Two Astrophysical Phenomena in Higher Dimensional Cosmology

S.Chatterjee
Department of Physics , New Alipore College , Kolkata 700 053, INDIA

Assuming a homogeneous matter energy content such that $p = p(t)$ *and* $\rho = \rho(t)$) we have obtained exact solutions in cosmological models in higher dimensions. The explicit dependence of the scale factor on time can be obtained via the assumption of an equation of state. Utilizing one of the solutions an expression .of the luminosity distance function is found in higher dimension and it is observed that the de Sitter space time sets an upper limit to the absolute distance of any cosmological source, where the generally accepted value of the deceleration parameter, $q > -1$ is taken. Further some remarks of general nature are made on the nucleosynthesis in higher dimensional world with the help of time temperature relation and it is conjectured that dimensionality may have significant impact, in principle at least, on the phenomena of nucleosynthesis. However, it is too premature to come to any definite conclusion in this regard at this stage.

1. INTRODUCTION

In the present work we have thought it worthwhile to extend the pioneering work of Barnes [1] in the field of *luminosity distance* and also of Tinsley [2] relating to *primordial nucleosynthesis* in higher dimensional spacetime. Over the years the interests in higher dimensional theories have stemmed up in their attempts to unify gravity with other forces in nature [3]. The recent spurt in activities is also due to its new field of application in brane cosmology. Moreover, in the cosmological context the higher dimensional theories are particularly relevant in the early universe when all the spatial dimensions including the extra ones are treated in the same footing before the universe underwent the compactification transition. Concerned with the problems of the origin of elements, pioneering work in this field was done by Gamow. Current interests in this field also arise from the observational results of Ia supernova as also from anisotropy studies of CMBR that the expansion of the Universe is at present accelerated. In this work we have taken the case of one important aspect of the nucleosynthetic process in the early universe, namely the decoupling phase of the neutrinos. The analysis is , however, of preliminary status and lacks, at once the depth and sophistication required in this type of investigations. But since in the early universe the higher dimensional phase is particularly relevant the question of nucleosynthesis should be pursued more seriously in future in higher dimensional spacetime. In the next section we have addressed some problems relating to the *Luminosity distance* which has undergone a revival of interests following the recent evidence coming from the supernovae studies that of late the expansion of the Universe is accelerating.

2. MATHEMATICAL FORMALISM

We take the generalised (n+2) dim. metric as

$$dS_{-}^2 = e^{\nu} dt^2 - e^{\alpha} dr^2 - e^{\omega} dX_n^2 \tag{1}$$

where

$$dX_n^2 = d\theta_1^2 + \sin^2\theta_1 d\theta_2^2 + \cdots \sin^2\theta_1 \sin^2\theta_2 \cdots \sin^2\theta_{n-1} d\theta_n^2 \tag{2}$$

We have shown in an earlier work [4] that the line element (1)reduces via the postulate of homogeneity in matter energy distribution to

$$dS^2 = dt^2 - R^2[\frac{dr^2}{1-kr^2} + r^2 dX_n^2] \tag{3}$$

where k is the spatial curvature. Thus we have generalised to higher dimension the well known result that a 4D spherically symmetric metric whose source is a homogeneous perfect fluid must be of RW type. In the dust case it follows from the equation (3) that the deceleration parameter $q = -\frac{R\ddot{R}}{\dot{R}^2}$ is

$$q > \frac{n-1}{2} \tag{4}$$

This relation is interesting in the sense that q is , in principle at least, observationally verifiable. But people are hopelessly divided on the range- magnitude of q. Otherwise this relation may act as the *window* of the existence of the extra dimensional space. If we assume that the 4D formulation of the laws of thermodynamics can be extended to higher dimensional spacetime and also the interactions at the initial stage of the expansion of the universe led to thermodynamic equilibrium then the energy momentum conservation relation $T^{ij}_{j} = 0$ in comoving coordinates yields

$$\dot{\rho} + (n+1)\frac{\dot{R}}{R}(\rho + p) = 0 \tag{5}$$

If we assume that the radiation dominated era is characterized by $p = \frac{\rho}{n+1}$ we get a relation between between the energy density and the temperature of the radiation [5] as

$$\rho = a\ T_{rad}^{n+2} \tag{6}$$

where a is the radiation density constant. From the equation(5) it further follows that

$$T_{rad} = [\frac{2n(n+1)}{a(n+2)^2}]^{\frac{1}{n+2)}} t^{-\frac{2}{n+2)}} \tag{7}$$

where $T_{rad}$is the temperature of radiation, 'a' is the radiation density constant and 't' is the age of the Universe. This equation reduces to the familiar $T_{kelvin} = 1.52 \times 10^{10}\ t^{-\frac{1}{2}}s$ in the 4D case. This equation is crucial for the analysis to follow next, and also to investigate if the dimensionality has any role to play in the equilibrium phase and also to the decoupling phase of the elementary particles (neutrinos, for example).

2.1. Neutrino Physics

One of the great successes of the standard model is that it almost correctly predicts the primordial nucleosynthesis , particularly the observed abundances of the light nuclei. One may look into the relation (7) more closely to see if the existence of the extra dimensions has any perceptible effect on this phenomena.Excellent outline on the subject may be found in many texts (see for example [7]). We start from the time when the elementary particles are already in existence such that the low temperature approximation $\kappa\ T \ll m_\mu c^2$ holds for the distribution function. Here m_μ is the mass of a particular species. From the equation (7) it follows that the temperature falls less rapidly in multidimensional spacetime compared to 4D one. As is well known , the reactions involving the neutrinos fall in the category of weak interaction and the cross section of a typical reaction is of the order $A\ f^2 h^{-4} c^{-4} (kT)^2$ where f is a weak coupling constant. Since the number density of the participating particles are of the order $(kT/cH)^{n+2}$ the key equation to the analysis is $Q/H = (\frac{T}{10^{10k}})^{\frac{n+4}{2}}$, where Q is the reaction rate and H is the expansion rate given by $H = t(kT)^{\frac{n+2}{2}}$. The two equations suggest an important difference from the analogous 4D case. Here it takes a relatively longer time for the elementary particles to fall below their critical temperature characterised by $T_c = m_0c^2/k$. For $T > 10^{10}K$ the reaction rate predominates over the expansion rate and the neutrinos are in equilibrium. But when $T < 10^{10}K$ the cosmological expansion is larger than the reaction rate which results in the decoupling of the neutrinos from the rest of the elementary particles. One may also include a damping factor, $exp(-\frac{T_\mu}{T})$ coming from other interacting particle, say muon. To summarise: as the cooling progresses less rapidly in the multidimensional phase the neutrinos (for that matter other particles as well) maintain the equilibrium phase for a relatively longer period.

2.2. Luminosity Distance Function

Following the recent evidence of the late acceleration of the Universe the luminosity distance function is one of the most useful and important concepts in astrophysics connecting the intrinsic and the apparent luminosities to some parameters like scale factor and curvature [6]. As is well known the equation (3) may be reduced to

$$dS^2 = dt^2 - R^2(t)[d\omega^2 + A_k^2 dX_n^2] \tag{8}$$

where A_k stands for $sin\ \omega$, ω and sinh ω correspoding to $k = 1, 0, -1$. The importance of the concept of Luminosity distance in the study of theoretical astrophysics need no further elaboration as the intrinsic luminosity of a source may be calculated with its help if the source's redshift and apparent luminosity are known. However as it is connected to the scale factor ,curvature etc it is definitely model dependent. In an earlier observation Barnes (1) showed that the de Sitter spacetime admits of the maximum luminosity distance. We address the similar situation in our model also described by the line element (8). If P_e is the total power emitted by a star then the flux density luminosity distance relation is given by

$$dh_i = \frac{P_e}{b_n D^n} \tag{9}$$

where b_n is a constant and the familiar luminosity distance is given by

$$D_n = \frac{R_0(1+z)^{2/n} r_i}{1 + kr_i^2/4} \tag{10}$$

Here the suffix 0 means that the evaluation has been made at $t = t_0$ Thus D would be less than the analogous 4D case for fixed z and R_0. For the metric (8) the last expression reduces to

$$D_n = R_0(1+z)^{2/n} A_k(\omega) \tag{11}$$

From the field equations it further follows that

$$kT_0^2/R_o^2 = (n+1)(1+\epsilon_0)\mu_0 \quad - q_0 \ - 1 \tag{12}$$

where $\mu_0 = \rho_0 T_0^2/n(n+1), \epsilon_0 = p_0/\rho_0, and \quad T_0 = R_0/\dot{R}_0$. If we assume at this stage a dust distribution we obtain via Bianchi identity the relation

$$\begin{aligned} 2\mu_0(R_0/R)^{n+1} = \\ R^{\dot{2}} T_0^2/R^2 + R_0^2/R^2[(n+1)\mu_0 - q_0 - 1] \\ +q_0 - (n-1)\mu_0 \end{aligned} \tag{13}$$

Introducing the dimensionless parameter $Y = R/R_0$ and $X = t/T_0$ the last equation yields $dY/dX = 1/Z(Y)$ where

$$Z^2(Y) = \frac{Y^{n-1}}{2\mu_0 + [q_0 + 1 - (n+1)\mu_0]Y^{n-1} - [q_0 - (n-1)\mu_0]Y^{n-1}} \tag{14}$$

where $1 + z = 1/Y$. Moreover if we assume a null radial geodesic coming from the source a little manipulation for our metric form (8) for flat spacetime gives

$\omega = \int \frac{dt}{R} = \frac{T_0}{R_0} I(z)$ where z is the familiar redshift factor and I(z) given by

$$I(z) = \int \frac{Z(z)dz}{1+z} \tag{15}$$

reduces to

$$I(z) = \int \frac{dz}{[2\mu_0(1+z)^{n+1} + \{q_0 1 - (n+1)\mu_0\}(1+z)^2 - \{q_0 - (n-1)\mu_0\}]^{1/2}} \tag{16}$$

For the flat space model ($k = 0$)with dust distribution it follows from (12) that

$$(n+1)\mu_0 = q_0 + 1 \tag{17}$$

So we finally have, skipping mathematical details the following relation

$$D(z, q_0, \mu_0) = T_0(1+z)^{2/n} \int \frac{dz}{[1 + 2\frac{q_0+1}{n+1}\{(1+z)^{n+1} - 1\}]^{1/2}} \tag{18}$$

It follows from the equation(18) that for $z > 0$ *and* $q_0 > -1$ the integrand consists of positive terms only and would be maximum for any fixed z when $q_0 = -1$. In that case we get from equation (12), $\mu_0 = 0$. Thus the deSitter universe admits of maximum luminosity distance of all flat space models. One obtains similar conclusion for $k \pm 0$ cases also. Thus dimensionality appears to play no role in this regard except at some quantitative level.

References

[1] R.C.Barnes, Mon. Not. R. Astro. Soc. **195**, 959 (1981).

[2] B M. Tinsley, Phys. Today **30**, 32, (1977).

[3] P. S. Wesson Space-Time - Matter (World Scientific), Singapore (1999).

[4] S. Chatterjee and B. Bhui , Mon. Not. R. Astron. Soc. **247**, 577, (1990).

[5] E. Alvarez and M. Gavela, Phys. Rev. Lett. **51**, 931 (1983).

[6] E. Malec, G. Wylezek and J. Karlowski , Gen. Rel. Grav. **36**, 2151 , (2004).

[7] E. Burbidge, G. Burbidge, W. Fowler and F. Hoyle , Rev. Mod. Phys. **29**, 547, (1957).