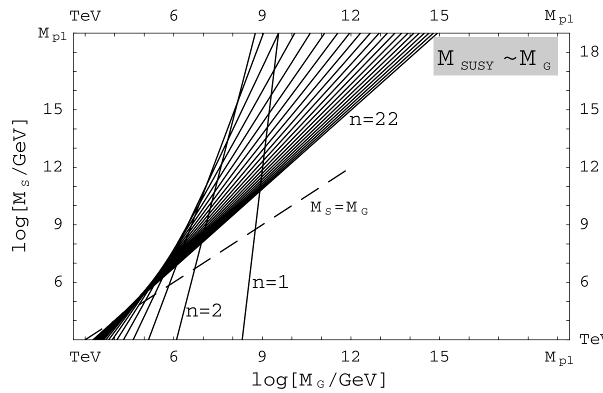

Figure 1: Constraint on $M_{\rm S}$ and $M_{\rm G}$ under the assumption of gravitino dominance: $\mu_{\rm MAX} \sim M_{\rm SUSY}^2/M_{\rm pl}$ (i.e. $\eta \sim M_{\rm SUSY}/M_{\rm pl}$). The solid curves correspond to solutions under the further assumption of $M_{\rm SUSY} = M_{\rm G}$ and the dashed line indicates the condition $M_{\rm S} = M_{\rm G}$

the Planck scale, we find that in case (a) $M_{\rm G}$ cannot exceed $10^{15}\,$GeV while the string scale $M_{\rm S}$ is barely restricted. In case (b), the values of these quantities are restricted in the range between TeV and $10^9\,$GeV. We note with interest that for large n, the solutions are approaching the TeV range, a soon-to-be testable scale in Large Hadron Colliders.

It is also possible that the dominant mass shift $\mu_{\rm MAX}$ is roughly of the same order of the SUSY-breaking scale $M_{\rm SUSY}$ (i.e. $\eta \sim 1$). So we repeat the same exercise but replace the condition $\eta \sim M_{\rm SUSY}/M_{\rm pl}$ by $\eta \sim 1$. The results are shown in Fig. 2. Case (a) retains similar qualitative features as the previous case regarding the gravitino mass shift. Namely, $M_{\rm G}$ is quite restricted (especially for small n) while $M_{\rm S}$ is barely restricted. In case (b), the constraint is so severe that only the case of $n = 2$ survives. In this case (n=2) all fundamental scales are merely of the order of a TeV and therefore can be tested in the near future. We note that in the case of one extra dimension, where $M_{\rm G}$ is required to be around $10^9\,$GeV (corresponding to the extra-dimension size around 100 meters), the solution is already ruled out.

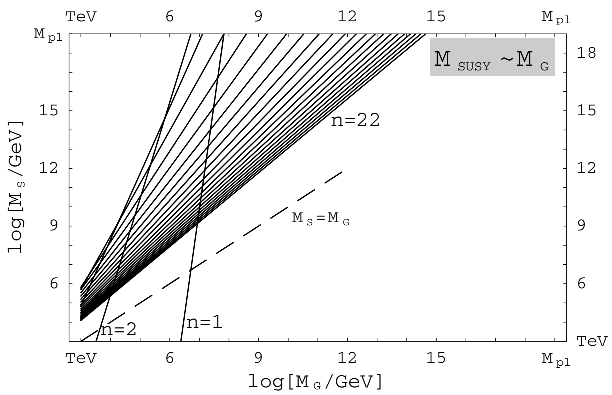

Figure 2: Constraint on $M_{\rm S}$ and $M_{\rm G}$ under the assumption $\mu_{\rm MAX} \sim M_{\rm SUSY}$ (i.e. $\eta \sim 1$). The solid curves correspond to solutions under the further assumption of $M_{\rm SUSY} = M_{\rm G}$ and the dashed line indicates the condition $M_{\rm S} = M_{\rm G}$

4. Perspectives

In our approach the smallness of the dark energy is attained through the sharp contrast between the volume of the supersymmetry-breaking region around the brane and that of the supersymmetry-preserving region in the bulk, where their ratio naturally arises when one integrates out the imbedding extra dimensions to obtain the Casimir energy on the 3D brane. To demonstrate the powerfulness of this new way of handling the post-dark-energy problem, we have examined such Casimir effect under a wide range of extra dimensions, without limiting ourselves to the specific 1 and 2 extra dimensions previously invoked in the self-tuning models.

There exist various hierarchy problems in physics, such as the weakness of gravity and the smallness of the cosmological constant. To reveal the fundamental laws of nature, it is often desirable to relate the origins of the hierarchies to more profound physics. For example, the weakness of gravity can be explained as a result of the largeness of extra dimensions (as in the Arkani-Hamed-Dimopoulos-Dvali (ADD) model [23]) or the warpage of higher-dimensional geometry (as in the Randall-Sundrum (RS) model [24]). In this regard our model follows the same spirit as that of the ADD and the RS models. Note that by further invoking $M_{\rm SUSY} \sim M_{\rm G}$, our model manages to solve both hierarchy problems at once. In addition, since our model is based on the similar ingredients as those in the self-tuning mechanism, it provides much hope that a synergy between these two concepts may be found for a complete solution to both pre- and post-dark-energy problems in one stroke.

In the current construction of our model the configuration of the extra dimensions is ADD-like [23]. It is interesting to further explore our scenario under the RS configuration [24].

The possibility of time evolution of the extra-dimension size is another important issue yet to be studied. In many models that invoke extra dimensions, their specific sizes are required to be stable in order to satisfy the experimental constraints from the solar-system tests of gravity. This is generally assumed by introducing a certain stabilization mechanism (e.g., see [22]). By invoking similar arguments, the Casimir energy on the brane in our model is exactly a cosmological constant in our 3-dimensional world. On the other hand, it is possible that at some cosmic epochs the stabilization mechanism was not effective so that the extra dimensions could evolve. In this case the Casimir energy in our model may change in time. This possibility of a time-varying dark energy is worthy of further investigation.

The notion of Casimir energy involves the concept of a reference energy. In our model this reference energy is associated with a Minkowskian, non-compact, higher-dimensional space-time without boundary condition, with no particles, and no other energy forms. Whether such a energy reference point is conceptually meaningful requires further investigation. A different energy reference point may be required, especially under the situation where certain extra dimensions stabilization mechanism is introduced to the system.

The ultimate challenge of the dark energy issue is evidently finding a complete, self-consistent solution to the problem. In our model we bypassed the pre-dark-energy problem and assumed that it would eventually be addressed by the self-tuning mechanism or its variant. It is known that the self-tuning idea has its own challenges[29, 32], and it clearly awaits further developments before the pre-dark-energy problem can be fully addressed. But even if that can be done, one still faces the challenge of reinstating the very small amount of vacuum energy self-consistently to comply with the observation. With this in mind, the further integration of our model with the self-tuning mechanism would be extremely interesting.

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