Primordial Black Holes - Recent Developments

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Recent developments in the study of primordial black holes (PBHs) will be reviewed, with particular emphasis on their formation and evaporation. PBHs could provide a unique probe of the early Universe, gravitational collapse, high energy physics and quantum gravity. Indeed their study may place interesting constraints on the physics relevant to these areas even if they never formed. In the "early Universe" context, particularly useful constraints can be placed on inflationary scenarios, especially if evaporating PBHs leave stable Planck-mass relics. In the "gravitational collapse" context, the existence of PBHs could provide a unique test of the sort of critical phenomena discovered in recent numerical calculations. In the "high energy physics" context, information may come from gamma-ray bursts (if a subset of these are generated by PBH explosions) or from cosmic rays (if some of these derive from evaporating PBHs). In the "quantum gravity" context, the formation and evaporation of small black holes could lead to observable signatures in cosmic ray events and accelerator experiments, providing there are extra dimensions and providing the quantum gravity scale is around a TeV.

1. INTRODUCTION

Black holes with a wide range of masses could have formed in the early Universe as a result of the great compression associated with the Big Bang [70, 143]. A comparison of the cosmological density at a time t after the Big Bang with the density associated with a black hole of mass M shows that PBHs would have of order the particle horizon mass at their formation epoch:

$$M_H(t) \approx \frac{c^3 t}{G} \approx 10^{15} \left(\frac{t}{10^{-23} \text{ s}} \right) g. \qquad (1)$$

PBHs could thus span an enormous mass range: those formed at the Planck time (10^{-43}s) would have the Planck mass (10^{-5}g), whereas those formed at 1 s would be as large as $10^5 M_\odot$, comparable to the mass of the holes thought to reside in galactic nuclei. By contrast, black holes forming at the present epoch could never be smaller than about $1 M_\odot$.

The realization that PBHs might be small prompted Hawking to study their quantum properties. This led to his famous discovery [71] that black holes radiate thermally with a temperature

$$T = \frac{\hbar c^3}{8\pi GMk} \approx 10^{-7} \left(\frac{M}{M_\odot} \right)^{-1} \text{K}, \qquad (2)$$

so they evaporate on a timescale

$$\tau(M) \approx \frac{\hbar c^4}{G^2 M^3} \approx 10^{64} \left(\frac{M}{M_\odot} \right)^3 \text{y}. \qquad (3)$$

Only black holes smaller than 10^{15}g would have evaporated by the present epoch, so eqn (1) implies that this effect could be important only for black holes which formed before 10^{-23}s.

Hawking's result was a tremendous conceptual advance, since it linked three previously disparate areas of physics - quantum theory, general relativity and thermodynamics. However, at first sight it was bad news for PBH enthusiasts. For since PBHs with a mass of 10^{15}g would be producing photons with energy of order 100 MeV at the present epoch, the observational limit on the γ-ray background intensity at 100 MeV immediately implied that their density could not exceed 10^{-8} times the critical density [122]. Not only did this render PBHs unlikely dark matter candidates, it also implied that there was little chance of detecting black hole explosions at the present epoch [125].

Nevertheless, it was soon realized that the γ-ray background results did not preclude PBHs playing other important cosmological roles [27] and some of these will be discussed in this review. Indeed the discovery of a PBH could provide a unique probe of at least four areas of physics: the early Universe; gravitational collapse; high energy physics; and quantum gravity. The first topic is relevant because studying PBH formation and evaporation can impose important constraints on primordial inhomogeneities and cosmological phase transitions. The second topic relates to recent developments in the study of "critical phenomena" and the issue of whether PBHs are viable dark matter candidates. The third topic arises because PBH evaporations could contribute to cosmic rays, whose energy distribution would then give significant information about the high energy physics involved in the final explosive phase of black hole evaporation. The fourth topic arises because it has been suggested that quantum gravity effects could appear at the TeV scale and this leads to the intriguing possibility that small black holes could be generated in accelerators experiments or cosmic ray events, with striking observational consequences. Although such black holes are not technically "primordial", this possibility would have radical implications for PBHs themselves.

2. PBHS AS A PROBE OF PRIMORDIAL INHOMOGENEITIES

One of the most important reasons for studying PBHs is that it enables one to place limits on the spectrum of density fluctuations in the early Universe. This is because, if the PBHs form directly from density perturbations, the fraction of regions undergoing collapse at any epoch is determined by the root-mean-square amplitude ϵ of the fluctuations entering the horizon at that epoch and the equation of state $p = \gamma\rho$ ($0 < \gamma < 1$). One usually expects a radiation equation of state ($\gamma = 1/3$) in the early Universe but it may have deviated from this in some periods.

2.1. Simplistic Analysis

Early calculations assumed that the overdense region which evolves to a PBH is spherically symmetric and part of a closed Friedmann model. In order to collapse against the pressure, such a region must be larger than the Jeans length at maximum expansion and this is just $\sqrt{\gamma}$ times the horizon size. On the other hand, it cannot be larger than the horizon size, else it would form a separate closed universe and not be part of our Universe [29].

This has two important implications. Firstly, PBHs forming at time t after the Big Bang should have of order the horizon mass given by eqn (1). Secondly, for a region destined to collapse to a PBH, one requires the fractional overdensity at the horizon epoch δ to exceed γ. Providing the density fluctuations have a Gaussian distribution and are spherically symmetric, one can infer that the fraction of regions of mass M which collapse is [26]

$$\beta(M) \sim \epsilon(M) \exp\left[-\frac{\gamma^2}{2\epsilon(M)^2}\right] \quad (4)$$

where $\epsilon(M)$ is the value of ϵ when the horizon mass is M. The PBHs can have an extended mass spectrum only if the fluctuations are scale-invariant (i.e. with ϵ independent of M). In this case, the PBH mass distribution is given by [26]

$$dn/dM = (\alpha - 2)(M/M_*)^{-\alpha}M_*^{-2}\Omega_{\text{PBH}}\rho_{\text{crit}} \quad (5)$$

where $M_* \approx 10^{15}$g is the current lower cut-off in the mass spectrum due to evaporations, Ω_{PBH} is the total density of the PBHs in units of the critical density (which itself depends on β) and the exponent α is determined by the equation of state:

$$\alpha = \left(\frac{1 + 3\gamma}{1 + \gamma}\right) + 1. \quad (6)$$

$\alpha = 5/2$ if one has a radiation equation of state. This means that the density of PBHs larger than M falls off as $M^{-1/2}$, so most of the PBH density is contained in the smallest ones.

Many scenarios for the cosmological density fluctuations predict that ϵ is at least approximately scale-invariant but the sensitive dependence of β on ϵ means that even tiny deviations from scale-invariance can be important. If $\epsilon(M)$ decreases with increasing M, then the spectrum falls off exponentially and most of the PBH density is contained in the smallest ones. If $\epsilon(M)$ increases with increasing M, the spectrum rises exponentially and - if PBHs were to form at all - they could only do so at large scales. However, the microwave background anisotropies would then be larger than observed, so this possibility can be rejected.

The current density parameter Ω_{PBH} associated with PBHs which form at a redshift z or time t is related to β by [26]

$$\Omega_{\text{PBH}} = \beta\Omega_R(1+z) \approx 10^6\beta\left(\frac{t}{s}\right)^{-1/2} \approx 10^{18}\beta\left(\frac{M}{10^{15}g}\right)^{-1/2} \quad (7)$$

where $\Omega_R \approx 10^{-4}$ is the density parameter of the microwave background and we have used eqn (1). The $(1 + z)$ factor arises because the radiation density scales as $(1 + z)^4$, whereas the PBH density scales as $(1 + z)^3$. Any limit on Ω_{PBH} therefore places a constraint on $\beta(M)$ and the constraints are summarized in Fig. 1, which is taken from Carr et al. [32]. The constraint for non-evaporating mass ranges above 10^{15}g comes from requiring $\Omega_{\text{PBH}} < 1$ but stronger constraints are associated with PBHs smaller than this since they would have evaporated by now. The strongest one is the γ-ray limit associated with the 10^{15}g PBHs evaporating at the present epoch [122]. Other ones are associated with the generation of entropy and modifications to the cosmological production of light elements [120]. The constraints below 10^6g are based on the (uncertain) assumption that evaporating PBHs leave stable Planck mass relics, an issue which is discussed in detail in Section 4 . Other constraints, not shown here, are associated with gravitino production [91], reionization [75], thermodynamics [101] and the holographic principle [44].

The constraints on $\beta(M)$ can be converted into constraints on $\epsilon(M)$ using eqn (4) and these are shown in Fig. 2. Also shown here are the (non-PBH) constraints associated with the spectral distortions in the cosmic microwave background induced by the dissipation of intermediate scale density perturbations and the COBE quadrupole measurement. This shows that one needs the fluctuation amplitude to decrease with increasing scale in order to produce PBHs and the lines corresponding to various slopes in the $\epsilon(M)$ relationship are also shown in Fig. 2.
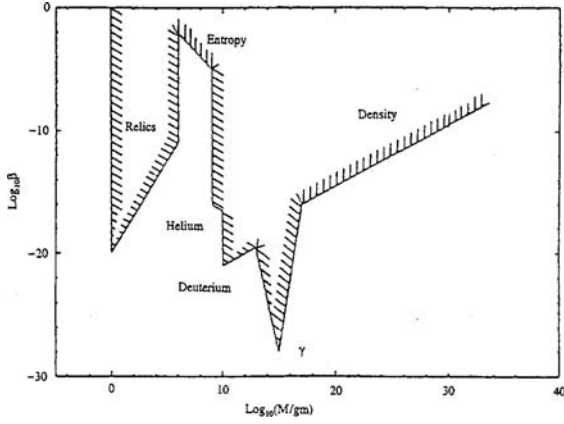
Figure 1: Constraints on $\beta(M)$
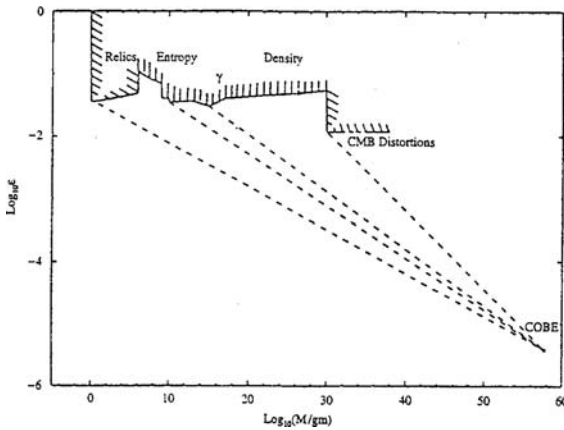
Figure 2: Constraints on $\epsilon(M)$

2.2. Refinements of Simplistic Analysis

The criterion for PBH formation given above is rather simplistic and needs to be tested with detailed numerical calculations. The first hydrodynamical studies of PBH formation were carried out by Nadezhin et al. [115]. These roughly confirmed the criterion $\delta > \gamma$ for PBH formation, although the PBHs could be be somewhat smaller than the horizon. In recent years several groups have carried out more detailed hydrodynamical calculations and these have refined the $\delta > \gamma$ criterion and hence the estimate for $\beta(M)$ given by eqn (4). Niemeyer & Jedamzik [118] find that one needs $\delta > 0.7$ rather than $\delta > 0.3$ to ensure PBH formation and they also find that there is little accretion after PBH formation, as expected theoretically [29]. Shibata & Sasaki [132] reach similar conclusions.

It should be stressed that the description of fluctuations beyond the horizon is somewhat problematic and it is clearer to use a gauge-invariant description which involves the total energy or metric perturbation [132]. Also the derivation of the mass spectrum given by eqn (4) is based on Press-Schechter theory

and it is more satisfactory to use peaks theory. Both these points have been considered by Green et al. [64]. They find that that the critical value for the density contrast is around 0.3, which (ironically) is close to the value advocated 30 years ago!

Another refinement of the simplistic analysis which underlies eqn (4) concerns the assumption that the fluctuations have a Gaussian distribution. Bullock & Primack [24] and Ivanov [81] have pointed out that this may not apply if the fluctuations derive from an inflationary period (discussed in more detail later). So long as the fluctuations are small ($\delta\phi/\phi \ll 1$), as certainly applies on a galactic scale, this assumption is valid. However, for PBH formation one requires $\delta\phi/\phi \sim 1$, and, in this case, the coupling of different Fourier modes destroys the Gaussianity. Their analysis suggests that $\beta(M)$ can be very different from the value indicated by eqn (4) but it still depends very sensitively on ϵ.

2.3. PBHs and Critical Collapse

A particularly interesting development has been the application of "critical phenomena" to PBH formation. Studies of the collapse of various types of spherically symmetric matter fields have shown that there is always a critical solution which separates those configurations which form a black hole from those which disperse to an asymptotically flat state. The configurations are described by some index p and, as the critical index p_c is approached, the black hole mass is found to scale as $(p - p_c)^\eta$ for some exponent η. This effect was first discovered for scalar fields [35] but subsequently demonstrated for radiation [49] and then more general fluids with equation of state $p = \gamma\rho$ [96, 112].

In all these studies the spacetime was assumed to be asymptotically flat. However, Niemeyer & Jedamzik [117] have applied the same idea to study black hole formation in asymptotically Friedmann models and have found similar results. For a variety of initial density perturbation profiles, they find that the relationship between the PBH mass and the the horizon-scale density perturbation has the form

$$M = KM_H(\delta - \delta_c)^\eta \qquad (8)$$

where M_H is the horizon mass and the constants are in the range $0.34 < \eta < 0.37$, $2.4 < K < 11.9$ and $0.67 < \delta_c < 0.71$ for the various configurations. Since $M \to 0$ as $\delta \to \delta_c$, this suggests that PBHs may be much smaller than the particle horizon at formation and it also modifies the mass spectrum [58, 61, 97, 141]. However, recently Miller et al. [114] have found that the critical overdensity lies in the lower range $0.43 < \delta_c < 0.47$ if one only allows growing modes at decoupling (which is more plausible if the fluctuations derive from inflation). They also

find that the exponent η is modified if there is a cosmological constant. Hawke & Stewart [69] claim that the formation of shocks prevents black holes forming on scales below 10^{-4} of the horizon mass but this has been disputed [114].

3. PBHS AS A PROBE OF THE EARLY UNIVERSE

Many phase transitions could occur in the early Universe which lead to PBH formation. In some of these one require pre-existing density fluctuations but in others the PBHs form spontaneously even if the Universe starts off perfectly smooth. In the latter case, $\beta(M)$ depends not on $\epsilon(M)$ but on some other cosmological parameter.

3.1. Soft Equation of State

Some phase transitions can lead to the equation of state becoming soft ($\gamma \ll 1$) for a while. For example, the pressure may be reduced if the Universe's mass is ever channelled into particles which are massive enough to be non-relativistic. In such cases, the effect of pressure in stopping collapse is unimportant and the probability of PBH formation just depends upon the fraction of regions which are sufficiently spherical to undergo collapse [90]. For a given spectrum of primordial fluctuations, this means that there may just be a narrow mass range - associated with the period of the soft equation of state - in which the PBHs form.

3.2. Collapse of Cosmic Loops

In the cosmic string scenario, one expects some strings to self-intersect and form cosmic loops. A typical loop will be larger than its Schwarzschild radius by the factor $(G\mu)^{-1}$, where μ is the string mass per unit length. If strings play a role in generating large-scale structure, $G\mu$ must be of order 10^{-6}. However, as discussed by many authors [25, 55, 73, 107, 124], there is always a small probability that a cosmic loop will get into a configuration in which every dimension lies within its Schwarzschild radius. This probability determines the collapse fraction β and depends upon both μ and the string correlation scale. Note that eqn (5) still applies since the holes are forming with equal probability at every epoch.

3.3. Bubble Collisions

Bubbles of broken symmetry might arise at any spontaneously broken symmetry epoch and various people have suggested that PBHs could form as a result of bubble collisions [43, 74, 99]. However, this happens only if the bubble formation rate per Hubble volume is finely tuned: if it is much larger than the Hubble rate, the entire Universe undergoes the phase transition immediately and there is not time to form black holes; if it is much less than the Hubble rate, the bubbles are very rare and never collide. The holes should have a mass of order the horizon mass at the phase transition, so PBHs forming at the GUT epoch would have a mass of 10^3g, those forming at the electroweak unification epoch would have a mass of 10^{28}g, and those forming at the QCD (quark-hadron) phase transition would have mass of around $1 M_\odot$.

3.4. PBHs and Inflation

Inflation has two important consequences for PBHs. On the one hand, any PBHs formed before the end of inflation will be diluted to a negligible density. Inflation thus imposes a lower limit on the PBH mass spectrum:

$$M > M_{\min} = M_P(T_{RH}/T_{Pl})^{-2} \qquad (9)$$

where T_{RH} is the reheat temperature and $T_P \approx 10^{19}$ GeV is the Planck temperature. The CMB quadrupole measurement implies $T_{RH} \approx 10^{16}$GeV, so $M_{\min}$ certainly exceeds 1 g. On the other hand, inflation will itself generate fluctuations and these may suffice to produce PBHs after reheating. If the inflaton potential is $V(\phi)$, then the horizon-scale fluctuations for a mass-scale M are

$$\epsilon(M) \approx \left(\frac{V^{3/2}}{M_P^3 V'}\right)_H \qquad (10)$$

where a prime denotes $d/d\phi$ and the right-hand side is evaluated for the value of ϕ when the mass-scale M falls within the horizon. In the standard chaotic inflationary scenario, one makes the "slow-roll" and "friction-dominated" asumptions:

$$\xi \equiv (M_P V'/V)^2 \ll 1, \quad \eta \equiv M_P^2 V''/V \ll 1. \quad (11)$$

Usually the exponent n characterizing the power spectrum of the fluctuations, $|\delta_k|^2 \approx k^n$, is very close to but slightly below 1:

$$n = 1 + 4\xi - 2\eta \approx 1. \qquad (12)$$

Since ϵ scales as $M^{(1-n)/4}$, this means that the fluctuations are slightly increasing with scale. The normalization required to explain galaxy formation ($\epsilon \approx 10^{-5}$) would then preclude the formation of PBHs on a smaller scale. If PBH formation is to occur, one needs the fluctuations to decrease with increasing mass ($n > 1$) and, from eqn (13), this is only possible if the scalar field is accelerating sufficiently fast that

$$V''/V > (1/2)(V'/V)^2. \qquad (13)$$

This condition is certainly satisfied in some scenarios [30] and, if it is, eqn (4) implies that the PBH density will be dominated by the ones forming immediately after reheating.

Since each value of n corresponds to a straight line in Fig.3, any particular value for the reheat time t_1 corresponds to an upper limit on n. This limit is indicated in Fig.3, which is taken from [32], apart from a correction pointed out by Green & Liddle [60]. Similar constraints have been obtained by several other people [18, 94]. The figure also shows how the constraint on n is strengthened if the reheating at the end of inflation is sufficiently slow for there to be a dust-like phase [62]. However, it should be stressed that not all inflationary scenarios predict that the spectral index should be constant. Hodges & Blumenthal [78] have pointed out that one can get any spectrum for the fluctuations whatsoever by suitably choosing the form of $V(\phi)$. For example, eqn (10) suggests that one can get a spike in the spectrum by flattening the potential over some mass range (since the fluctuation diverges when V' goes to 0). This idea was exploited by Ivanov et al. [82], who fine-tuned the position of the spike so that it corresponds to the mass-scale associated with microlensing events observed in the Large Magellanic Cloud.
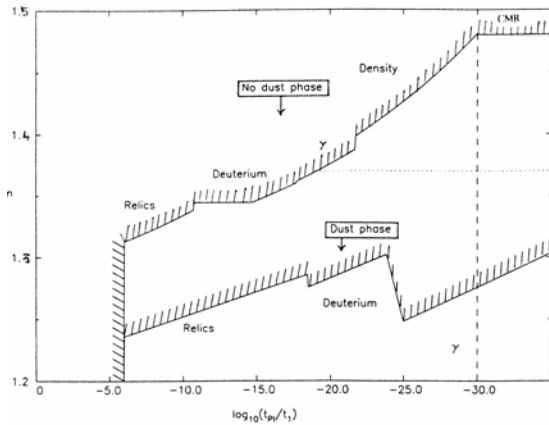

Figure 3: Constraints on spectral index n in terms of reheat time t_1

It should be noted that the relationship between the variance of the mass fluctuations relevant for PBHs and the present day horizon-scale density fluctuations in the inflation scenario is not trivial. For scale-free fluctuations, there is a simple relationship between them but it is more complicated otherwise. Blais et al. [19] find that the mass fluctuations are reduced by 34% for a spectral index in the range $1 < n < 1.3$, while Polarski [123] finds a further 15% reduction if there is a cosmological constant.

Even if PBHs never actually formed as a result of inflation, studying them places important constraints on the many types of inflationary scenarios. Besides

the chaotic scenario discussed above, there are also the variants of inflation described as supernatural [126], supersymmetric [57], hybrid [54, 87], multiple [140], oscillating [134], preheating [13, 47, 52, 63], running mass [100] and saddle [48]. There are also scenarios in which the inflaton serves as dark matter [102]. PBH formation has been studied in all of these models.

3.5. Varying Gravitational Constant

The PBH constraints would be severely modified if the value of the gravitational "constant" G was different at early times. The simplest varying-G model is Brans-Dicke (BD) theory [22], in which G is associated with a scalar field ϕ and the deviations from general relativity are specified by a parameter ω. A variety of astrophysical tests currently require $|\omega| > 500$, which implies that the deviations can only ever be small [137]. However, there exist generalized scalar-tensor theories [16, 119, 136] in which ω is itself a function of ϕ and these lead to a considerably broader range of variations in G. In particular, it permits ω to be small at early times (allowing noticeable variations of G then) even if it is large today. In the last decade interest in such theories has been revitalized as a result of early Universe studies. Extended inflation explicitly requires a model in which G varies [99] and, in higher dimensional Kaluza-Klein-type cosmologies, the variation in the sizes of the extra dimensions also naturally leads to this [53, 110].

The consequences of the cosmological variation of G for PBH evaporation depend upon how the value of G near the black hole evolves. Barrow [10] introduces two possibilities: in scenario A, G everywhere maintains the background cosmological value (so ϕ is homogeneous); in scenario B, it preserves the value it had at the formation epoch near the black hole even though it evolves at large distances (so ϕ becomes inhomogeneous). On the assumption that a PBH of mass M has a temperature given by eqn (2) with $G = G(t)$ in scenario A and $G = G(M)$ in scenario B, Barrow & Carr [11] calculate how the evaporation constraints summarized in Fig.1 are modified for a wide range of varying-G models. The question of whether scenario A or scenario B is more plausible has been considered in several papers [28, 83] but recent studies of the collapse of a scalar field to a PBH suggest that gravitational memory is unlikely [67, 68].

4. PBHS AND DARK MATTER

Roughly 30% of the total density of the Universe is now thought to be in the form of "cold dark matter". Recently there has been a lot of interest in whether PBHs could provide this, since those larger than 10^{15}g would not have evaporated yet and would

certainly be massive enough to be dynamically "cold". They might also play a role in the formation of large-scale structure [2, 92, 109]. However, there are some ranges in which this is excluded. For example, femtolensing of gamma-ray bursts by PBHs precludes those in the mass range $10^{17} - 10^{20}$g from having a critical density and searches for microlensing of stars in the Large Magellanic Cloud, while allowing a tenth-critical-density at around a solar mass, exclude $10^{26} - 10^{34}$g PBHs [3]. However, there are no constraints in the intermediate (sublunar) mass range $10^{20} - 10^{26}$g [18].

4.1. PBH Formation at Phase Transitions

One possibility is that PBHs with a mass of around $1 M_\odot$ could have formed at the quark-hadron phase transition at 10^{-5}s because of a temporary softening of the equation of state then [43]. Such PBHs would naturally have the sort of mass required to explain the MACHO microlensing results [84]. If the QCD phase transition is assumed to be 1st order, then hydrodynamical calculations show that the value of δ required for PBH formation is indeed reduced below the value which pertains in the radiation case [85]. This means that PBH formation will be strongly enhanced at the QCD epoch, with the mass distribution peaking at around the horizon mass then. Another possibility is that PBHs with a mass of around $10^{-7} M_\odot$ could form in TeV quantum gravity scenarios [79].

One of the interesting implications of these scenarios is the possible existence of a halo population of *binary* black holes [116]. With a full halo of such objects, there could be a huge number of binaries inside 50 kpc and some of these could be coalescing due to gravitational radiation losses at the present epoch [20]. If the associated gravitational waves were detected, it would provide a unique probe of the halo distribution [80]. Gravity waves from binary PBHs would be detectable down to $10^{-5} M_\odot$ using VIRGO, $10^{-7} M_\odot$ using EURO and $10^{-11} M_\odot$ using LISA [79]. Note that LISA could also detect isolated PBHs by measuring the gravitational impulse induced by any nearby passing ones [1, 131]. However, this method would not work below 10^{14}g (because the effect would be hidden by the Moon) or above 10^{20}g (because the encounters would be too rare).

4.2. PBHs and Supermassive Black Holes

Several people have suggested that the $10^6 - 10^8 M_\odot$ supermassive black holes thought to reside in galactic nuclei could be of primordial origin [46, 86, 127]. However, since no PBHs are likely to form after 1 s, corresponding to a maximum formation mass of $10^5 M_\odot$, this requires a large amount of accretion. Bean and Magueijo have proposed [14] that PBHs may accrete from the quintessence field which is invoked to explain the acceleration of the Universe. However, they invoke a Newtonian formula for the accretion rate [143] and this is known to be questionable [29]. Recent studies of the accretion of a scalar field by a PBH also indicate that this is unlikely [67].

4.3. Planck Mass Relics

Some people have speculated that black hole evaporation could cease once the hole gets close to the Planck mass [21, 41]. For example, in the standard Kaluza-Klein picture, extra dimensions are assumed to be compactified on the scale of the Planck length. This means that the influence of these extra dimensions becomes important at the energy scale of 10^{19}GeV on which quantum gravity effects become significant. Such effects could influence black hole evaporation and could conceivably result in evaporation ceasing at the Planck mass. Various non-quantum-gravitational effects (such as higher order corrections to the gravitational Lagrangian or string effects) could also lead to stable relics [32] but the relic mass is usually close to the Planck scale.

Another possibility, as argued by Chen & Adler [34], is that stable relics could arise if one invokes a "generalized uncertainty principle". This replaces the usual uncertainty principle with one of the form

$$\Delta x > \frac{\hbar}{\Delta p} + l_P^2 \frac{\Delta p}{\hbar}, \qquad (14)$$

where the second term is supposed to account for self-gravity effects. This means that the black hole temperature becomes

$$T_{BH} = \frac{Mc^2}{4\pi k} \left(1 - \sqrt{1 - \frac{M_P^2}{M^2}} \right). \qquad (15)$$

This reduces to the standard Hawking form for $M \gg M_P$ but it remains finite instead of diverging at the Planck mass itself.

Whatever the cause of their stability, Planck mass relics would provide a possible cold dark matter candidate [103]. Indeed this leads to the "relics" constraints indicated in Fig.1, Fig.2 and Fig.3. In particular, such relics could be left over from inflation [12, 18]. If the relics have a mass κM_P, then the requirement that they have less than the critical density implies [32]

$$\beta(M) < 10^{-27} \kappa^{-1} (M/M_P)^{3/2} \qquad (16)$$

for the mass range

$$(T_{RH}/T_P)^{-2} M_{Pl} < M < 10^{11} \kappa^{2/5} M_P. \qquad (17)$$

The upper mass limit arises because PBHs larger than this dominate the total density before they evaporate. Producing a critical density of relics obviously

requires fine-tuning of the index n. Also one needs $n \approx 1.3$, which is barely compatible with the WMAP results [5, 9]. Nevertheless, this is possible in principle. In particular, Chen [33] has argued that hybrid inflation could produce relics from 10^5g PBHs formed at 10^{-32}s.

5. PBHS AS A PROBE OF HIGH ENERGY PHYSICS

A black hole of mass M will emit particles like a black-body of temperature [72]

$$T \approx 10^{26} \left(\frac{M}{g}\right)^{-1} \text{ K} \approx \left(\frac{M}{10^{13}g}\right)^{-1} \text{ GeV}. \quad (18)$$

This assumes that the hole has no charge or angular momentum. This is a reasonable assumption since charge and angular momentum will also be lost through quantum emission but on a shorter timescale than the mass [121]. This means that it loses mass at a rate

$$\dot{M} = -5 \times 10^{25} (M/g)^{-2} f(M) \text{ g s}^{-1} \quad (19)$$

where the factor $f(M)$ depends on the number of particle species which are light enough to be emitted by a hole of mass M, so the lifetime is

$$\tau(M) = 6 \times 10^{-27} f(M)^{-1} (M/g)^3 \text{ s}. \quad (20)$$

The factor f is normalized to be 1 for holes larger than 10^{17} g and such holes are only able to emit "massless" particles like photons, neutrinos and gravitons. Holes in the mass range 10^{15} g $< M < 10^{17}$ g are also able to emit electrons, while those in the range 10^{14} g $< M < 10^{15}$ g emit muons which subsequently decay into electrons and neutrinos. The latter range includes, in particular, the critical mass for which τ equals the age of the Universe.

Once M falls below 10^{14}g, a black hole can also begin to emit hadrons. However, hadrons are composite particles made up of quarks held together by gluons. For temperatures exceeding the QCD confinement scale of $\Lambda_{\text{QCD}} = 250 - 300$ GeV, one would therefore expect these fundamental particles to be emitted rather than composite particles. Only pions would be light enough to be emitted below Λ_{QCD}. Since there are 12 quark degrees of freedom per flavour and 16 gluon degrees of freedom, one would also expect the emission rate (i.e. the value of f) to increase dramatically once the QCD temperature is reached.

The physics of quark and gluon emission from black holes is simplified by a number of factors [104]. Firstly, one can show that the separation between successively emitted particles is about 20 times their wavelength, which means that short range interactions between them can be neglected. Secondly, the condition

$T > \Lambda_{\text{QCD}}$ implies that their separation is much less than $\Lambda_{\text{QCD}}^{-1} \approx 10^{-13}$cm (the characteristic strong interaction range) and this means that the particles are also unaffected by strong interactions. The implication of these three conditions is that one can regard the black hole as emitting quark and gluon jets of the kind produced in collider events. The jets will decay into hadrons over a distance which is always much larger than the size of the hole, so gravitational effects can be neglected. The hadrons may then decay into astrophysically stable particles through weak and electomagnetic decays.

To find the final spectra of stable particles emitted from a black hole, one must convolve the Hawking emission spectrum with the jet fragmentation function. This gives the instantaneous emission spectrum shown in Fig.4 for a $T = 1$ GeV black hole [106]. The direct emission just corresponds to the small bumps on the right. All the particle spectra show a peak at 100 MeV due to pion decays; the electrons and neutrinos also have peaks at 1 MeV due to neutron decays. In order to determine the present day background spectrum of particles generated by PBH evaporations, one must first integrate over the lifetime of each hole of mass M and then over the PBH mass spectrum [106]. In doing so, one must allow for the fact that smaller holes will evaporate at an earlier cosmological epoch, so the particles they generate will be redshifted in energy by the present epoch.
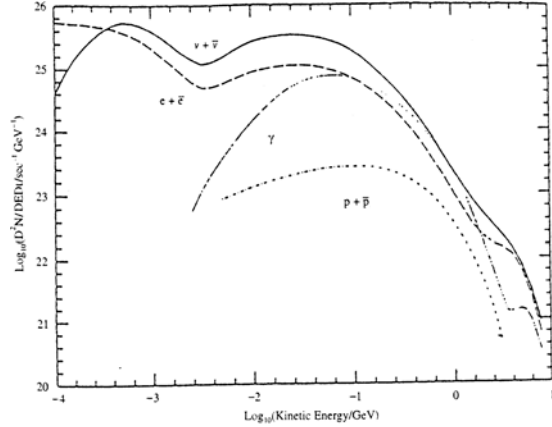

Figure 4: Instantaneous emission from a 1 GeV black hole

If the holes are uniformly distributed throughout the Universe, the background spectra should have the form indicated in Fig.5. All the spectra have rather similar shapes: an E^{-3} fall-off for $E > 100$ MeV due to the final phases of evaporation at the present epoch and an E^{-1} tail for $E < 100$ MeV due to the fragmentation of jets produced at the present and earlier epochs. Note that the E^{-1} tail generally masks any effect associated with the mass spectrum of smaller

PBHs which evaporated at earlier epochs [27].

The situation is more complicated if the PBHs evaporating at the present epoch are clustered inside our own Galactic halo (as is most likely). In this case, any charged particles emitted after the epoch of galaxy formation (i.e. from PBHs only somewhat smaller than M_*) will have their flux enhanced relative to the photon spectra by a factor ξ which depends upon the halo concentration factor and the time for which particles are trapped inside the halo by the Galactic magnetic field. This time is rather uncertain and also energy-dependent. At 100 MeV one has $\xi \sim 10^3$ for electrons or positrons and $\xi \sim 10^4$ for protons and antiprotons. MacGibbon & Carr [105] first used the observed cosmic ray spectra to constrain Ω_{PBH} but their estimates have recently been updated.
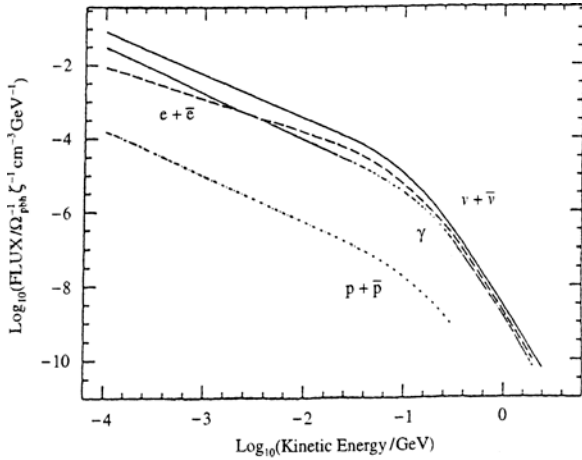

Figure 5: Spectrum of particles from uniformly distributed PBHs

5.1. Gamma-Rays

EGRET observations [133] give a γ-ray background of

$$\frac{dF_\gamma}{dE} = 7.3 \times 10^{-14} \left(\frac{E}{100 MeV} \right)^{-2.10} \text{cm}^{-3}\text{GeV}^{-1}$$
(21)

between 30 MeV and 120 GeV. This leads to an upper limit [31]

$$\Omega_{\text{PBH}} \leq (5.1 \pm 1.3) \times 10^{-9} h^{-2},$$
(22)

where h is the Hubble parameter in units of 100. This is a refinement of the original Page-Hawking limit, but the form of the spectrum suggests that PBHs do not provide the dominant contribution. If PBHs are clustered inside our own Galactic halo, then there should also be a Galactic γ-ray background and, since this would be anisotropic, it should be separable from the extragalactic background. The ratio of the anisotropic

to isotropic intensity depends on the Galactic longitude and latitude, the Galactic core radius and the halo flattening. Wright claims that such a halo background has been detected [138]. His detailed fit to the EGRET data, subtracting various other known components, requires the PBH clustering factor to be $(2 - 12) \times 10^5 h^{-1}$, comparable to that expected.

5.2. Antiprotons and Antideuterons

Since the ratio of antiprotons to protons in cosmic rays is less than 10^{-4} over the energy range $100 \text{ MeV} - 10 \text{ GeV}$, whereas PBHs should produce them in equal numbers, PBHs could only contribute appreciably to the antiprotons [135]. It is usually assumed that the observed antiproton cosmic rays are secondary particles, produced by spallation of the interstellar medium by primary cosmic rays. However, the spectrum of secondary antiprotons should show a steep cut-off at kinetic energies below 2 GeV, whereas the spectrum of PBH antiprotons should increase with decreasing energy down to 0.2 GeV. Also the antiproton fraction should tend to 0.5 at low energies, so these features provide a distinctive signature [95].

MacGibbon & Carr originally calculated the PBH density required to explain the interstellar antiproton flux at 1 GeV and found a value somewhat larger than the γ-ray limit [105]. More recent data on the antiproton flux below 0.5 GeV comes from the BESS balloon experiment [142] and Maki et al. [111] have tried to fit this data in the PBH scenario. They model the Galaxy as a cylindrical diffusing halo of diameter 40 kpc and thickness 4-8 kpc and then use Monte Carlo simulations of cosmic ray propagation. A comparison with the data shows no positive evidence for PBHs (i.e. there is no tendency for the antiproton fraction to tend to 0.5 at low energies) but they require the fraction of the local halo density in PBHs to be less than 3×10^{-8} and this is stronger than the γ-ray background limit. A more recent attempt to fit the observed antiproton spectrum with PBH emission comes from Barrau et al. [7] and is shown in Fig.6. PBHs might also be detected by their antideuteron flux [8].

5.3. PBH Explosions

One of the most striking observational consequences of PBH evaporations would be their final explosive phase. However, in the standard particle physics picture, where the number of elementary particle species never exceeds around 100, the likelihood of detecting such explosions is very low. Indeed, in this case, observations only place an upper limit on the explosion rate of $5 \times 10^8 \text{pc}^{-3}\text{y}^{-1}$ [4, 129]. This compares to Wright's γ-ray halo limit of 0.3 pc^{-3}y^{-1} [138] and the Maki et al. antiproton limit of 0.02 pc^{-3}y^{-1} [111].
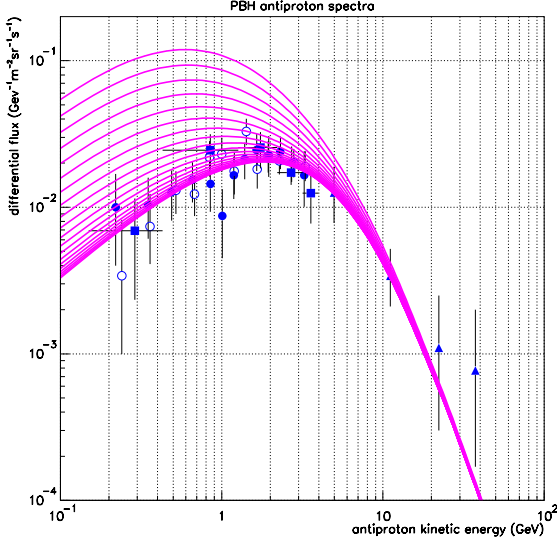
Figure 6: Comparison of PBH emission and antiproton data from Barrau et al.

However, the physics at the QCD phase transition is still uncertain and the prospects of detecting explosions would be improved in less conventional particle physics models. For example, in a Hagedorn-type picture, where the number of particle species exponentiates at the the quark-hadron temperature, the upper limit is reduced to 0.05 $pc^{-3}y^{-1}$ [51]. Cline and colleagues have argued that one might expect the formation of a QCD fireball at this temperature [37, 38] and this might even explain some of the short period γ-ray bursts observed by BATSE [40]. They claim to have found 42 candidates of this kind and the fact that their distribution matches the spiral arms suggests that they are Galactic [39]. Although this proposal is speculative and has been disputed [59], it has the attraction of making testable predictions (eg. the hardness ratio should increase as the duration of the burst decreases). A rather different way of producing a γ-ray burst is to assume that the outgoing charged particles form a plasma due to turbulent magnetic field effects at sufficiently high temperatures [15].

Some people have emphasized the possibility of detecting very high energy cosmic rays from PBHs using air shower techniques [42, 66, 98]. However, this is refuted by the claim of Heckler [76] that QED interactions could produce an optically thick photosphere once the black hole temperature exceeds $T_{crit} = 45$ GeV. In this case, the mean photon energy is reduced to $m_e(T_{BH}/T_{crit})^{1/2}$, which is well below T_{BH}, so the number of high energy photons is much reduced. He has proposed that a similar effect may operate at even lower temperatures due to QCD effects [77]. Several groups have examined the implications of this proposal for PBH emission [40, 88]. However, these ar-

guments should not be regarded as definitive since MacGibbon et al. claim that QED and QCD interactions are never important [108].

6. PBHS AS A PROBE OF QUANTUM GRAVITY

In "brane" versions of Kaluza-Klein theory, some of the extra dimensions can be much larger than the usual Planck length and this means that quantum gravity effects may become important at a much smaller energy scale than usual. If the internal space has n dimensions and a compact volume V_n, then Newton's constant G_N is related to the higher dimensional gravitational constant G_D and the value of the modified Planck mass M_P is related to the usual 4-dimensional Planck mass M_4 by the order-of-magnitude equations:

$$G_N \sim G_D/V_n, \quad M_P^{n+2} \sim M_4^2/V_n. \quad (23)$$

The same relationship applies if one has an infinite extra dimension but with a "warped" geometry, provided one interprets V_n as the "warped volume". In the standard model, $V_n \sim 1/M_4^n$ and so $M_P \sim M_4$. However, with large extra dimensions, one has $V_n \gg 1/M_4^n$ and so $M_P \ll M_4$.

6.1. Black Hole Production at Accelerators

One exciting implication of these scenarios is that quantum gravitational effects may arise at the experimentally observable TeV scale. If so, this would have profound implications for black hole formation and evaporation since black holes could be generated in accelerator experiments, such as the Large Hadron Collider (LHC). Two partons with centre-of-mass energy $\sqrt{s}$ will form a black hole if they come within a distance corresponding to the Schwarzschild radius r_S for a black hole whose mass M_{BH} is equivalent to that energy [45, 56, 128]. Thus the cross-section for black hole production is

$$\sigma_{BH} \approx \pi r_S^2 \Theta(\sqrt{s} - M_{BH}^{min}) \quad (24)$$

where M_{BH}^{min} is the mass below which the semiclassical approximation fails. Here the Schwarzschild radius itself depends upon the number of internal dimensions:

$$r_S \approx \frac{1}{M_P} \left(\frac{M_{BH}}{M_P} \right)^{1/(1+n)}, \quad (25)$$

so that $\sigma_{BH} \propto s^{2/(n+1)}$. This means that the cross-section for black hole production in scattering experiments goes well above the cross-section for the standard model above a certain energy scale and in a way which depends on the number of extra dimensions.

The evaporation of the black holes produced in this way will produce a characteristic signature [45, 56, 128] because the temperature and lifetime of the black holes depend on the number of internal dimensions:

$$T_{BH} \sim \frac{n+1}{r_S}, \quad \tau_{BH} \sim \frac{1}{M_P}\left(\frac{M_{BH}}{M_P}\right)^{(n+3)/(n+1)}. \tag{26}$$

Thus the temperature is decreased relative to the standard 4-dimensional case and the lifetime is increased. The important qualitative effect is that a large fraction of the beam energy is converted into transverse energy, leading to large-multiplicity events with many more hard jets and leptons than would otherwise be expected. In principle, the formation and evaporation of black holes might be observed by the LHC when it turns on in 2007 and this might also allow one to experimentally probe the number of extra dimensions. On the other hand, this would also mean that scattering processes above the Planck scale could not be probed directly because they would be hidden behind a black hole event horizon.

Similar effects could be evident in the interaction between high energy cosmic rays and atmospheric nucleons. Nearly horizontal cosmic ray neutrinos would lead to the production of black holes, whose decays could generate deeply penetrating showers with an electromagnetic component substantially larger than that expected with conventional neutrino interactions. Several authors have studied this in the context of the Pierre Auger experiment, with event rates in excess of one per year being predicted [6, 50, 128]. Indeed there is a small window of opportunity in which Auger might detect such events before the LHC.

6.2. PBHs and Brane Cosmology

The black holes produced in accelerators should not themselves be described as "primordial" since they do not form in the early Universe. On the other hand, it is clear that the theories which predict such processes will also have profound implications for PBHs because, at sufficiently early times, the effects of the extra dimensions must be cosmologically important. Such effects are not fully understood but studies of PBH formation and evaporation in brane cosmology are beginning to explore them.

Brane cosmology modifies the standard PBH formation scenario. In particular, in Randall-Sundrum type II scenarios, a ρ^2 term appears in the Friedmann equation and this dominates the usual ρ term for

$$T > 10^{18}(l/l_P)^{-1/4}\text{GeV} \tag{27}$$

where l is the scale of the extra dimension. This exceeds 1 TeV for $l < 0.2$ mm. Black holes which are smaller than l are effectively 5-dimensional. This

means that they are cooler and evaporate more slowly than in the standard scenario:

$$T_{BH} \propto M^{-1/2}, \quad \tau \propto M^2. \tag{28}$$

The critical mass for PBHs evaporating at the present epoch is in the range 3×10^9g to 4×10^{14}g. This modifies all the standard PBH evaporation constraints [36, 65, 89] and, if some cosmic rays come from PBHs, it means that these could probe the extra dimensions [130]. Another complication is that accretion could dominate evaporation during the era when the ρ^2 term dominates the ρ term in the Friedmann equation [113].

7. CONCLUSIONS

We have seen that PBHs could be used to study the early Universe, gravitational collapse, high energy physics and quantum gravity. In the "early Universe" context, useful constraints can be placed on inflationary scenarios and gravitational memory. In the "gravitational collapse" context, the existence of PBHs could provide a test of critical phenomena. In the "high energy physics" context, information may come from observing cosmic rays from evaporating PBHs since the constraints on the number of evaporating PBHs imposed by gamma-ray background observations do not exclude their making a significant contribution to the Galactic flux of electrons, positrons and antiprotons. Evaporating PBHs may also be detectable in their final explosive phase as gamma-ray bursts if suitable physics is invoked at the QCD phase transition. In the "quantum gravity" context, the formation and evaporation of small black holes could be observable in cosmic ray events and accelerator experiments if the quantum gravity scale is around a TeV.

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