A Simple Accelerating Model of the Universe in Higher Dimensional Spacetime

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We have, in this work, discussed a scenario in homogeneous 5D spacetime which admits a decelerating expansion in the early epoch along with an accelerated phase at present in line with the current observational results. The most important finding, in our opinion, is the result that it is possible to achieve this acceleration without introducing any external quintessence -like scalar field or vacuum energy into the theory- the presence of the extra dimension , so to say, seems to cause the expansion to accelerate.This result is quite interesting *vis a vis* the current attempts to construct accelerating models as in quintessential universe. Encouraging to point out that as time evolves the extra dimension shows the desirable feature of dimensional reduction so that the model finally ends up as a 4D world. We have further noted that asymptotically our model mimics a steady state type of universe as advocated by Gold and Bondi as also by Hoyle and Narlikar although it originates from a big bang type of singularity.

1. INTRODUCTION

Recent advance based on measurements of anisotropies in the cosmic microwave background by DASI, BOOMERANG [1] and MAXIMA [2] groups along with the observational results coming from the type Ia Supernova [3] suggest that most of the energy of the universe consists of some form of dark energy that is gravitationally repulsive and is causing the expansion of the universe to accelerate. One is tempted to believe that in the universe there exists an important matter component, in its most simple description , has the characteristic of a cosmological constant i.e., a vacuum energy density which contributes to a large component of a negative pressure. However, the inability of the particle theorists to compute the energy of the quantum vacuum - contributions from well understood physics amount to 10^{55} times the critical density - casts a dark shadow on the feasibility or otherwise of the presence of the constant. A rather important issue is the coincidence problem: dark energy seems to start dominating the energy budget, and accelerating the expansion of the universe, just around the present time. A number of other candidates have also been proposed: rolling scalar field (a quintessence) [4], Brans-Dicke type of scalar fields [5] and a network of frustrated topological defects, to name a few. While these and other models have some motivation and also attractive features none of them are compelling. In view of what has been stated above we have thought it worthwhile to examine a scenario in multidimensional spacetime where the accelerated expansion of the universe at the current epoch may be made possible without forcing ourselves to invoke any extraneous scalar field or vacuum energy.We have here taken a five dimensional spatially flat, homogeneous spacetime with perfect fluid as matter field and assuming a specific form of the 3D scale factor we have been able to show that the universe decelerates at the early era (a good news for structure formation) and after a certain instant starts accelerating in conformity with the present day observations. Higher dimensional spacetime is now an active field of activity in both general relativity and particle physics in its attempts to unify gravity with all other forces of nature [6]. These theories include kaluza-klein , induced matter, super string, supergravity and string. In these (4+d) dimensional models the d-spacelike dimensions are generally spontaneously compactified and the symmetries of this space appear as gauge symmetries of the 4D theory. At present these extra dimensions are not observed presumably because with time they shrink to an unobservably small length, say plankian. However standard cosmology indicate that the scale factor for the extra dimensions at some epoch in the past could have been comparable with or even larger than, that of the usual three dimensional space. Renewed interests to these models also stem from their recent applications to brane-cosmology. Our paper is organised as follows: After Introduction in section 1 we discussed the mathematical formalism and its implications in section 2 and the paper ends with a short discussion in section 3.

2. FIELD EQUATIONS AND ITS INTEGRALS

We here discuss a spatially flat 5D homogeneous cosmological model with the topology $M^1 \times R^3 \times S^1$ where S^1 is taken in the form of a circle such that

$$ds^2 = dt^2 - R^2(t)\left(dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2\right) - A^2(t) dy^2 \tag{1}$$

where $R(t)$ is the scale factor for the 3D space and $A(t)$, that for the extra dimension and y is the fifth dimensional coordinate. The independent field equations for our metric (1) and energy momentum tensor

$$T_{ij} = (\rho + p + p_5) v^i v^j - (p + p_5) g_{ij} \tag{2}$$

are given by

$$3\frac{\dot{R}^2}{R^2} + 3\frac{\dot{R}}{R}\frac{\dot{A}}{A} = \rho \quad (3)$$

$$2\frac{\ddot{R}}{R} + \frac{\dot{R}^2}{R^2} + 2\frac{\dot{R}}{R}\frac{\dot{A}}{A} + \frac{\ddot{A}}{A} = -p \quad (4)$$

$$3\frac{\ddot{R}}{R} + 3\frac{\dot{R}^2}{R^2} = -p_5 \quad (5)$$

where p_5 is the pressure in the fifth dimension while that for the 3D space is isotropic and is given by p. As we have five unknowns (R, A, ρ , p and p_5) with three independent equations we are at liberty to choose two connecting equations. We assume that $p = p_5$. From this condition we get from the field equations

$$\ddot{A} + 2\frac{\dot{A}\dot{R}}{R} - A(\frac{\ddot{R}}{R} + 2\frac{\dot{R}^2}{R^2}) = 0 \quad (6)$$

To solve this equation we put $A = Ru(t)$ such that

$$R\ddot{u} + 4\dot{R}\dot{u} = 0 \quad (7)$$

such that

$$\dot{u} = \frac{\beta}{R^4} \quad (8)$$

where β is an arbitrary constant. As we have another choice we make the ansatz that $R = sinh^n\omega t($ where n is an arbitrary constant) such that the above equation reduces to

$$\dot{u} = \frac{\beta}{sinh^{4n}t} \quad (9)$$

We here consider only two cases corresponding to $n = 1/2 \; and \; 1/4$. As we are considering an expanding model any negative values of n are ruled out.

2.1. CASE I

Skipping intermediate mathematical steps for economy of space we give the final expressions for $n = 1/2$ as

$$A = sinh^{\frac{1}{2}}\omega t(\gamma - \beta coth\omega t) \quad (10)$$

$$R = sinh^{\frac{1}{2}}\omega t \quad (11)$$

With the above form of the metric coefficients

$$p = p_5 = -3\omega^2/2 \quad (12)$$

$$\rho = \frac{3}{2}\frac{cosh^2\omega t}{sinh^2\omega t} + \frac{3\beta cosh\omega t}{2sinh^3\omega t(\gamma - \beta coth t)} \quad (13)$$

Interestingly as $t \sim 0$, $\rho = \rho_0 = \frac{3\omega^2}{2}$ and as $t \sim \infty$, $\rho_\infty = \frac{3\omega^2}{2}$ So asymptotically the mass density tends to assume a constant value and the cosmology mimics a steady state type of behavior though not exactly following the type advocated by Bondi and Gold or Hoyle and Narliker. Further as $t \sim 0$, $R \sim t^{1/2}$ and $A \sim t^{-1/2}$ So at the early era the spacetime resembles the well known form given by Chodos and Detweiler[7]. The temporal behavior of the model depends critically on the initial conditions. If the arbitrary constant$a$ is made zero the extra dimension starts from an infinite extension, shrinks to a minimum and then expands again indefinitely so that there is no dimensional reduction in this case However the large extra dimensions in the theory are not that much out of favor these days as it attempts to address the well known hierarchical problem in quantum field theory.Further with time the model isotropises in this case and the 4D volume expands.On the other hand with non vanishing a the extra dimension contracts and ultimately vanishes exhibiting the desirable feature of dimensional reduction.In this case the model again becomes singular. Theorists try to save the situation via assuming varied stabilizing mechanisms [8]like quantum gravity, Casimir effect so that they produce a sort of repulsive potential to halt the shrinkage at a very small constant value,say planckian length so that the extra dimensions essentially decouple as all its derivatives vanish in the field equations. The cosmology now enters the standard 4D phase following the FRW model without having any reference to the extra dimensions.
A serious shortcoming of this analysis is that there is no dynamical evolution in the expression for pressure. We shall shortly see that this defect is overcome in our next section.

2.2. CASE II

In this case $n = 1/4$ and skipping the intermediate mathematical steps we write the final results as

$$R = sinh^{\frac{1}{4}}\omega t \quad (14)$$

$$A = sinh^{\frac{1}{4}}\omega t(\beta \; ln \; tanh\frac{\omega t}{2} + \gamma) \quad (15)$$

where γ is an arbitrary constant. With the above values of the R and A we get the following expressions for the pressure matter density as

$$p = p_5 = -\frac{3\omega^2}{8sinh^2\omega t}(sinh^2\omega t - 1) \quad (16)$$

$$\rho = \frac{3\omega^2}{8}\frac{cosh\omega t}{sinh^2\omega t}(cosh\omega t + \frac{2\beta}{\beta \; ln \; tanh\omega t + \gamma}) \quad (17)$$

As commented earlier this model does not suffer from the disqualification of a constant pressure. Here both pressure and mass density are evolving and start from an infinite value as a big bang singularity.But it has not escaped our notice that both the physical quantities assume steady values asymptotically at $\frac{3\omega^2}{8}$. So unlike the big bang type it resembles more a steady state type cosmology.But the pressure changes signature from positive to negative at $sinh\omega t = 1$. However it is not difficult to to explain the asymptotically steady value of the matter field because a little algebra shows that with time the 4D volume,

$$V = R^3 A = sinh\omega t(\beta ln\ tanh\omega t + \gamma) \qquad (18)$$

stabilizes at some finite value. Another striking difference from the earlier case is that here both the scales start from zero and depending on the signature of the arbitrary constant γ the fifth dimension either expands indefinitely or collapses at a finite time.

3. ACCELERATED UNIVERSE

As commented in our introduction this model admits of both deceleration at the early phase and acceleration at present. The early deceleration is physically relevant in the sense that it allows structure formation while the present day acceleration is in conformity with the current observations. For the general case $R = sinh^n \omega t$, we get for deceleration parameter $q = -\frac{\ddot{R}R}{\dot{R}^2} = -\frac{ncosh^2\omega t - 1}{ncosh^2\omega t}$. Thus for $n \geq 1$, $q < 0$. Hence always accelerating.

For case I, $n = \frac{1}{2}$ and $q = \frac{1 - sinh^2\omega t}{1 + sinh^2\omega t}$. Let $sinh\omega t_c = 1$. Hence for $t < t_c$, $q > 0$ (deceleration) and for $t > t_c$, $q < 0$ (acceleration).

For case II, $n = \frac{1}{4}$ and $q = \frac{3 - sinh^2\omega t}{cosh^2\omega t}$. Let $sinh\omega t_c = \frac{1}{\sqrt{3}}$. Hence for $t < t_c$, $q > 0$ (deceleration) and for $t > t_c$, $q < 0$ (acceleration).

Hence acceleration starts earlier in the second case.

4. DISCUSSION

While vast literature exists to address the observational fact of the current accelerated expansion of the universe we are not aware of models of similar kind in the framework of higher dimensional spacetime.We have here discussed a scenario where in homogeneous 5D spacetime which admits a decelerating expansion in the early epoch along with an accelerated phase at present in line with the current observational results. The most important finding, in our opinion is the result that it is possible to achieve this acceleration without introducing any external quintessence=like scalar field or vacuum energy into the theory-the presence of the extra dimension , so to say, seems to cause the expansion to accelerate. Another desirable feature is the phenomenon of dimensional reduction so that the model finally reduces to an effective 4D one. This takes place in both the cases discussed here.But the most serious shortcoming is the absence of any mechanism to achieve the stabilization of the extra dimension at a very short length. It is not apparent from our analysis how that stabilization work in our model. However in an earlier work Guendelman and Kaganovich [9] studied the Wheeler-Dewitt equation in presence of a negative cosmological constant and dust.They got the interesting result that the quantum effects do stabilize the volume of the universe,thus providing a mechanism of quantum avoidance of the singularity.

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