

## Recent Results of Muon g-2 Collaboration

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The anomalous magnetic moment of  $\mu^-$  has been measured precisely at Brookhaven National Laboratory (BNL). Data analysis was done blindly and independently for magnetic field  $B$  and anomalous spin precession frequency  $\omega_a$ . The result is based on data collected in 2001. The anomalous magnetic moment is  $a_{\mu^-} = 11\,659\,214(8)(3) \times 10^{-10}$  (0.7 ppm), where the first uncertainty is statistical and the second is systematic. The result is consistent with our previous measurement for positive muon. The average value for  $a_\mu$  is  $11\,659\,208(6) \times 10^{-10}$  (0.5 ppm).

## 1. INTRODUCTION

Particle with the electric charge  $e$  and mass  $m$  can have a dipole magnetic moment  $\vec{\mu}$  via its intrinsic angular momentum vector  $\vec{s}$  (spin):

$$\vec{\mu} = g \frac{e\hbar}{2mc} \vec{s}; \quad \mu = (1 + a) \frac{e\hbar}{2mc}; \quad a \equiv \frac{g - 2}{2} \quad (1)$$

where  $g$  is called gyromagnetic ratio. The quantity  $a$  is the anomalous dipole magnetic moment (or anomaly).

In 1928 Dirac successfully quantized Einstein's relativistic energy equation to show theoretically that  $g \equiv 2$  for spin 1/2 point-like particle.

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\*Deceased

The measurement of the electron gyromagnetic ratio was done in 1948 by Foley and Kirsh [1]. The result for the  $g_e$  value was not exactly 2 ( $\approx 2.002$ ). Schwinger published in 1948 his famous result [2] for lowest-order vertex correction (emission and absorption of virtual photon),  $a = \frac{\alpha}{2\pi} \approx \frac{1}{860}$ , in excellent agreement with Foley and Kusch experiment. The electron anomaly has been measured to a precision of four parts per billion (ppb). Next order vertex correction and vacuum polarization were included into the theoretical calculation of the electron anomalous magnetic moment. The remarkable agreement between measured electron anomaly and the theoretical prediction was a triumph of the modern physics.

The muon anomalous magnetic moment can be calculated similarly to the electron one. However, the greater muon mass leads to the measurable contribution of the massive local field fluctuations (quark loops,  $W$  and  $Z^0$  bosons, etc.). The relative contribution of any existing massive field fluctuation goes as  $(\frac{m_\mu}{m_e})^2$ , so muon anomaly is 40 000 times more sensitive to any new massive field contribution compared to the electron anomaly.

The Standard Model (SM) calculations for the muon anomalous magnetic moment includes QED term and small contributions from hadronic and weak interactions. The largest uncertainty for SM  $a_\mu$  calculation comes from the hadronic interactions.

## 2. MEASUREMENT PRINCIPLE

Muon ( $g-2$ ) experiment is based on the fact that spin precesses faster than momentum vector for the muon orbiting in the transverse uniform magnetic field  $B$ . The spin precession frequency  $\omega_s$ , the momentum precession (cyclotron) frequency  $\omega_c$  and the difference frequency  $\omega_a$  are given by

$$\omega_s = g \frac{eB}{2mc} + (1 - \gamma) \frac{eB}{mc\gamma}; \quad \omega_c = \frac{eB}{mc\gamma}; \quad \omega_a = \omega_s - \omega_c = \frac{(g - 2) eB}{2 mc} \quad (2)$$

The difference frequency  $\omega_a$  (or anomalous precession frequency) is the frequency of the spin precession relative to the momentum vector. The spin precession frequency  $\omega_a$  does not depend on a  $\gamma$  factor, so relativistic muon beam can be used for the anomaly measurement leading to the increased number of spin precession cycles detected.

Quadrupole electric field was used for vertical beam focusing. Muon spin motion is affected by that focusing field, so in the presence of vertical focusing electric field  $E$  and magnetic field  $B$  spin precession frequency is modified by

$$\vec{\omega}_a = - \frac{e}{mc} [a_\mu \vec{B} - \left( a_\mu - \frac{1}{\gamma^2 - 1} \right) \vec{\beta} \times \vec{E}] \quad (3)$$

where  $a_\mu$  is the muon anomaly. By choosing the so-called “magic”  $\gamma$ , satisfying the relation,

$$\left( a_\mu - \frac{1}{\gamma^2 - 1} \right) = 0 \quad (4)$$

the  $\vec{\beta} \times \vec{E}$  term is vanished. The “magic”  $\gamma = 29.3$  defines muon momentum  $p = 3.09$  GeV/ $c$  and dilated lifetime  $\tau_\mu = 64.4 \mu s$ .

The measurement of the spin precession frequency  $\omega_a$  and magnetic field  $B$  (measured in terms of the proton NMR frequency  $\omega_p$ ) determines the anomaly  $a_\mu$  as follows:

$$a_\mu = \frac{\frac{\omega_a}{\omega_p}}{\lambda - \frac{\omega_a}{\omega_p}} \quad (5)$$

where  $\lambda = \mu_\mu / \mu_p = 3.183\ 345\ 39(10)$  [3].

## 3. MUON $g-2$ EXPERIMENT AT BNL

Longitudinally polarized muons were injected into a superconducting storage ring. The storage ring is a single C-shape dipole magnet, 14.22 m in diameter. Four superconducting coils provided uniform 1.45 T dipole magnetic field.

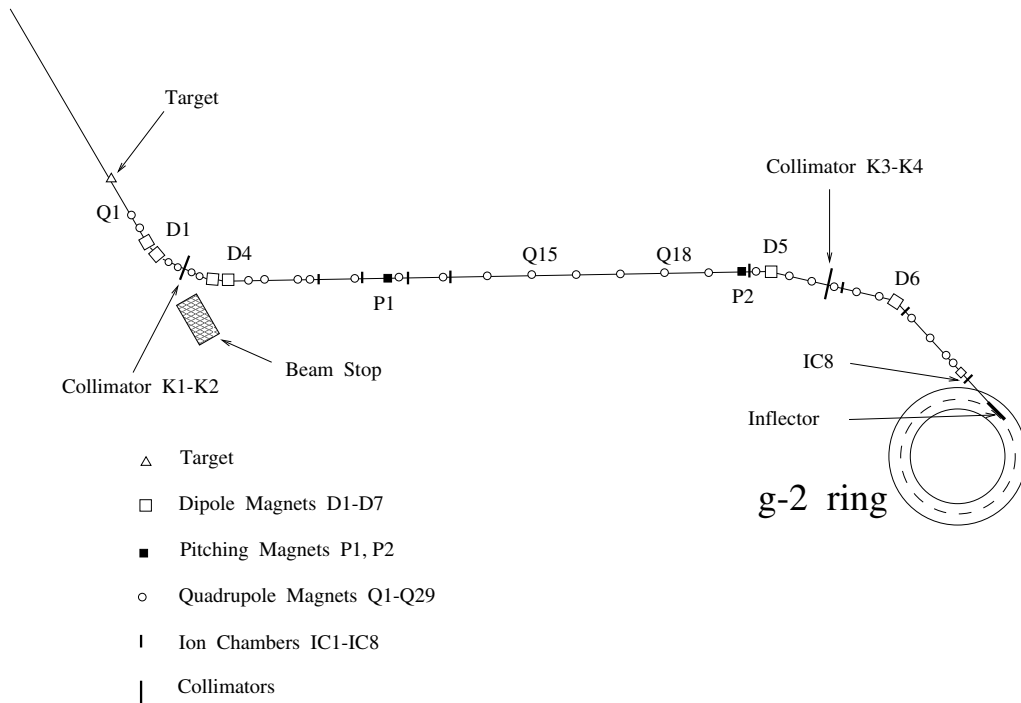


Figure 1: Beam line

Polarized muon beam was produced by Alternating Gradient Synchrotron (AGS) at BNL. 24 GeV/c momentum protons stroke a nickel target to produce pions. Pions were directed into a decay channel where they decay to muons. Pion decay is parity violating process, so muons were naturally polarized. The average muon polarization was 95% for a small forward momentum bite selected. The muon beam line schematic is shown in Figure 1.

The beam injection scheme is shown in Figure 2. Muon beam entered the storage ring through a hole in main magnet yoke. Next superconducting inflector [5] was used to cancel the field of the main magnet. The injection point is 7.7 cm radially outward of the central storage orbit. To store muons, three fast kicker magnets [6] were used to give the designed  $7.7\text{cm}/711.2\text{cm} = 10.8$  mrad outward kick needed to move muons from injection onto the storage orbit.

Electrostatic quadrupole field [7] was used to stabilize muon orbit in vertical direction. Four groups of the quadrupoles were located in the ring, each was pulsed with  $\pm 24$  kV totally covering 43% of the storage ring azimuth.

Muon decay electrons were detected using 24 Pb-Scintillating fiber calorimeters [8] located symmetrically around the inside of the ring (see Figure 3). Decay electron signals above the certain threshold were digitized using 400 MHz waveform digitizer (WFD) and stored for the further analysis.

## 4. 2001 DATA ANALYSIS

### 4.1. Magnetic Field Analysis

The magnetic field  $B$  was measured in terms of nuclear magnetic resonance (NMR) frequency for free protons  $\omega_p$  [9]. The field inside the storage ring was mapped 3-4 times a week using 17 NMR probes housed inside vacuum tight trolley. The trolley was pulled through the muon storage region along the rails. Before and after data taking, the trolley probes were calibrated with respect to a spherical water probe [10].

Two independent magnetic field analyses were performed. Figure 4 (left plot) shows a two-dimensional multi-pole

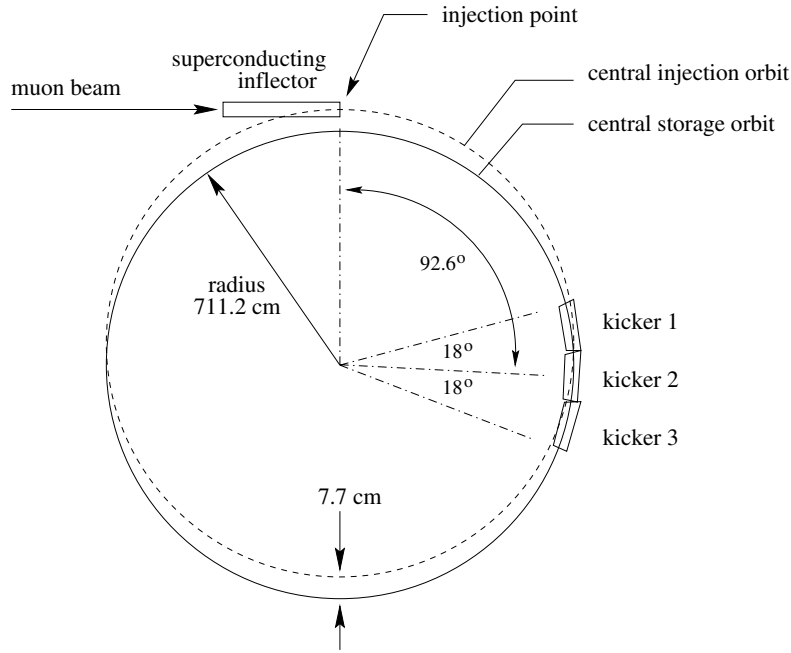


Figure 2: Muon beam injection

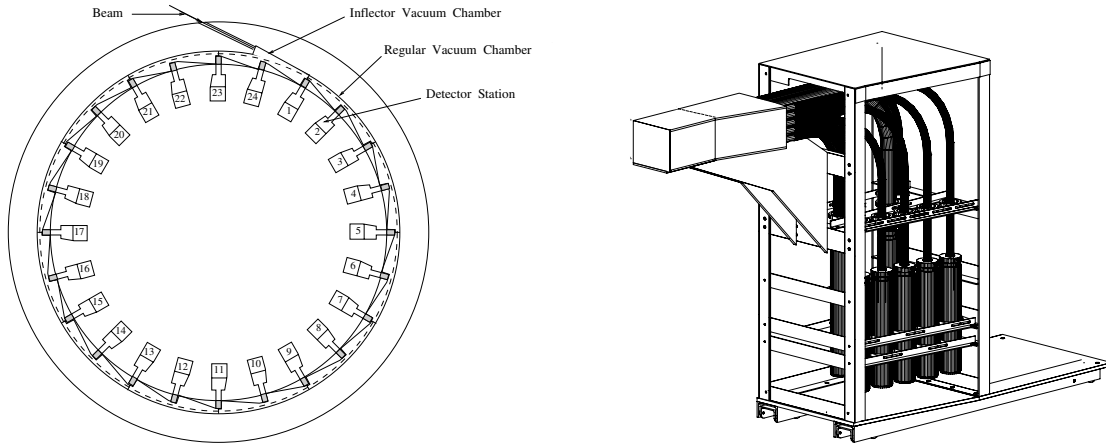


Figure 3: Calorimeters lay-out (left). One of the 24 electromagnetic calorimeters (right).

expansion of the azimuthally averaged readings of the 17 NMR probes from a trolley measurement. To monitor the magnetic field during the data taking, continuous readings from 150 fixed NMR probes distributed around the storage ring were used. The difference between trolley field measurement and reading of the fixed NMR probes is shown in Figure 4 (right plot).

The systematic uncertainties for the  $\omega_p$  analysis are listed in Table I. The total systematic uncertainty for the magnetic field measurement is 0.17 ppm.

## 4.2. $\omega_a$ Analysis

The  $\omega_a$  frequency was determined by fitting the time spectrum of the muon decay electrons. The 2001 data set consisted of 4 billion electrons with energy above 1.8 GeV collected for two different field focusing indexes  $n = 0.122$  and  $n = 0.142$ , which respectively resulted in a coherent betatron oscillation (CBO) frequencies  $f_{cbo} = 419$  kHz and

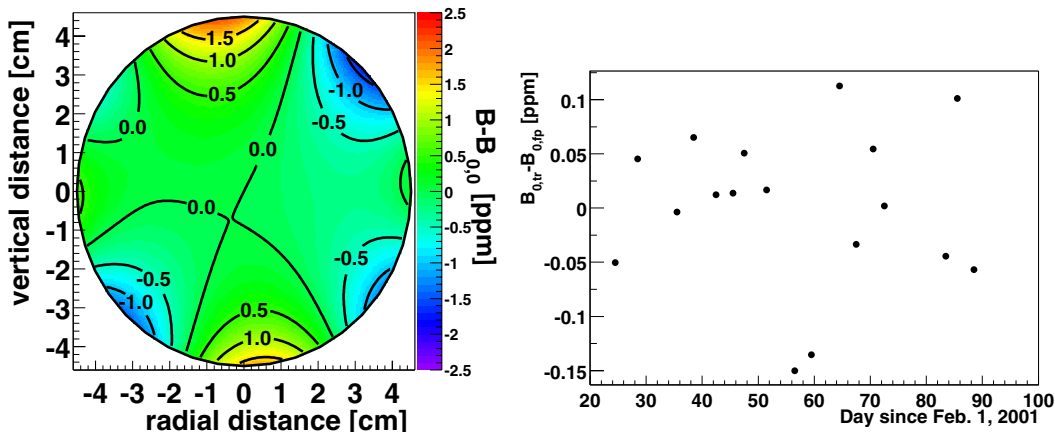


Figure 4: Two-dimensional contour plot of the 2001 magnetic field multi-pole expansion (left plot). Stability of the trolley measurements from the day since the data collection started (right plot).

Table I: Error table for magnetic field measurement.

Effect	2000 [ppm]	2001 [ppm]
Absolute calibration of standard probe	0.05	0.05
Calibration of trolley probes	0.15	0.09
Trolley measurements of $B_o$	0.10	0.05
Interpolation with fixed probes	0.10	0.07
Uncertainty from muon distribution	0.03	0.03
Others	0.10	0.10
Total	0.24	0.17

$f_{cbo} = 491$  kHz.

Two independent algorithms were implemented to reconstruct the energies and times of the decay electrons detected by calorimeters. The time spectrum of the decay electrons is shown in Figure 5.

Five independent analyses of  $\omega_a$  were performed for 2001 data set. Only one of them [11] is briefly described below.

The counting rate  $N_e(t)$  of the decay electrons with energy above the threshold can be ideally described using 5 parameter function

$$N_e(t) = N_o e^{-t/\gamma\tau} [1 + A \cos(\omega_a t + \phi)] \quad (6)$$

where  $N_o$  is normalization,  $\tau_\mu$  is muon lifetime,  $A$  is energy dependent asymmetry (or amplitude of the spin precession oscillations) and  $\phi$  is the phase (the angle between the spin and momentum vectors at the moment of injection). To minimize the statistical uncertainty of the spin precession frequency, 1.8 GeV energy threshold was used.

The described analysis is based on the Ratio Method [12]. The data were randomly assigned to the four independent subsets  $n_1(t)$ ,  $n_2(t)$ ,  $n_3(t)$ ,  $n_4(t)$  and recombined to form the ratio spectrum

$$r(t) \equiv \frac{n_1(t + \frac{\tau_a}{2}) + n_2(t - \frac{\tau_a}{2}) - n_3(t) - n_4(t)}{n_1(t + \frac{\tau_a}{2}) + n_2(t - \frac{\tau_a}{2}) + n_3(t) + n_4(t)} \quad (7)$$

where  $\tau_a = \frac{2\pi}{\omega_a}$  is an estimate of the muon spin precession period. Exponential muon decay and effects slowly-depending on time are absent or significantly reduced in the ratio spectrum, while (g-2) oscillations are clearly seen. The ratio spectrum for the sum of the detectors is shown in Figure 6.

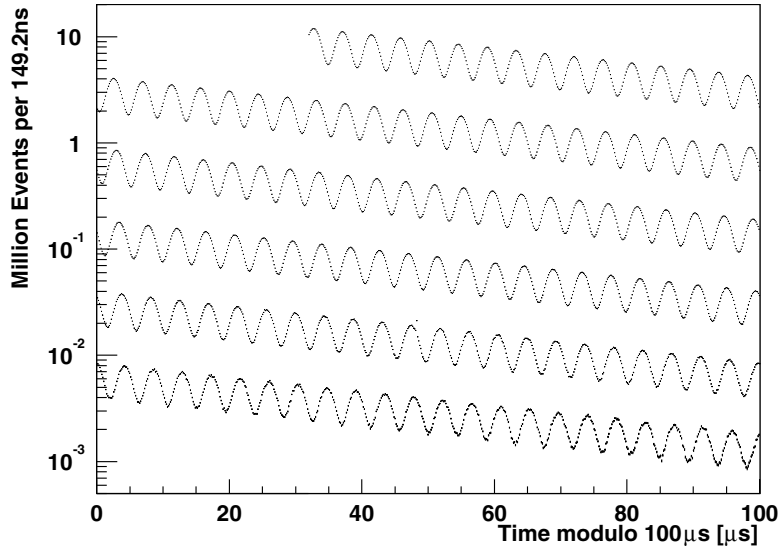


Figure 5: Time spectrum of detected electrons. The period of  $(g-2)$  oscillations is  $4.365\mu\text{s}$  and dilated muon lifetime is  $64.4\mu\text{s}$ .

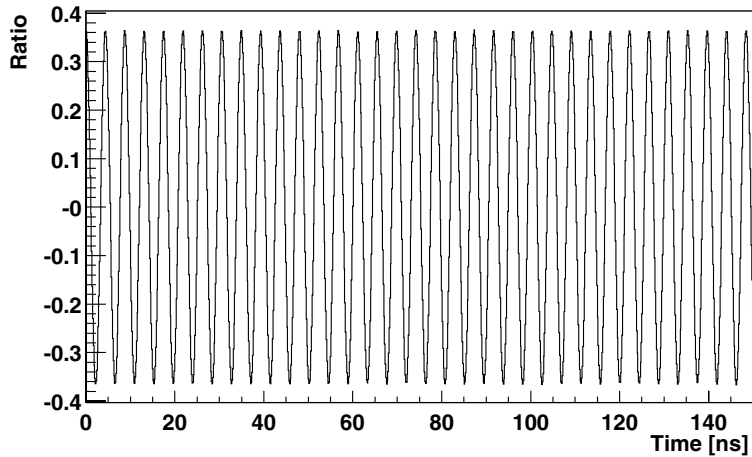


Figure 6: Ratio spectrum  $r(t)$ .

Spectrum was fitted using a three-parameter fit function  $F_r(t)$

$$F_r(t) \approx A \sin(\omega_a t + \phi) + \frac{1}{16} \left( \frac{\tau_a^2}{\tau_\mu^2} \right) \quad (8)$$

The statistical error on the spin precession frequency was determined precession frequency with 0.7 ppm. The systematic uncertainty was studied for different effects. The most significant contributions to the systematic error were beam dynamic effects (0.19 ppm), events overlapping (0.07 ppm), gain changes (0.09 ppm) and lost muons (0.08 ppm).

The internal consistency of the fit result was verified in several ways. The fitted frequency  $\omega_a$  was found constant with the fit start time, decay electron energy, detector and run number (see Figure 7).

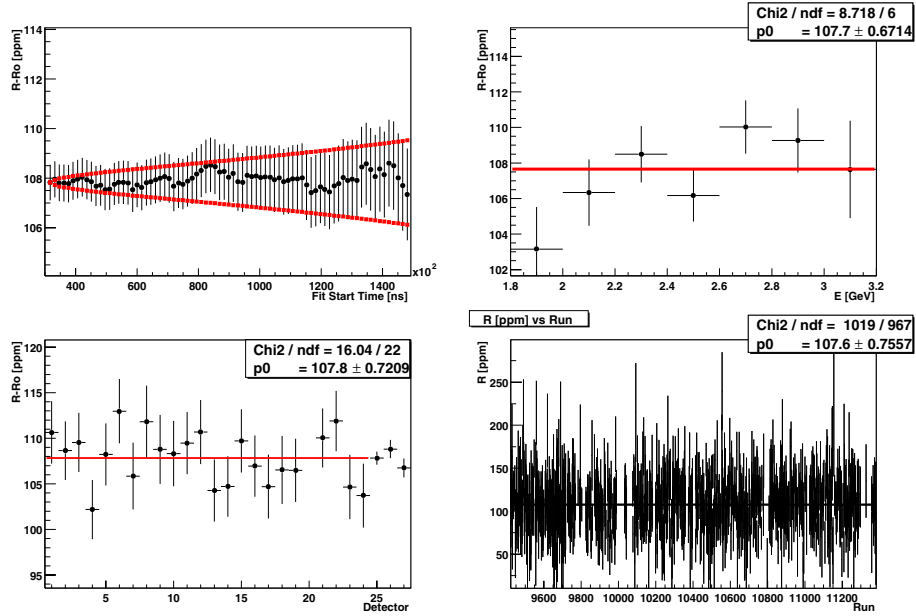


Figure 7: Internal consistency of the fit result versus fit start time (top left plot), decay electron energy (top right plot), detector (bottom left) and run number (bottom right). Red lines on the top left plot indicates the allowable statistical variation for  $\omega_a$ .

Table II: Error table for the combined  $\omega_a$  measurements.

Effect	Uncertainty [ppm]	
	$\mu^+$	$\mu^-$
Statistics	0.62	0.66
Overlapping pulses (pileup)	0.13	0.08
Gain changes	0.12	0.12
Lost muons	0.10	0.09
Beam dynamics	0.21	0.07
Other	0.08	0.11
Total systematics	0.31	0.21
Total uncertainty	0.69	0.72

### 4.3. Result

The values of  $\omega_a$  from the five analyses were in a good agreement. Small variations were expected from the differences in the analyzed data samples and analysis methods used.

Five resulting  $\omega_a$  values were combined in a simple arithmetic mean to obtain the final value for  $\omega_a$ .

Table II lists statistical and systematic uncertainties for the combined result.

The anomaly  $a_{\mu^-}$  was obtained from the results of independently measured frequencies  $\omega_p$  and  $\omega_a$ , as determined by Eq. 5 [20]

$$a_{\mu^-} = 11\,659\,214(8)(3) \times 10^{-10} \quad (0.7 \text{ ppm}) \quad (9)$$

this result is in a good agreement with positive muon anomaly [19]

$$a_{\mu^+} = 11\,659\,203(8) \times 10^{-10} \quad (0.7 \text{ ppm}) \quad (10)$$

as predicted by the CPT theorem. The difference  $\Delta R = R_{\mu^-} - R_{\mu^+} = (3.5 \pm 3.4) \times 10^{-9}$  where  $R_{\mu^-} \equiv \frac{\omega_a}{\omega_p}$ .

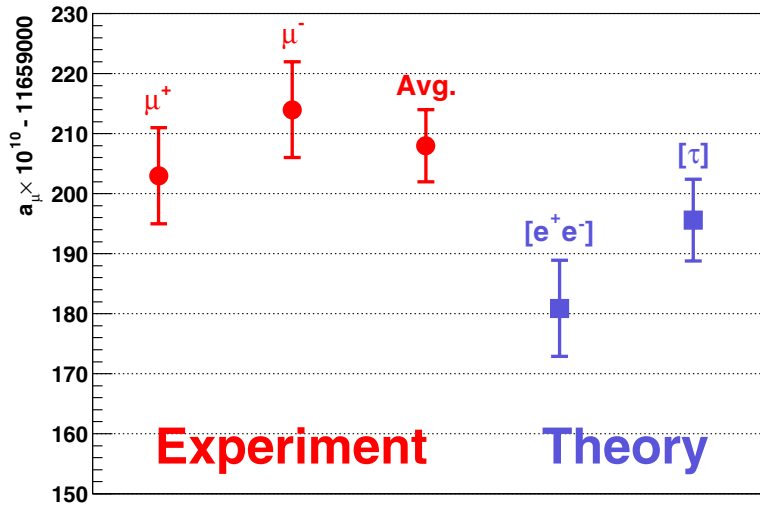


Figure 8: Results of Muon g-2 experiment for  $\mu^+$ ,  $\mu^-$  and average anomaly compared to SM predictions.

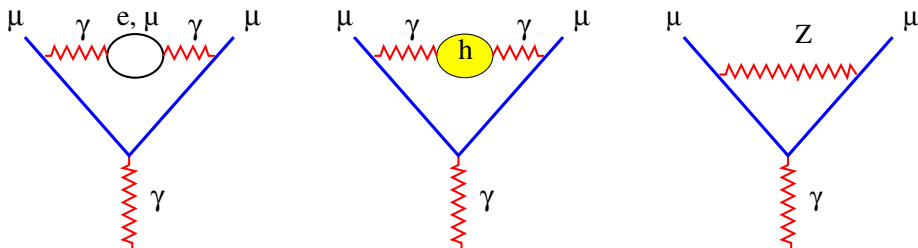


Figure 9: QED, hadronic and weak contributions to the SM  $a_\mu$  prediction.

The new world average value is

$$a_\mu = 11\,659\,208(6) \times 10^{-10} \quad (0.5 \text{ ppm}) \tag{11}$$

where the total error was combined from statistical (0.4 ppm) and systematic (0.3 ppm) uncertainties.

Figure 8 shows the comparison of the BNL results for  $\mu^+$ ,  $\mu^-$  and the average value with SM theoretical predictions.

## 5. THEORETICAL PREDICTION FOR $a_\mu$

### 5.1. Standard Model Prediction

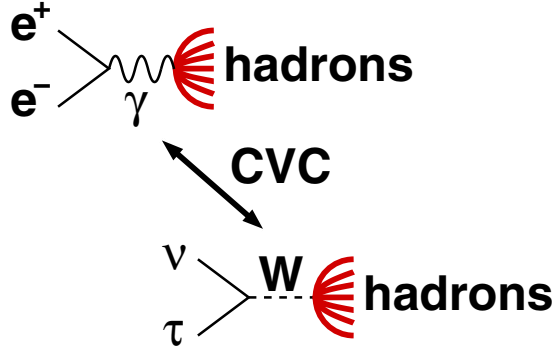
The SM prediction for  $a_\mu(SM)$  consists of three terms: QED, hadronic and weak contributions (see Figure 9).

$$a_\mu(SM) = a_\mu(QED) + a_\mu(had) + a_\mu(weak) \tag{12}$$

The total QED term in Eq. 12 has been evaluated to order  $\alpha^4$ , with the  $\alpha^5$  term estimated.

$$a_\mu(QED) = 11\,658\,472.07(0.04)(0.1) \times 10^{-10} \tag{13}$$




 Figure 10: Hadronic  $e^+e^-$  annihilation and  $\tau$  decay.

The direct coupling of muons to the massive vector bosons,  $W^\pm$  and  $Z^0$ , and the scalar Higgs boson, contributes to the  $a_\mu(\text{weak})$  term. Calculation for one and two loops gives [21]

$$a_\mu(\text{weak}) = 15.4(0.1)(0.2) \times 10^{-10} \quad (14)$$

The hadronic vacuum polarization term  $a_\mu(\text{had})$  consists of low order, high order and light-by-light contributions

$$a_\mu(\text{had}) = a_\mu^{\text{had},LO} + a_\mu^{\text{had},HO} + a_\mu^{\text{had},LBL} \quad (15)$$

Lowest-order hadronic term  $a_\mu^{\text{had},LO}$  can be calculated directly from the measured cross section for  $e^+e^- \rightarrow \text{hadrons}$  annihilation using optical theorem and dispersion relations

$$a_\mu^{\text{had},LO} = \left(\frac{\alpha m_\mu}{3\pi}\right)^2 \int_{(2m_\pi)^2}^{\infty} ds \frac{K(s)}{s^2} R(s) \quad \text{where } R(s) \equiv \frac{\sigma_{\text{tot}}(e^+e^- \rightarrow \text{hadrons})}{\sigma_{\text{tot}}(e^+e^- \rightarrow \mu^+\mu^-)} \quad (16)$$

and  $K(s)$  is a kinematic factor.

The lowest-order contribution for hadronic vacuum polarization carries the largest uncertainty in the  $a_\mu(\text{SM})$  (7.3 ppm).

The value for the lowest-order hadronic contribution [24] [25] is determined using the most recent data of the CMD2 experiment  $e^+e^-$  data [22] [23]

$$a_\mu^{\text{had},LO} = 696.3(7.2) \times 10^{-10} \quad (17)$$

KLOE Collaboration (Frascati) has recently published their result for the hadronic cross section  $e^+e^- \rightarrow \pi^+\pi^-$  below 1 GeV using initial state radiation to vary the center of mass energy [26]. The result agrees with the CMD2 data [27].

The value of  $a_\mu^{\text{had},LO}$  can be indirectly obtained from hadronic  $\tau$  decay data. Hadronic decay  $\tau^- \rightarrow \pi^-\pi^0\nu_\tau$  can be related to  $e^+e^- \rightarrow \pi^+\pi^-$  using conserved vector current (CVC) hypothesis and isospin conservation (see Figure 10).

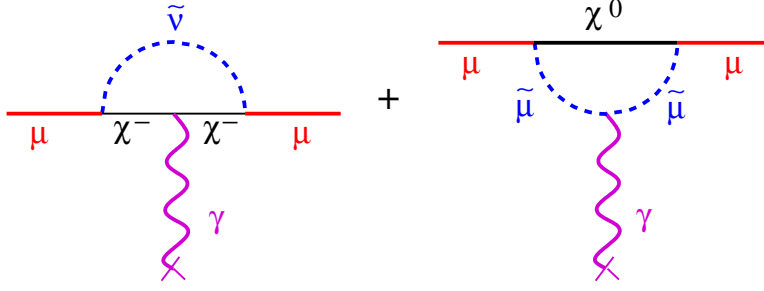
The calculated value for the lowest-order hadronic contribution (from hadronic  $\tau$  decay data [25]) is

$$a_\mu^{\text{had},LO} = 701.9(6.2) \times 10^{-10} \quad (18)$$

Two different analyses of  $e^+e^-$  data show a good agreement, the  $a_\mu^{\text{had},LO}$  value obtained from  $\tau$  decay data disagree with  $e^+e^-$  data. Additional corrections must be included in the  $\tau$  data before they can be used for reliable  $a_\mu^{\text{had},LO}$  calculation.

The higher-order hadronic term calculated by [28] is

$$a_\mu^{\text{had},HO} = -10.0(0.6) \times 10^{-10} \quad (19)$$


 Figure 11: SUSY particles contribution to  $a_\mu^{SM}$ .

The most recent light-by-light scattering contribution calculated by Melnikov and Vainshtein [29] is

$$a_\mu^{had, LBL} = 13.6(2.5) \times 10^{-10} \quad (20)$$

Using the  $e^+e^-$  based SM value from Davier and Marciano [21]

$$a_\mu^{SM} = 11\,659\,182.8(8) \times 10^{-10} \quad (0.7 \text{ ppm}) \quad (21)$$

and the average  $a_\mu$  value from Muon (g-2) experiment [20]

$$a_\mu^{exp} = 11\,659\,208(6) \times 10^{-10} \quad (0.5 \text{ ppm}) \quad (22)$$

the difference between experimental and theoretical values is

$$\Delta a_\mu = 25.5(9.2) \times 10^{-10} \quad (23)$$

or  $2.7 \sigma$ .

## 5.2. SUSY Contribution to $a_\mu$

The difference between experimental measurement and SM prediction can be interpreted as the additional contribution to  $a_\mu^{SM}$  from SUSY particles [30] see Figure 11

$$\Delta a_\mu^{SUSY} \approx \text{sign}(\mu_o) \cdot 13 \times 10^{-10} \cdot \left( \frac{100 \text{ GeV}}{\tilde{M}} \right)^2 \cdot \tan \beta \quad (24)$$

where  $\tilde{M}$  is the SUSY mass scale and  $\tan \beta$  is the ratio of two Higgs vacuum expectation values,  $\tan \beta \equiv \frac{\langle \phi_1 \rangle}{\langle \phi_2 \rangle}$ .

For large  $\tan \beta$  in the range  $4 \sim 40$  one can find SUSY particle mass limits (assuming  $\tilde{M} > 100 \text{ GeV}$  from other experimental constrains)

$$\Delta a_\mu^{exp-SM} \approx 25.2 \times 10^{-10} \rightarrow \tilde{M} \approx 150 - 475 \text{ GeV} \quad (25)$$

## 6. SUMMARY

The recent BNL result for the muon anomalous magnetic moment is  $a_{\mu^-} = 11\,659\,214(8)(3) \times 10^{-10}$  (0.7 ppm) in a good agreement with  $\mu^+$  measurement as predicted by CPT theorem. The difference  $\Delta R = R_{\mu^-} - R_{\mu^+} = (3.5 \pm 3.4) \times 10^{-9}$ . The new average value for muon anomaly is  $a_\mu = 11\,659\,208(6) \times 10^{-10}$  (0.5 ppm) where the total uncertainty was combined from statistical (0.4 ppm) and systematic (0.3 ppm) errors. The statistical error is dominating.

This is the final analysis of the anomalous magnetic moment from Muon (g-2) experiment at BNL.

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