Modification at Large Distances?

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We discuss an idea that the cosmic acceleration of the Universe may be caused by modification of gravity at very large distances, and not by a dark energy source. Such a modification could, in particular, be triggered by extra space dimensions, to which gravity spreads over cosmic distances. This idea is testable by distant supernovae and other cosmological surveys. Also modified gravity predicts testable deviations in planetary motions. For the moon this effect would show up in anomalous perihelion precessions of its orbit, and can be detected by lunar ranging experiments.

1. Modified Gravity Versus Dark Energy

In this talk I shall briefly discuss an idea that the observed accelerated expansion of the Universe[1] may be the consequence of large distance modification of gravity. As an example of a consistent and predictive effective theory of such modification, I will closely discuss the model [2], but qualitative conclusions are very general, and apply to a large class of modified gravity theories. The most important of these results is that the distance at which the new corrections set in is not universal but rather source dependent. For sources as heavy as the Universe, new dynamics sets in at the horizon scale[4], but for the lighter sources, such as the earth, the corrections appear already at much shorter distances[5]. Because of this the modified gravity theories are potentially testable not only by cosmology but also by the precision gravitational measurements at very different scales.

The cosmological evolution of the Universe is described by Friedmann equation, which for the spatially-flat Universe has the following form

$$H^2 = \frac{8\pi}{3} G_N \rho \tag{1}$$

The left hand side of this equation describes the energy density budget of the Universe, which acts as a source for gravity. The right hand side describes how gravity, or equivalently the geometry of the Universe, responds to this energy source.

Here H is the Hubble parameter, which is related to the scale factor of the Universe by $H = (\frac{\dot{a}}{a})$, where dot stands for the time derivative. Because the scale factor describes how the distance between the two points evolves in time, the Hubble parameter encodes the information about the expansion rate of the Universe.

For instance, in the standard picture of the decelerating Friedmann-Robertson-Walker (FRW) Universe H would decrease as inverse cosmic time. However, the current observations indicate[1] that we are entering the cosmic era in which H approaches an approximately constant value. That is, the Universe is entering the epoch of an accelerated expansion. The origin of this acceleration is a big mystery, because of the following reason.

The energy density source that could cause acceleration should have very peculiar properties. It must be *undilutable* (or very slowly-dilutable) by Universe's expansion, and have a negative pressure. All the known energy sources in the Universe, such as the cosmic radiation, baryons, planets, or even dark matter, all have positive pressures. So their gravitational pull can only slow down the expansion of the Universe, much in the same way as the earth's gravitational pull slows down an object that was thrown up.

So one logical possibility is to assume that the energy budget of the Universe is dominated by a new hypothetical source with negative pressure, which appears on the left hand side of Friedmann equation, and causes the accelerated expansion. This source is referred to as the *dark energy.* So far, any independent observational evidence or a well established theoretical motivation for such sources is absent, and this is why it is crucial to explore the alternative and testable ideas. This is the subject of the present talk.

Einstein's General Relativity, has its applicability range of distances. For example, we know that in the vicinity of a black hole singularity the standard laws of gravity break down, and General Relativity must be *completed* by a larger theory that incorporates quantum gravity, such as String Theory.

The purpose of this talk is to advocate an idea that the acceleration of the Universe indicates that the laws of General Relativity get modified not only at very short but also at very large distances. It is this modification, and not the dark energy, that is responsible for the accelerated expansion of the Universe. In the other words the key to the mystery is in modification of the right hand side of Friedmann equation, as opposed to modification of its left hand side in case of dark energy hypothesis.

It should be stressed that, such a modification is not merely a re-arrangement of entries between the

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different sides of Friedmann equation. Modified gravity predicts qualitatively new dynamical phenomena, and is testable also by experiments other than the large distance cosmological observations. The reason for this is the following *discontinuity*. Any theory of modified gravity has a characteristic length-scale r_c. This marks a crossover distance beyond which the cosmological evolution gets dramatically modified. Then, schematically we have

$$\begin{aligned}&\text{Action of Modified Gravty}\\ &= \text{Einstein} + \frac{1}{r_c}(\text{New Dynamics})\end{aligned} \tag{2}$$

The crucial point is that, contrary to the naive intuition, in the limit $r_c \to \infty$ we don't recover exactly the Einstein's theory. In the other words,

$$\lim_{r_c \to \infty} (\text{Modified Gravity}) \neq (\text{Einstein}) \tag{3}$$

The reason is that *only* in Einstein's theory the mediator of gravitational force is a *massless* spin-2 particle, graviton. The converse is also true. It can be proven that a generally covariant theory of massless spin-2 graviton is unique and is Einstein's General Relativity. The massless graviton, just like the massless photon, has only two degrees of freedom. These are the two polarizations of a gravitational wave, which just like the electromagnetic waves is transversely polarized. It can be proven (in fact it follows from the above) that any deviation from Einstein's action, no matter how small, changes this property. In modified gravity, the mediator is no longer described by a massless state with two degrees of freedom, but inevitably acquires new degrees of freedom and new dispersion relation. These additional degrees of freedom lead to testable predictions of the new theory. Historically, the firs example of the above discontinuity was observed in [8] in Pauli-Fierz theory of linearized massive gravity. However, as we shall see it is a generic property of modified gravity theories.

In studying modification of gravity, we will only be interested in theories that maintain all the elegance and the consistency of Einstein's generally covariant theory, as well as its perfect agreement with all the existing observations. This requirement, extremely narrows the class of the interesting theories, and makes them strongly constrained.

Let us now discuss why a large distance modification of gravity can lead to the cosmic acceleration without any dark energy source, whereas the standard gravity cannot. First, note that because the Friedmann equation is linear in H^2, it automatically implies that without a proper source, no evolution of the Universe is possible. No source = no expansion. One way to think about this is the following. Consider an Universe with a constant acceleration, that is an Universe with $H^2 = constant$. A constant H implies that the Universe is filled with a coherent time-dependent gravitational field, or equivalently with a *graviton condensate*. Hence, an uniformly-accelerating Universe can be described as a result of graviton condensation. Because in Einstein theory the gravitons are massless bosons, they cannot condense without a proper source. This is a general property of massless bosons. For instance, to have an uniform electric field, one needs a source as well. Without the source, massless bosons can only form freely travelling waves. This is also the reason why in Einstein's General Relativity the Universe's acceleration requires a dark energy source.

Now consider a gravity that is modified at distances $\gg r_c$. As said above, in modified theory gravitons are no longer the massless bosons, and acquire a different dispersion relation. Since they are no longer massless, gravitons can condense without an external dark energy source, and hence, lead to an accelerated cosmic expansion on their own!

In the language of Friedmann equation, modification implies that its r.h.s will contain lower powers of H^2 and its derivatives. In the simplest case such a modification can be parameterized as [6]

$$H^2 - \frac{1}{r_c^{\alpha}} H^{2-\alpha} = \frac{8\pi}{3} G_N \rho \tag{4}$$

where $\alpha > 0$ is a parameter. It is not surprising that there is an uniformly-accelerated solution $H = r_c$ even with $\rho = 0$.

Let me now address the following question. How could a quantum gravity theory such as String Theory, which is usually associated with modification of the laws of gravity at very small distances, give new dynamics at large distances? One way in which such a large distance dynamics can set in, is through extra dimensions[2, 4, 7, 9]. (Some alternative ideas can be found in [19–21]). String theory (at least the way we know it) can only be formulated in more than three space dimensions. Where are these dimensions? For further discussion I shall adopt a particular framework, called the "brane-world". According to it, the reason why we do not see these extra dimensions through the ordinary interactions, like electromagnetism, is because all the known forces (other than gravity) and particles are confined to a three-dimensional surface, called *brane*. Brane is floating in higher dimensional space, but ordinary particles are confined and can only move along three dimensions. This creates an illusion of three-dimensionality, but only for the confined forces. This picture is natural in string theory, in which the ordinary elementary particles (such as quarks, leptons, photons) come about as excitations of *open strings*. Consistency of the theory requires the open strings to be stack with their endpoints to the hyper-surfaces called *D*-branes, and cannot move in the extra space[22]. Note that in contrast gravity comes from the vibrations of the closed strings, which have no endpoints and cannot be confined.

In such a picture, at low energies the only indicator of extra dimensions would be gravity, which would reveal its high-dimensional nature through the Gauss' law. According to it, the Newton's inverse-square gravitational force would become $1/r^{2+n}$ -force, where n is the number of large extra dimensions. One way to hide these extra dimensions is through curling them up in small compact spaces. The modification of the Newton's law would then be "postponed" until the very short distances, smaller than the size of the compact dimensions. At large distances however gravity and cosmological evolution of the Universe would look completely normal.

It turns out that there is an alternative way, to hide extra dimensions, but now at very large distances as opposed to the very small[2]. I shall explain first how this case works on a simple analogy. Consider a metallic sheet submerged in water. The two dimensional sheet will be the analog of the brane, and water will play the role of the extra dimension. Waves will play the role of gravity. Now imagine hitting the sheet with a hammer. This will excite waves that will propagate in all the directions and carry away the energy. Now imagine a two-dimensional observer that measures the dispersion of waves on the surface, and suspects nothing about the existence of the extra dimension. Could this observer nevertheless discover the additional (water) dimension? Because the metal is much denser than water, close to the source propagation of waves will look two-dimensional. Only far away from the source, when the energy leakage to extra dimension becomes significant, can the observer detect the extra dimension. Qualitatively this is very close to what happens to gravity in our scenario. Newton's law interpolates between the four-dimensional and five dimensional regimes at the distance r_c.

The reason for the nearly four-dimensional behavior at short distances is that the brane breaks translational invariance in the extra dimensions, and alters the gravitational action by inducing the terms that would not be there in its absence. The most important of such terms is the four-dimensional brane-localized curvature term. The strength of this term depends on the fundamental high energy dynamics[2, 3, 14, 15]. Since our discussion is at the level of effective field theory, we shall treat it as a parameter. Reduced to the bare essentials, the relevant part of the action takes the following form

$$S \;=\; \frac{M_P^2}{4r_c}\int d^4x\;dy\;\sqrt{|g^{(5)}|}\;R_5 + \frac{M_P^2}{2}\int d^4x\;\sqrt{|g|}\;R. \qquad (5)$$

The first term is just an usual 5D Einstein action, with the five dimensional Plank mass given by $M_*^3 = \frac{M_P^2}{4r_c}$. $g_{\mu\nu}(x)$ is the induced metric on the brane, which in the approximation of the "rigid" brane is simply given by the value of the five-dimensional metric at the position of the brane. For instance, if we locate the brane at the origin of the fifth coordinate y, the induced metric takes the form $g_{\mu\nu}(x) = g^{(5)}_{\mu\nu}(x, y = 0)$. The 4D Einstein term on the brane (R) plays the crucial role in generating 4D gravity on the brane at intermediate distances, despite the fact that the space is actually five-dimensional. Here is how this effect comes about. Consider a gravitating source localized on the brane, and let us ask what kind of gravitational field will it create on the brane. That is, what type of a Newtonian attractive force will it exert on the test bodies that are also localized on the brane? The Newtonian force is mediated by the virtual graviton exchange, which in our case (after some gauge fixing) satisfy the following linearized equation

$$\left(\frac{1}{r_c}(\nabla^2 \,-\, \partial_y^2) \,+\, \delta(y)\,\nabla^2\right)\,h_{\mu\nu} = -\frac{1}{M_P^2}\left\{T_{\mu\nu} - \frac{1}{3}\eta_{\mu\nu}T^\alpha_\alpha\right\}\delta(y) + \delta(y)\,\partial_\mu\partial_\nu h^\alpha_\alpha, \qquad (6)$$

where $T_{\mu\nu}$ is the brane-localized energy-momentum source. The unusual thing about this graviton is that its propagation is governed by two kinetic terms. The 4D kinetic term localized on the brane forces the graviton to propagate according to four-dimensional laws, which would result into the 4D $1/r^2$-Newton's force. The 5D kinetic term, however, allows graviton to also propagate off the brane, and this term alone would of course result into a 5D Newton's force, which scales as $1/r^3$. In the presence of both kinetic terms graviton "compromises", and the force law exhibits the crossover behavior. For $r \ll r_c$ the potential is $1/r$, and for $r \gg r_c$ it turns into $1/r^2$.

Now suppose we have an observer, that lives on the brane and knows nothing about the existence of extra dimensions. Based on cosmological observations this observer derives an effective cosmological equation for the four-dimensional scale factor $a(t)$, of the effective 4D FRW metric. The question is what would be the analog of the 4D Friedmann equation derived in such a way? The answer turns out to be the following[4]

$$H^2 \,\pm\, H/r_c = \frac{8\pi G_N}{3}\rho \qquad (7)$$

The $1/r_c$ correction comes from large-distance modification of gravity, due to fifth dimension, and is negligible at early times. So for $H \gg 1/r_c$ the standard FRW cosmology is reproduced. However, for late times modification is dramatic. In particular, the cosmological expansion admits a "self-accelerating" branch with constant $H = 1/r_c$, without need of any matter source[4, 7]. Hence, the large-distance modification of gravity may be a possible explanation for the late time acceleration of the Universe, that is suggested by the current observations[1]. Because of the specific nature of the transition, the upcoming precision cosmological studies can potentially differentiate between the modified gravity and more conventional dark energy scenarios[4, 6].

As said above, It is expected that such a late time accelerated (deSitter) phase is a generic property of large-distance modified gravity theories, since such a modifications should result in lower powers of H^2 in the modified Friedmann equation, and thus, in general could admit new solutions with constant H, even for $\rho = 0$.

2. Short Distance Signatures

What are the other possible tests of these idea? As we said above, modified gravity implies new degrees of freedom that mediate the additional gravitational force on top of the standard one. This extra force is one of the key points for constraining and testing theories of modified gravity. In fact in any ghost-free local theory of large-distance modified gravity which admits Lorentz-invariant linearized expansion, there is an inevitable presence of additional scalar polarization(s) of the graviton. This new degree of freedom in the linearized approximation mediates an additional scalar attraction incompatible with the solar system observations. Thus, we are driven to a very important conclusion that *any* modified gravity theory from the above class, is automatically ruled out, unless the linearized approximation (in G_N-expansion) for the gravitational field of the sun in this theory breaks down at the solar system distances. That is, not G_N but rather $1/r_c$ must be a good expansion parameter for the gravitational field at short distances. This implies that in sensible theories of modified gravity one should expect that the additional degrees of freedom must become strongly coupled at some intermediate energy scale between M_P and $1/r_c$. This effect was indeed discovered in [9]. Imagine that we wish to study the motion of a planet in the modified theory. We can try to find the metric in form of the G_N-expansion. But in order to trust this expansion, one has to be sure that the leading effect is indeed given by one-graviton exchange, that is by the contribution of order G_N. However, this is not always the case, because the additional polarizations of massive graviton have couplings singular in $1/r_c$, and blow up at certain distance. This can be seen from the effective brane-to-brane graviton propagator that can be written as

$$D_{\mu\nu;\alpha\beta}(q) = \left(\frac{1}{2}\,\tilde{\eta}_{\mu\alpha}\tilde{\eta}_{\nu\beta} + \frac{1}{2}\,\tilde{\eta}_{\mu\beta}\tilde{\eta}_{\nu\alpha} - \frac{1}{3}\,\tilde{\eta}_{\mu\nu}\tilde{\eta}_{\alpha\beta}\right)\frac{1}{q^2 + q/r_c - i\epsilon}\,, \quad (8)$$

where q is four-momentum and

$$\tilde{\eta}_{\mu\nu} = \eta_{\mu\nu} + \frac{q_\mu q_\nu}{q/r_c}\,. \quad (9)$$

The additional polarizations are identical (one spin zero and two spin one), to the one of massive gravity, and the role of the "mass" is played by q/r_c term. This is not surprising since the 5D graviton from the point of view of the 4D theory looks like a resonance that can be spectrally expanded in continuum of massive spin-2 states, each of which has five polarizations. It can be directly checked that the tree-graviton vertex exhibits the following singularity [9]

$$\frac{q^3 r_c^2}{M_P} \quad (10)$$

and hence the longitudinal polarizations become strongly coupled at the scale $q_s = (M_P/r_c^2)^{\frac{1}{3}}$. (For massive graviton this was also found in [13]). What is the physical meaning of this scale? The breakdown of the linearized approximation and existence of the "strong coupling" signals that G_N is not any more a good expansion parameter, and series have to be resummed. If the resummation is possible and the answer does not explicitly contain the "strong coupling" scale, then this scale is simply an artifact of the incorrect perturbative expansion in powers of G_N.

At the tree-level there indeed is a complete resummation for different cases of interest[9–12]. For instance, Schwarzchild solution can be found in terms of $1/r_c$-expansions and exhibits a complete continuity. This solution has the following form [11]

$$\nu(r) = -\frac{r_g}{r} + \mathcal{O}\left(\frac{1}{r_c r}\sqrt{r_g r^3}\right),$$
$$\lambda(r) = \frac{r_g}{r} + \mathcal{O}\left(\frac{1}{r_c r}\sqrt{r_g r^3}\right) \quad (11)$$

Here r_g is the gravitational radius of the body, and the functions $\nu(r)$ and $\lambda(r)$ parameterize the spherically-symmetric metric in the usual way $g_{00} = \mathrm{e}^{\nu(r)}$, $g_{rr} = \mathrm{e}^{\lambda(r)}$. The validity of the above expansion is also confirmed by the exact solution of [12] As we see, any reference to the scale q_s has disappeared from the resumed solution. Which indicates that the "strong coupling" scale could indeed be an artifact of the perturbative expansion in terms of G_N.

It is interesting to understand what happens when the additional states run in quantum gravity loops. This issue will not be discussed here, and instead we shall turn to phenomenological implications of the above effect.

As it is obvious from (11), the additional force is suppressed relative to the Einstein gravity near the massive objects, but "catches up" at large distances. The distance at which the new force becomes significant depends on the mass of a gravitating object and is large for massive bodies and small for the light ones. For a gravitating body of gravitational radius r_g, this distance is given by

$$r_* = (r_c^2 r_g)^{\frac{1}{3}} \tag{12}$$

In the other words, every gravitating source carries an "umbrella" of size r_*, beyond which corrections to Einstein gravity become strong. At closer distances, the relative corrections to the gravitational potential Ψ are small and are given by[11]

$$\frac{\delta\Psi}{\Psi} \sim \frac{r}{r_c}\sqrt{\frac{r}{r_g}} \tag{13}$$

corrections are suppressed, but *not negligible*. This gives an exciting possibility of detecting new physics in gravitational experiments at short distances. For instance, for the Earth, r_g ~cm, and one has to move away by $r_*^{earth} \sim 10^{19}$ cm, before the corrections become comparable to the earths gravitational force computed in Einstein's theory. This would require going beyond the solar system, at which point the Earth's gravitational effects become washed out by sun's gravity, which has a roughly ten times bigger "umbrella", $r_*^{sun} \sim 10^{20}$cm. This fact, however, should not create a false impression that the corrections are negligible. Fortunately, Einstein's gravity is an extremely well-tested theory, and so, we do not need to travel that far for detecting the difference. For instance, at the earth-moon distance $r \sim 10^{10}$cm, the relative correction to the Earth's gravity is $\sim 10^{-12}$, which is just an order of magnitude away from the current accuracy. This opens up an exciting possibility of testing the modified gravity theory by measuring the lunar perihelion precession!

3. Lunar Ranging Test

The experiments that would be sensitive to such a deviation are the improved-accuracy lunar ranging experiment. These experiment perform the precision measurements of lunar orbit, by sending the laser beam to the moon. Beam gets reflected from the lunar surface by the *retroreflector* mirrors. The firsts of these mirrors were placed on the lunar surface by the astronauts of Apollo 11 mission, and are used since then for lunar ranging. The detection of the reflected beam gives then a possibility to monitor the lunar orbit with a great accuracy. Current accuracy is approximately one cm, but the planned new experiments will be capable of improving accuracy by order of magnitude or so. Such an improvement would enable the test of modified gravity theory as shown in [5]. We shall briefly reproduce these arguments here. Let ϵ be the fractional change of the gravitational potential

$$\epsilon \equiv \frac{\delta\Psi}{\Psi}, \tag{14}$$

where $\Psi = -GM/r$ is the Newtonian potential. The anomalous perihelion precession (the perihelion advance per orbit due to gravity modification) is

$$\delta\phi = \pi r(r^2(r^{-1}\epsilon)')', \tag{15}$$

where $' \equiv d/dr$. In our modified gravity theory

$$\epsilon = -\sqrt{2} r_c^{-1} r_g^{-1/2} r^{3/2}. \tag{16}$$

The numerical coefficient deserves some clarification. The above coefficient was derived in [11] on Minkowski background. However, non-linearities created by cosmological expansion can further correct the coefficient. One would expect these corrections to scale as powers of $r_c H$, where H is the observed value of the Hubble parameter. On the accelerated branch, as its obvious from (7), $H \sim 1/r_c$ and thus, one would expect the corrections to be of order one. We will restrict ourselves to order of magnitude estimate, but the sign may be important if the effect is found, since according to[16] it could give information about the cosmological branch. We get

$$\delta\phi = (3\pi/4)\epsilon. \tag{17}$$

Numerically, the gravitational radius of the Earth is $r_g = 0.886$cm, the Earth-Moon distance is $r = 3.84 \times 10^{10}$cm, the gravity modification parameter that gives the observed acceleration without dark energy $r_c = 6$ Gpc. We get the theoretical precession

$$\delta\phi = 1.4 \times 10^{-12}. \tag{18}$$

This is to be compared to the accuracy of the precession measurement by the lunar laser ranging. Today the accuracy is $\sigma_\phi = 4 \times 10^{-12}$ and no anomalous precession is detected at this accuracy, in the near future a tenfold improvement of the accuracy is expected[17]. Then a detection of gravity modification will be possible.

4. Discussions

The accelerated Universe can be a window of opportunity for looking into the most fundamental properties of gravitation, and may signal the modification of standard laws of gravity at very large distances. Because of the extremely restrictive nature of gravity, this hypothesis is potentially experimentally testable, both by cosmology, as well as by gravitational measurements at much shorter scales, such as the earth-moon distance scale. These experiments together with cosmological studies and supernova surveys, provide an unique possibility to learn about the nature of gravity at very large scales.

Acknowledgments

It is pleasure to thank the organizers of the Texas at Stanford Symposium for the exciting meeting.

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