Formation and Evolution of Primordial Black Holes After Hybrid Inflation

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We examine the formation and evolution of primordial black holes (PBH's) after hybrid inflation. Our goal is to assess the effects of various theoretical uncertainties on the extrapolation from a given inflation model to a spectrum of primordial black hole masses. The context of our work is an examination of the possibility[1, 2] that the dark matter is comprised of Planck-mass black hole remnants (BHR's). As an example we focus on a particular scenario[3, 4] in which the black holes form from quantum perturbations that were generated during hybrid inflation. We find the correspondence between hybrid inflation parameters and the range of initial PBH masses that would allow BHR's to comprise the dark matter, taking account of the possible early presence of radiation and its accretion onto the PBH's.

1. INTRODUCTION

The main goal of this paper is to assess of the effects of various theoretical uncertainties on extrapolation from a given inflation model to a spectrum of primordial black holes. These uncertainties include initial size of the PBH's, efficiency of accretion of ambient radiation onto the PBH's, and possible presence of radiation due to reheating mechanisms acting prior to PH formation. As a particular example we examine a scenario[3, 4] in which: (1) significant numbers of PBH's form just after an epoch of hybrid inflation, (2) reheating of the Universe occurs primarily through Hawking evaporation of these PBH's to Planck scale black hole remnants (BHR's), and (3) the dark matter consists of these BHR's. In this BHR dark matter (BHRDM) scenario the problem is further simplified because there is a fairly sharply defined characteristic mass scale at which the PBH's form. (BHR's as a dark matter candidate were first proposed by MacGibbon[2]; other authors have also explored this possibility[5, 6]).

The endstage of Hawking evaporation is not well understood. However, heuristic arguments[7, 8] suggest that black holes might not evaporate completely, but instead may leave behind a stable Planck mass remnant, and that a black hole's temperature is given by

$$T_{BH} \approx \frac{1}{4\pi M[1+\sqrt{1-\frac{1}{M^2}}]} \ . \qquad (1)$$

rather than the Hawking value

$$T_{BH} \approx \frac{1}{8\pi M} \ . \qquad (2)$$

We use Planck units: Planck's constant $\hbar$, the speed of light c, Newton's gravitational constant G, and the Boltzmann constant k_b are all set to 1. Thus masses are in units of Planck mass $m_P \equiv \sqrt{\hbar c/G} \approx 2.2 \times 10^{-5}$ g ($\approx 1.2 \times 10^{19}$ GeV), lengths in units of Planck length $l_P \equiv \sqrt{G\hbar/c^3} \approx 1.6 \times 10^{-33}$ cm, times in units of Planck time $t_P \equiv \sqrt{G\hbar/c^5} \approx 5.4 \times 10^{-44}$ sec, etc.

We use a hybrid inflation[9, 10] potential given by:

$$V(\phi,\psi) = (M^2 - \frac{\sqrt{\lambda}}{2}\psi^2)^2 + \frac{1}{2}m^2\phi^2 + \frac{1}{2}\gamma\phi^2\psi^2 \quad . \qquad (3)$$

The effective mass of field ψ goes from positive to negative as ϕ decreases from large values. This shift to a negative ("tachyonic") mass for ψ occurs when ϕ decreases to the critical value $\phi_c = \left(\frac{2\sqrt{\lambda}M^2}{\gamma}\right)^{1/2}$ There are two inflation regimes in this model: (1) slow rolling of ϕ down "trough" where $\phi > \phi_c$, with $\psi \approx 0$, (2) Rapid fall of ψ to $\psi_\pm$, beginning when ϕ reaches ϕ_c. Large perturbations occuring at the "phase transition between these two regimes later produce PBH's at a fairly sharply defined mass when they re-enter the horizon.

As discussed in Ref. [11, 12], one finds two evolution equations for the simple "PBH+radiation epoch" of a homogeneous, isotropic, flat, Friedmann Universe:

$$f'_{BH} + \frac{f'_r}{\bar{a}} = 0,$$
$$f'_{BH} = \frac{\alpha_1 f_{BH}^2 \frac{f_r}{\bar{a}^4} - \frac{\alpha_2}{f_{BH}^2}}{\sqrt{\frac{f_{BH}}{\bar{a}} + \frac{f_r}{\bar{a}^2}}}, \qquad (4)$$

where

$$\alpha_1 \equiv F \cdot 27\pi\sqrt{\frac{3}{8\pi}}\sqrt{n_i}, \quad \alpha_2 \equiv \frac{g}{120\cdot 16^2\pi}\sqrt{\frac{3}{8\pi n_i}} \ . (5)$$

Here $\bar{a} \equiv \frac{a}{a_i}$, a prime $'$ means $d/d\bar{a}$, and we use scaled black hole and radiation energy densities defined by:

$$f_{BH} \equiv \frac{\rho_{BH}\bar{a}^3}{n_i} = M \ , \qquad f_r \equiv \frac{\rho_r \bar{a}^4}{n_i} \quad . \qquad (6)$$

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where n_i is the initial number density of PBH's, and $\rho_{BH} = n_i \bar{a}^{-3} M$ is the energy density in PBH's. Note that f_{BH} is also equal to the mass M of the PBH's. The first term in the numerator on the right hand side of Eq.(4) represents the accretion of radiation (with energy density ρ_r) onto the black holes, and the second term repesents Hawking evaporation. Here we've used the simpler result in Eq.(2), and in simulations discussed later will simply put in a cut-off at the Planck mass at the end of evaporation. The factor $g \sim 100$ gives the multiplicity of particles at the high temperatures characteristic of black holes near the end of their evaporation. In obtaining these equations we used the high frequency ("geometrical optics") limit[13] for the absorption cross section of the black holes, which is $27\pi M^2$ for all relativistic particles. This limit is well-satisfied for the situations considered in our scenarios. However, for reasons to be discussed later, we have also introduced into Eq. 5 an "efficiency factor" for accretion, F.

2. PBH FORMATION

2.1. "Standard picture"

We begin by reviewing the treatment of PBH formation used in Refs. [3, 4] which is based on the Press-Schechter[14] type of formalism put forward by Carr[15]. Assuming spherically symmetric density perturbations with Gaussian radial profile and rms amplitude $\delta(M)$, and an equation of state $p = w\rho$ with $0 < w < 1$, Carr argued that the probability $P(m)$ of a region of mass M collapsing to form a PBH is given by

$$P(m) \approx \delta(M) \exp\left(- \frac{w^2}{2\delta^2(M)} \right) \quad . \tag{7}$$

Assuming a "hard" equation of state (in our case, $p = \rho/3$) and a flat Universe the PBH's are expected to form with approximately the horizon mass.

The dependence of initial conditions $f_{BH,i}$, $f_{r,i}$ (as well as the initial number density of PBH's n_i appearing in α_1 and α_2) upon the hybrid inflation potential of Eq. 3 can be reduced to two parameters H_* and s, where $H_* \approx \sqrt{8\pi/3}M^2$ is the Hubble parameter during inflation and $s \equiv -\frac{3}{2} + \sqrt{\frac{9}{4} + \frac{2\sqrt{\lambda}M^2}{H_*^2}}$. The evolution of ψ during the waterfall regime is given by:

$$\psi(t) = \psi_{ie} \exp[-sH_*(t - t_{ie})] \quad , \tag{8}$$

where the subscript "ie" denotes the end of inflation.

The number of e-foldings of inflation between the "phase transition" near $\phi \sim \phi_c$ and the end of inflation is given by

$$N_c \approx \left[\frac{2}{sH_*}\right]^{1/s} \quad . \tag{9}$$

The PBH's form when the fractional density perturbations δ occurring at the "phase transition" near $\phi \sim \phi_c$ reenter the horizon. This density perturbation is closely related to the "curvature perturbation" $\mathcal{R}$ at horizon re-entry[16]:

$$\delta \equiv \frac{\delta\rho}{\rho} = \frac{2 + 2w_{reent}}{5 + 3w_{reent}}\mathcal{R} \quad . \tag{10}$$

Here $p = w\rho$ is the equation of state, and w at the time of re-entry is denoted by w_{reent}. We also have

$$\mathcal{R} = \left[\frac{H}{\dot{\psi}}\delta\psi\right]_{t_{reent}} \quad , \tag{11}$$

where t_{reent} denotes that the quantity in brackets is to be evaluated at the time when the perturbation of interest enters the horizon. Furthermore, the average spectral perturbation due to quantum fluctuations is

$$|\delta\psi| \sim \frac{H(\psi)}{2\pi} \quad . \tag{12}$$

If the Universe is approximately matter-dominated between the end of inflation and horizon re-entry, then

$$H_i \approx \frac{2}{3}e^{-3N_c}H_* \quad , \tag{13}$$

while if it is more nearly radiation-dominated between the end of inflation and horizon re-entry, we have

$$H_i \approx \frac{1}{2}e^{-2N_c}H_* \quad . \tag{14}$$

Regardless of where between the two extremes the actual evolution falls, we assume that the Universe has become radiation dominated by the time of re-entry, i.e. $w_{reent} = 1/3$.

The standard assumption is that PBH's form with the horizon mass. As will be discussed below, this may be only roughly true, and changes from this are important when considering the effects of accretion onto the PBH's. So we introduce a factor $x_{BH} (\leq 1)$ which gives the ratio of the initial PBH Schwarzschild radius to the horizon radius H^{-1} at the time of formation:

$$f_{BH,i} = x_{BH}H_i^{-1}/2 \tag{15}$$

Finally, the initial proportions of PBH's and radiation (where everything except the PBH's falls into the category of "radiation") is obtained as follows. From Eqs. 11, 12, and 8 we have $\mathcal{R} \sim 1/s$. Putting this into Eq. 10, gives

$$\delta_{reent} \equiv \frac{\delta\rho}{\rho} = \frac{2 + 2w_{reent}}{5 + 3w_{reent}}\frac{1}{s} = \frac{4}{9s} \quad . \tag{16}$$

Assume the probability $P(M)$ of formation is given by Eq. 7, where $\delta(M)$ is given by δ_{reent} from Eq. 16. (Recall M is the mass inside the horizon at re-entry, and thus approximately equal to the mass of the PBH's that form.) The initial proportions of PBH's and radiation is then just

$$y \equiv \frac{f_{r,i}}{f_{BH,i}} = \frac{1 - P(M)x_{BH}}{P(M)x_{BH}} \quad . \tag{17}$$

2.2. Uncertainties in PBH formation threshold and initial mass

There remain significant uncertainties in both the threshold overdensity for PBH formation and in the size of the PBH's that do form[17–23]. Early work by Nadejin, Novikov, and Polnarev[20] and Novikov and Polnarev[22] gave result that black holes form with order 10% the horizon mass and also that threshold δ is higher. Work by some authors[21, 23] indicates critical phenomena in gravitational collapse, leading to the possibility of black hole formation with significantly less than the horizon mass. Green, et.al.[17] concluded as the results of a study comparing the Press-Schechter based formalism with a peaks formalism that the "standard" formulation is fairly good if the threshold is in the range 0.3 to 0.5. We do not attempt to settle these uncertainties here, but note for later use that the initial mass of the PBH's may be at least somewhat less than the horizon mass.

3. SIMULATION AND RESULTS

Given a choice of the hybrid inflation parameters s, H_*, we can calculate initial values $f_{r,i}$, $f_{BH.i}$ and then solve Eqns. 4 for $f_r(a)$ and $f_{BH}(a)$ up to the time when the PBH's have evaporated to remnants. After that, f_r and f_{BH} remain constant in time (note in particular that $f_{BH} = 1$). Matter-radiation equality occurs at

$$\bar{a}_{eq} = f_{r,e} \quad . \tag{18}$$

Here the subscript "eq" denotes matter-radiation equality and the subscript "e" denotes the end of PBH evaporation. Evolution then continues into the era which is dominated first by dark matter and eventually, as the present time is approached, by dark energy. We assume the dark energy behaves like a cosmological constant, so that $f_{de} \equiv \frac{\rho_{de}}{n_i m_P}$ is constant in time. (We ignore baryons, in effect just lumping them in with the dark matter since they have the same equation of state, $p \approx 0$.)

The present age of the universe is given by

$$t_0 = t_{eq} + \int_{\bar{a}_{eq}}^{\bar{a}_0} \frac{du}{\sqrt{\frac{8\pi}{3} n_i \left(\frac{f_{BH,e}}{u} + \frac{f_{r,e}}{u^2} + f_{de} u^2 \right)}} \ . \tag{19}$$

Since the influence of dark energy is negligible until relatively late, the time of matter-radiation equality is given by

$$t_{eq} = t_e + \int_{\bar{a}_e}^{\bar{a}_{eq}} \frac{du}{\sqrt{\frac{8\pi}{3} n_i \left(\frac{f_{BH,e}}{u} + \frac{f_{r,e}}{u^2} \right)}} \ . \tag{20}$$

where t_e is the PBH evaporation time.

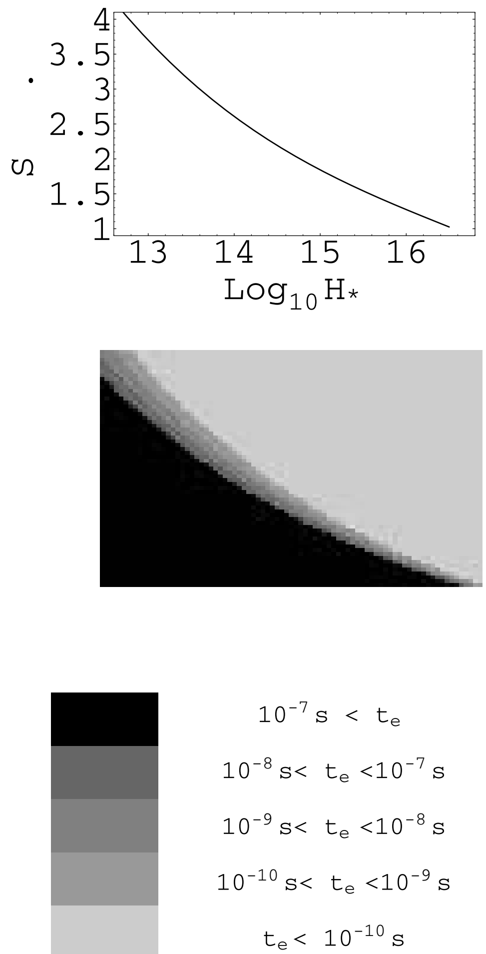

Figure 1: Case of no accretion, $w = 0$ between the end of inflation and the time of PBH formation, and $x_{BH} = 1$ (baseline case discussed in Refs. [3, 4]). The top plot shows the line in H_*, s parameter space satisfying the constraint that matter-radiation equality occurs at the observed redshift ($z = 3234$). (Here we display H_* in conventional units of GeV rather than Planck units.) The next plot covers the same region in the H_*, s plane and shows the time t_e at which the evaporation of the PBH's to BHR's is completed; the gray scale coding in the latter plot is shown at the bottom of the figure.

Provided that matter-radiation equality occurs in our simulations at the observed redshift $z \approx 3234$ (and the PBH's do indeed complete their evaporation), the constraint that t_0 is the observationally inferred value 13.5 Gyr is also satisfied.

3.1. Baseline model

We begin by showing results for the case discussed in Refs. [3, 4], for which accretion was not included and to which we shall refer as the "baseline case".

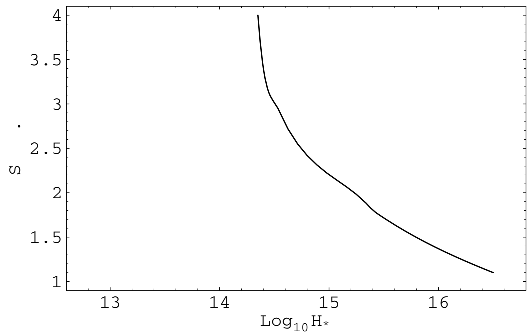

Figure 2: Case with accretion included, $w = 0$ between the end of inflation and the time of PBH formation, $x_{BH} = 1$, and $F = 1$. The meaning of the plot is as described for the top plot in Figure 1. *Note that the result in this figure cannot be exactly correct, since the PBH's initially grow to be larger than the horizon – see Figure 3 and discussion in text.*

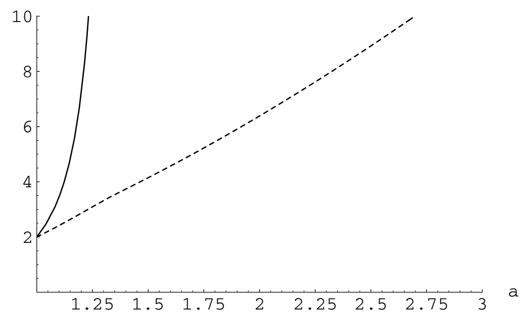

Figure 3: For particular point on the line shown in Figure 2 ($s = 2.45$, $\log_{10}[H_*/GeV] = 14.8$), we show the PBH radius $2f_{BH}$ (solid line) and the Hubble horizon H^{-1} (dashed line), both normalized by $f_{BH,i}$; this is as obtained from formulation of accretion used in Eq. 4 (with $F = 1$ in Eq. 5).

Thus it was assumed that (for whatever reason) the "accretion efficiency factor" F is essentially zero. We also assume (1) black holes form with horizon size ($x_{BH} = 1$), and (2) $w \approx 0$ between the end of inflation and the time of PBH formation. The result is shown in Figure 1. The top plot shows the line in H_*, s parameter space satisfying the constraint that matter-radiation equality occurs at the observed redshift ($z = 3234$). The next plot covers the same region in the H_*, s plane and shows the time t_e at which the evaporation of the PBH's to BHR's is completed; the gray scale coding in the latter plot is shown at the bottom of the figure. A safe criterion would be that they have evaporated by the time of the electroweak phase transition, expected to be around a TeV, or 10^{-10} sec. The actual values are typically somewhat more than this (by one or two orders of magnitude), but the criterion is somewhat uncertain anyway.

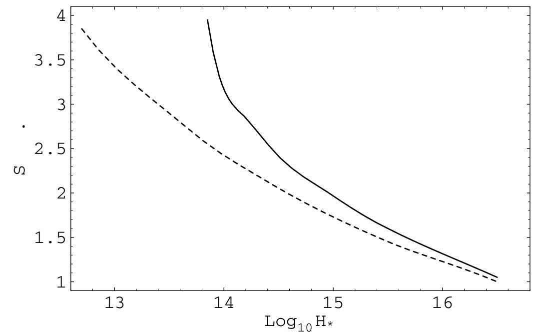

Figure 4: Loci in H_*, s parameter space satisfying the constraint that matter-radiation equality occurs at the observed redshift ($z = 3234$), for $x_{bh} = 0.4$, with accretion (solid line) and without accretion (dashed line); i.e., same type of plot as in top part of Figure 1. Other parameters are: $w = 0$ between the end of inflation and the time of PBH formation, and $F = 1$ for the case with accretion.

3.2. Accretion with $\mathbf{F = 1}$ and $\mathbf{x_{bh} = 1}$

In Figure 2 we include accretion in accordance with Eq. 4 (with $F = 1$ in Eq. 5). There is significant change from the case of no accretion. Closer examination shows that in this formulation, where the black holes are assumed to form with the horizon mass (i.e., $x_{bh} = 1$) and the accretion efficiency $F = 1$, the black holes accrete essentially all the available radiation background, before the scale factor grows appreciably, and the early growth rate of the black holes is faster than the growth of the horizon. This is illustrated in Figure 3 which shows (as a function of scale factor a) the PBH radius (solid line) and the Hubble horizon H^{-1} (dashed line) for the case $s = 2.45$, $\log_{10}[H_*/GeV] = 14.8$ (which is near the middle of the line in Figure 2).

As is obvious from causality (and was long ago pointed out by Zeldovich and Novikov[24]) the mass of the PBH cannot increase faster than the amount of mass within the sound horizon after formation, which in turn is less than the optical horizon (exactly equal to H^{-1} assuming radiation domination, and in any case equal up to factor of order 1)

The faster-than-horizon growth indicates a breakdown of one or more of the assumptions in our formulation of accretion, for example, (1) that the radiation background remains smooth and uniform, and (2) that the black holes form with the horizon mass.

3.3. Cases with $\mathbf{x_{bh} = 0.4}$

As was also pointed out by Zeldovich and Novikov[24], if PBH's form with significantly less than the horizon mass, accretion is unimportant. To get a limit on the possible effect of accretion in the BHRDM scenario, we reduce x_{bh} just enough to avoid having

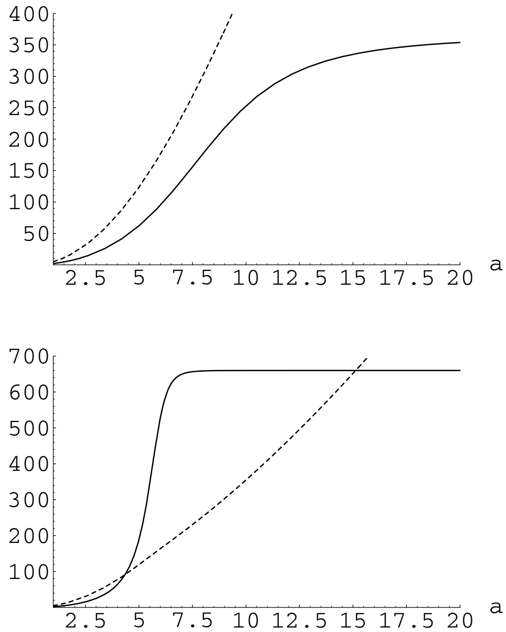

Figure 5: The PBH radius $2f_{BH}$ (solid line) as a function of scale factor a, along with the Hubble horizon H^{-1} (dashed line), both normalized by $f_{BH,i}$, at $s = 3.95$, $\log_{10}[H_*/GeV] = 13.85$, for $x_{BH} = 0.4$ (top plot) and $x_{BH} = 0.41$ (bottom plot).

PBH's grow to be larger than the Hubble horizon H^{-1}. The required value is $x_{bh} = 0.4$, and Figure 4 shows the loci in H_*, s parameter space satisfying the constraint that matter-radiation equality occurs at the observed redshift ($z = 3234$), for this case. The result with accretion is the solid line, and for comparison the dashed line shows the result without accretion. The effect of accretion is greatest at larger s (upper part of plots) because the parameter $y \equiv f_{r,i}/f_{BH,i}$ is larger, i.e. there is initially more radiation available for accretion. In Figure 5 we illustrate the growth of the PBH's (solid line) and the growth of the horizon (dashed line) for a case near the top of the solid line in Figure 4, at $s = 3.95$, $\log_{10}[H_*/GeV] = 13.85$. The top plot is for $x_{bh} = 0.4$; here the PBH's gain significant mass by accretion, but never grow faster than the horizon. The bottom plot is for $x_{bh} = 0.41$; here the PBH's do exceed H^{-1} for a short time.

3.4. Reduction of accretion efficiency

The solid line in Figure4 represents only an upper limit on the PBH growth for this $x = 0.4$ case, as there is no guarantee that our accretion formulation is valid even though the growth is not faster than the horizon. For example, it could well be the case that the assumption that the radiation background remains uniform is still violated in reality, i.e. the region around

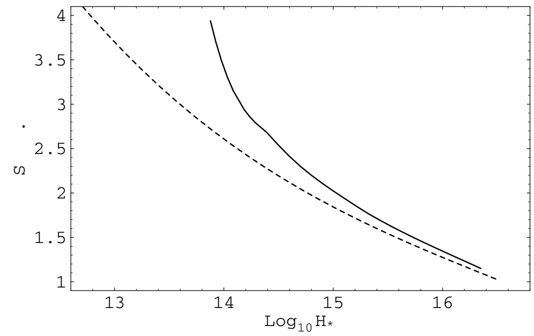

Figure 6: Loci in H_*, s parameter space satisfying the constraint that matter-radiation equality occurs at the observed redshift ($z = 3234$), for accretion with $F = 0.4$ (solid line) and without accretion (dashed line, same as in Figure 1).

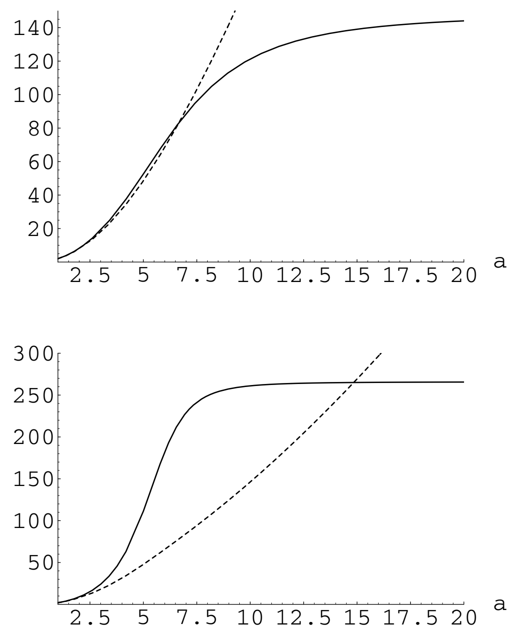

Figure 7: The PBH radius $2f_{BH}$ (solid line) as a function of scale factor a, along with the Hubble horizon H^{-1} (dashed line), both normalized by $f_{BH,i}$, at $s = 3.95$, $\log_{10}[H_*/GeV] = 13.85$, for $F = 0.4$ (top plot) and $F = 0.41$ (bottom plot).

the black holes could become depleted of radiation, making $F < 1$.

It is of course possible to avoid the faster-than-horizon growth in the $x_{bh} = 1$ case as well, if the accretion efficiency is reduced. Reduction to $F = 0.4$ is sufficient, and results are shown in Figures 6 and 7. (We would expect $F \rightarrow 1$ eventually, but by that time accretion becomes neglibible anyway.)

We note Hacyan[25] found (in a general-relativistic treatment based on Einstein-Strauss vacuole model) that the initial growth of PBH's by radiation accretion

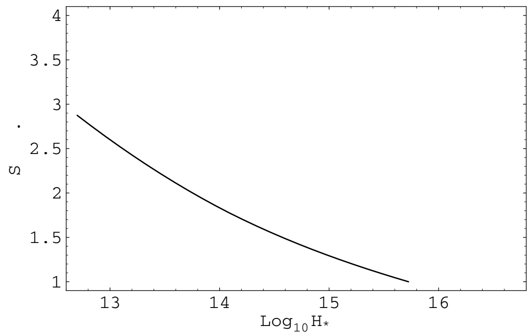

Figure 8: Locus in H_*, s parameter space satisfying the constraint that matter-radiation equality occurs at the observed redshift ($z = 3234$), for case with no accretion included, $w = 1/3$ between the end of inflation and the time of PBH formation, and $x_{BH} = 1$.

could be near the horizon rate if the initial PBH size near horizon size, but also interpreted this result as a generous upper limit on growth rate.

3.5. Possible additional reheating mechanisms

The baseline case assumed that the Universe is approximately matter dominated from the end of inflation until nearly the formation time of the PBH's. However PBH evaporation is not necessarily the only reheating mechanism. We need not exclude the possibility of additional reheating mechanism(s) that could result in approximate radiation domination well before PBH formation. Thus we compare also a case (Figure 8) where $w = 1/3$ between the end of inflation and the PBH formation time. Here we assume other parameters are as in Figure 1. For a given initial s, H_*, the $w = 1/3$ case gives smaller initial mass of PBH's than $w = 0$, since $M \propto 2e^{2N_c}$ instead of $M \propto \frac{3}{2}e^{3N_c}$.

4. SUMMARY AND CONCLUSIONS

We examined effects associated with early presence of radiation upon extrapolation from hybrid inflation parameters to a spectrum of PBH's in the BHRDM model. It appears that accretion can produce some early rapid growth. This may occur even if the requirement that the PBH's not grow to be larger than the horizon H^{-1} at any time is enforced by reducing either the fraction x_{bh} of the horizon mass going into the initial PBH mass or by reducing the accretion efficiency F. However our results should be taken as an upper limit on the effects of accretion, as there is no guarantee that the efficiency factor is close to $F = 1$ during the early rapid-accretion phase. Both these accretion effects and possible radiation production well before PBH formation affect the correspondence between the hybrid inflation parameters s, H_* and initial PBH masses, but do not seriously change the results of the BHRDM scenario.

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