Introduction to Extra Dimensions

- Why study Extra Dimensions ?
- Survey of Basic ED Concepts
- Some Well-known Models
    - Large E.D.
    - Warped E.D.
    - Universal E.D.
- Summary / Conclusion

TGRizzo
SSI 04

Why study extra dimensions ?

- Many problems / issues of the SM can be addressed in this context....

  → hierarchy problem ⟷ STRINGS

  → electroweak symmetry breaking (w/o a Higgs !!)

  → fermion masses / CKM / $\not{C}\not{P}$

  → unification w/o SUSY

  → new DM candidate + new cosmology perspective

  ⋮

- They lead to predictions that can be tested in future (and current !) experiments..

- Of course, the REAL reason is that they are <u>fun</u> to think about + would change our worldview if <u>discovered</u> !

→ Some basic ideas behind E. Dims...

Can we learn anything about Extra dims from 'classical' considerations? YES!

$\boxed{\text{e.g.,}}$

Consider a particle of mass M in 5D 'cartesian' co-ords + assume Lorentz Invariance holds in 5D

$\longrightarrow$ Then $\boxed{p^2 = M^2}$ but $p^2 = ?$

$$\boxed{p^2 = \underbrace{p_0^2}_{\text{Energy}} - \underbrace{\vec{p}^{\,2}}_{\text{3-momentum}} \underbrace{\pm}_{\text{sign ?}} \underbrace{p_5^2}_{\text{momentum in 5th dim}}}$$

$\{ O(4,1)$ or $O(3,2)$? $\}$

+ time-like
− space-like
Extra dim

Is there any preference? Yes ... rewrite..

$$p^2 = M^2 \rightarrow \quad p_0^2 - \vec{p}^{\,2} = p_\mu p^\mu = M^2 \mp p_5^2 \Rightarrow$$

- If $p_5^2 > M^2$ (why not?) AND I choose a time-like E.D. then $\boxed{p_\mu p^\mu < 0}$

$\rightarrow$ a $\boxed{\text{tachyon}}$ w/ possible causality problems!

To avoid tachyons we generally choose all extra dims space-like $\therefore$ $\boxed{\text{only 1 time dimension}}$

• Thinking about <u>one</u> extra dimension $\rightarrow$

∵ A simple extension of 4D ?

$$ds^2 = \eta_{\mu\nu} dx^\mu dx^\nu - dy^2 \, [dx_5^2]$$

A <u>flat</u>, <u>space-like</u> <u>extra dim</u>

(no complicated metric tensors etc)

5D Klein Gordon Equation : $\underbrace{\partial_A \partial^A}_{\partial_\mu \partial^\mu - \partial_y^2} \Phi(x_\mu, y) = 0$ $\quad \boxed{A = (\mu, y)}$

(real scalar)

∴ do sep. of variables $\quad \Phi \equiv \sum_n \chi_n(y)\, \phi_n(x_\mu)$

$\Rightarrow$ Kaluza-Klein (KK) decomposition

$\rightarrow$

$\rightarrow$ $\quad \sum_n \left(\chi_n \partial_\mu \partial^\mu \phi_n - \phi_n \partial_y^2 \chi_n \right) = 0$

now <u>if</u> $\partial_y^2 \chi_n = -m_n^2 \chi_n$ then

$$\sum_n \chi_n \underbrace{\left(\partial_\mu \partial^\mu + m_n^2 \right) \phi_n}_{0} = 0 \quad \Big\} \text{ a set of indep. equations}$$

∞ set of $\boxed{\text{massive}}$ scalar states : $\boxed{\text{A KK tower}}$

• m_n takes on <u>discrete</u> values $\Leftrightarrow$ extra dimension is <u>COMPACT</u> - of <u>finite</u> size !!

$\rightarrow$ They are <u>small</u> + that's why we don't 'see' them

Action Approach

$$S = \int d^4x \int_{y_1}^{y_2} dy \quad \underbrace{\frac{1}{2}\partial_A \Phi \, \partial^A \Phi}_{\frac{1}{2}\partial_\mu \Phi \, \partial^\mu \Phi \; - \frac{1}{2}\partial_y \Phi \, \partial_y \Phi}$$

let $\Phi \equiv \sum_n \chi_n(y)\, \phi_n(x_\mu)$ Then

$$S = \int d^4x \int_{y_1}^{y_2} dy \left\{ \frac{1}{2} \sum_{nm} \chi_n \chi_m \, \partial_\mu \phi_n \, \partial^\mu \phi_m \right. \quad (1)$$

$$\left. - \frac{1}{2} \sum_{nm} \phi_n \phi_m \, \partial_y \chi_n \, \partial_y \chi_m \right\} \quad (2)$$

To 'Diagonalize'

(1) $\int_{y_1}^{y_2} dy \; \chi_n \chi_m = \delta_{nm}$

(2) integrate-by-parts : $\int_{y_1}^{y_2} dy \; \partial_y \chi_n \, \partial_y \chi_m$

$$= \underbrace{\chi_m \partial_y \chi_n \Big|_{y_1}^{y_2}}_{\overset{!!}{0}} - \int_{y_1}^{y_2} dy \; \chi_m \underbrace{\partial_y^2 \chi_n}_{-m_n^2 \chi_n}$$

Boundary Conditions !

now <u>if</u>

<u>Then</u> $S = \int d^4x \sum_n \left(\frac{1}{2}\partial_\mu \phi_n \partial^\mu \phi_n - \frac{1}{2} m_n^2 \phi_n^2 \right)$

– n <u>massive scalars</u>

$+ \boxed{(\partial_y^2 + m_n^2)\chi_n = 0}$, $\chi_n = A_n e^{i m_n y} + B_n e^{-i m_n y}$

$m_n = ?$

non - compact $\sim e^{i p_x x}$

p_x momentum along an infinite dimension

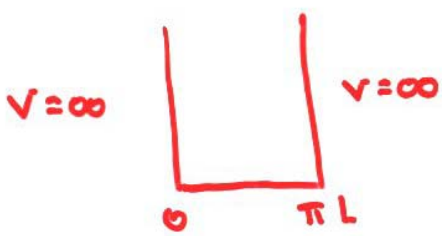
p is continuous

$\boxed{\text{Compact cases}}$ $\leftrightarrow$ quantization of momentum $\leftrightarrow$ mass

$(\partial_y^2 + m_n^2)\chi_n = 0$ $\qquad \chi_n = A_n e^{i m_n y} + B_n e^{-i m_n y}$

general solution

From QM : (1) "particle in a box": BC's ??



$\chi_n(0, \pi L) = 0 \rightarrow C_n \sin ny/L$

and $\boxed{m_n = n/L}$; $n = 1, 2, \ldots$

(2) motion in a circle (or orbital angular momentum)

periodic BC's

$S^1 \equiv$

$\chi_n(y + 2\pi R) = \chi(y)$

$\Downarrow$

MAP to a finite interval

$\chi_n = A_n \cos ny/R + B_n \sin ny/R$

$n = 0, 1, 2 \ldots$

$\boxed{m_n = n/R}$

$\boxed{\text{Observe}}$: K.K masses $\sim \frac{1}{\text{size}}$ of extra dim

$\rightarrow$ a slight variation ...

S^1/Z_2 ORBIFOLD

a manifold w/ discrete symmetr(ies)

Z_2: $y \leftrightarrow -y$

here Z_2 is just parity on the interval...

$$\chi_n = \begin{cases} A_n \cos ny/R \\ B_n \sin ny/R \end{cases} \quad \underline{\text{OR}} \quad (m_n = n/R)$$

.... even vs odd under Z_2

- Higher Dim Field Decomposition :

we saw a 5D scalar → a tower of 4D scalars ....

what about a 5-vector A^M or symmetric tensor h^{MN} ???

- We've 'done' this before .... !!

Lorentz (4D)		Rotations (3D)
scalar	$\longleftrightarrow$	scalar
4-vector A^μ	$\longrightarrow$	$\vec{A}, \phi$
tensor $F^{\mu\nu}$	$\longrightarrow$	$\vec{E}, \vec{B}$

do it again for 5D...

→ 5D scalar → 4D scaler(s) (this we know)

5D vector : $A^M \longrightarrow (A^\mu, A^5)_n$ — tower of vectors + scalars in 4D

Symmetric tensor : $h^{MN} \rightarrow (h^{\mu\nu}, h^{\mu 5}, h^{55})_n$ — tensors + vectors + scalars generalize to 4 + n dimensions

$$A^{L} = (A^{\mu}, A^{i})_{n_i} \quad (i = 1 \dots n)$$

4-vector

4-scalars

$$h^{LP} = (h^{\mu\nu}, h^{\mu i}, h^{ij})_{n_i}$$

n 4-vectors

$\frac{1}{2}n(n+1)$ 4-scalars

Messages: there are many ways to pick BC's and ways to compactify - the number only grows as we add more dimensions ...

Common Choices: S^1, S^2 ... S^n n-sphere in n+1 dim space

(circle) usual sphere

$$\underbrace{S^1 \times S^1}_{T^2} \times \dots S^1 \rightarrow \text{torus or 'doughnut'}$$

KK - Goldstone Mechanism:

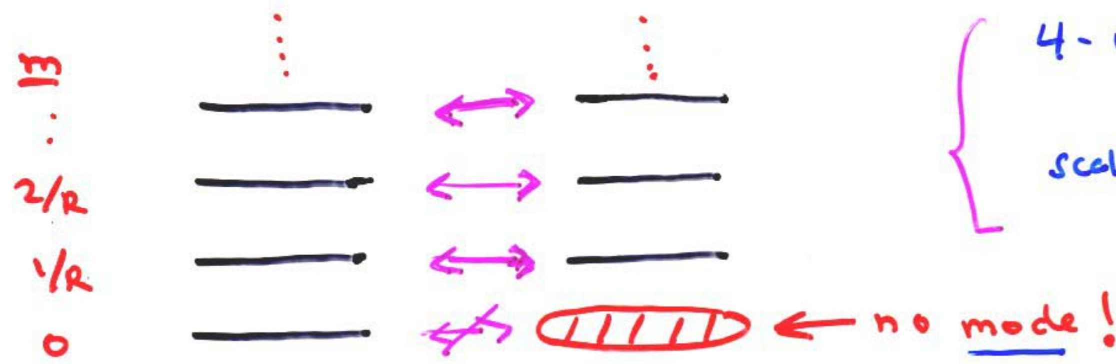
$\therefore$ consider a gauge field in 5D $\to A^N \ldots$

a massless field in 5D has 3 polarizations

$\Updownarrow$

U(1) gauge invariance

$\Rightarrow$ put it on S^1/Z_2:



4-vectors are Z_2-even... scalars are odd

$A^\mu_n \sim \cos ny/R$ $\quad$ $A^5 \sim \sin ny/R$

$\uparrow$ vectors w/ 2 polarizations (massless!) $\quad$ scalars

For $n \geq 1$ the vector eats the corresponding scalar to obtain a mass

$\therefore$ 3 polarisations in 4D

$\therefore$ in Physical (Unitary) gauge......

$\to$ massive tower $\oplus$ 1 massless mode

$\equiv$ zero mode

- The zero mode is massless since $A_5^{(0)}$ does not exist to be eaten $\therefore$ U(1) is maintained

But : it was the **BC's** that killed

$A_5^{(0)} \longleftrightarrow$ link between BC's and breaking of gauge invariance

$\Rightarrow$ { non-ORBifold BC's can be used to break gauge symmetries !!

$\boxed{\text{e.g.,}}$

$y=0$ $\qquad\qquad$ $y=\pi R$

$$\begin{cases} A^\mu = 0 & \partial A^\mu = 0 \\ \partial A_5 = 0 & A_5 = 0 \end{cases}$$

$A^\mu \sim a_n \cos m_n y + b_n \sin m_n y$

$$\rightarrow \begin{cases} a_n = 0 \\ m_n = (n + 1/2)/R \end{cases}$$

$\Rightarrow m_0 = 1/2R \neq 0$!!

$U(1)$ symmetry is broken! $\quad A_5^{(0)}$ exists + is eaten by $A_\mu^{(0)}$

$\therefore$ The physics of extra dimensions is the physics of the KK excitations

$\rightarrow$ Some Models..

Large Extra Dimensions

(Arkani-Hamed, Dimopoulos + Dvali)

- Suppose gravity lives in $(4+n)$ dimensional space while we (ie, the SM) are confined to 4D...

bulk

gravity is everywhere

we're on a 'brane'

Gauss' Law tell us that

$$\boxed{M_{pl}^2 = V_n \, M_*^{n+2}}$$

M_{pl}: Planck scale $\sim G_N^{-1}$

V_n: volume of n compact dims

M_*: n+4 dim. Planck scale

(IF we assume All n extra dims have the SAME Size, e.g, T^n then $V_n = (2\pi R)^n$)

Could M_* be $\sim$ few TeV ? $\rightarrow$ 'no' hierarchy!

$\rightarrow$ gravity strong at M_*...

IS This nuts ? NO !

$F_{grav} \sim \frac{1}{r^2} \quad r \gg R$ $\boxed{\text{but}}$

$\sim \frac{1}{r^{2+n}} \quad r \ll R$!! $\Leftarrow$

How large is R if $M_* \sim$ few TeV ?

(a bit sensitive to compactification)

$\underline{n=1}$ $\boxed{R \approx 10^{13}\ \text{cm}}$ excluded ! Huge - solar system size

$\boxed{R^{-1} \sim 10^{-4}\,\text{eV}}$ $\underline{n=2}$ $R \approx 100\ \mu m \Rightarrow$ $\boxed{\text{Table-top experiments}}$ $^{+}$

$\underline{n=3}$ $R \approx 10^{-9}\ m \Rightarrow$ too small to be seen directly

$\vdots \qquad \vdots$

** * + Adelberger et al $\rightarrow$ $R \lesssim 200\ \mu m$

$\rightarrow$ probing TeV scale M_*

- with a factor of $\approx$ 5-10 improvement $n=2$ case will be highly constrained by direct measurements

However, $n=2$ is already disfavored by other constraints.. what do the graviton KK's do?

Recall $m_{\vec{n}}^2 = \vec{n}^2/R^2 \ldots$

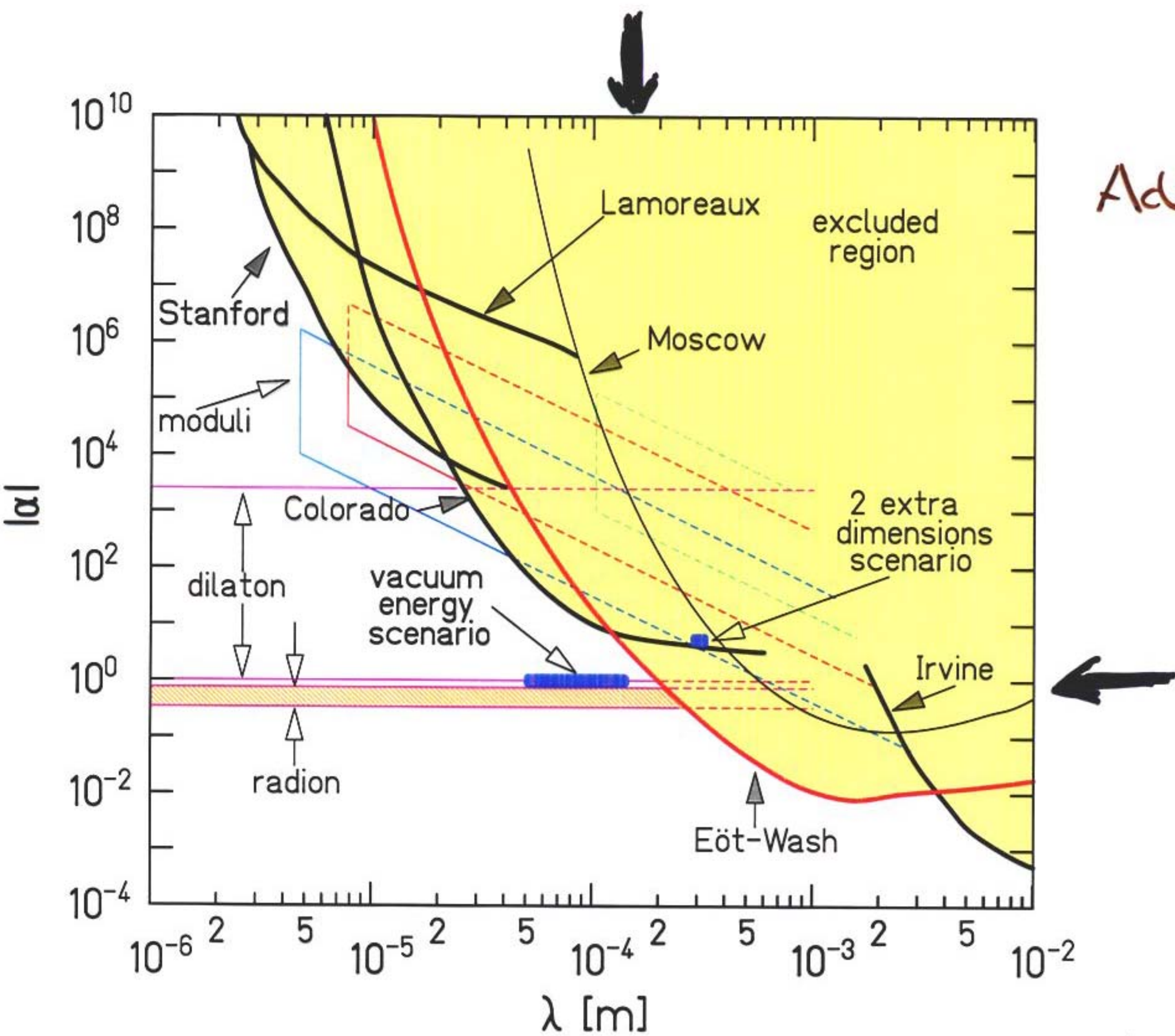
Table-top deviations from Newtonian Gravity

Parameterized as

$$\Delta V = -G_N M m \left\{ \alpha \frac{e^{-\lambda r}}{r} \right\}$$

α - strength

λ - length scale



Adelberger et al

Note: in this scheme the SM is 4D - no KK excitations .. why? $\underline{m_{\gamma_{KK}} = m_{gr_{KK}}}$!

→ this is NOT observed. $\Longrightarrow$

$\boxed{\text{Variations on a Theme}}$ → Recall: $\boxed{M_{Pl}^2 \sim R^n M_*^{n+2}}$

who says all R^n have the SAME size?

e.g.

(e.g) two different sizes

$$M_{Pl}^2 \sim R_1^{n-p} R_2^p M_*^{n+2}$$

if $R_2 \sim 1/M_*$ this LOOKS like

$$\Longrightarrow \boxed{M_{Pl}^2 \sim R_1^{(n-p)} M_*^{(n-p)+2}}$$

$(n-p)$ effective dimensions

For $M_* \sim$ TeV and $R_2 \sim 1/M_*$, the SM can live in the p TeV size dims

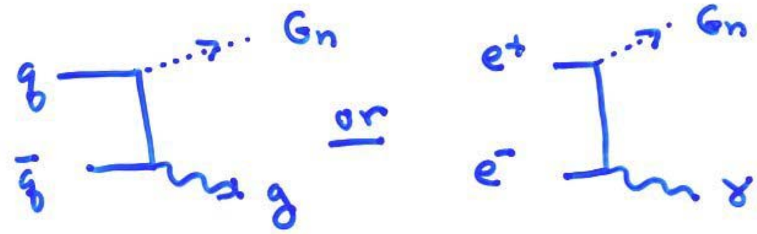
$$\Longrightarrow \boxed{m_{KK}^{SM} \sim \text{TeV}} !$$

OK phenomenologically!

⇒ Whole model building effort! SM fields in $\boxed{\text{TeV}^{-1} \text{ size}}$ extra dims (later)

ADD graviton KK's couple to the SM just like the zero mode - the conventional graviton - 2 possible signatures

emission



- each G_n is very weakly coupled BUT there are a Lot of them in the KK tower .... for fixed $\sqrt{s}$ all states up to $m_n = \sqrt{s}$ can be emitted, e.g.,

$\Rightarrow$ e.g. for $n = 2$ $\quad \Delta m \sim 10^{-4}$ eV $\quad \sqrt{s} = 200$ GeV $= 2 \cdot 10^{11}$ eV

$\therefore \sim 10^{16}$ states ! $\Rightarrow$ KK tower sum

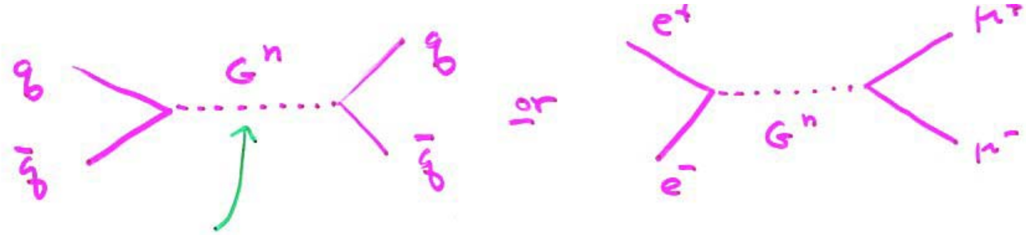
- Each state produces the same signature - $\boxed{\text{missing } E} \oplus \gamma$ or jet - incoherent sum

$$\sigma = \sum_n \hat{\sigma}(m_n) \sim \int^{\Lambda} dM\, M^{n-1}\, \hat{\sigma}(M) \,, \quad \Lambda = \sqrt{s}$$

$\rightarrow$ the integral is cutoff by the kinematics

- These can be seen + have been looked for at LEP2 + the Tevatron $\xrightarrow[\text{future}]{}$ LHC / LC (Landsberg)

Exchange



unique { spin-2 KK tower exchange

$$\sigma \sim \int^{\Lambda} M^{n-1}\, dM \; \hat{\sigma}(M)$$

But there is no upper limit here – all states contribute ... in fact, $\hat{\sigma}(M) = \dfrac{A}{s - M^2}$

so $$\sigma \underset{\Lambda\to\infty}{\sim} \int^{\Lambda} dM \; \frac{M^{n-1}}{M^2} A$$ converges for $n=1$

BUT

diverges for $n \geq 2$! ⇒ expect $\Lambda \sim M_*$

→ graviton exchange induces effective dim-8 operators

$$\Rightarrow \quad \Theta = \frac{4\lambda}{\Lambda_H^4} \, T^{(1)}_{\mu\nu} \, T^{\mu\nu}_{(2)}$$

SM stress-energy tensors of the initial and final SM fields

$\boxed{\text{Unique}}$ signatures at colliders
– unlike other contact interactions

The signal is the little guy on top...

Vacavant + Hinchliffe

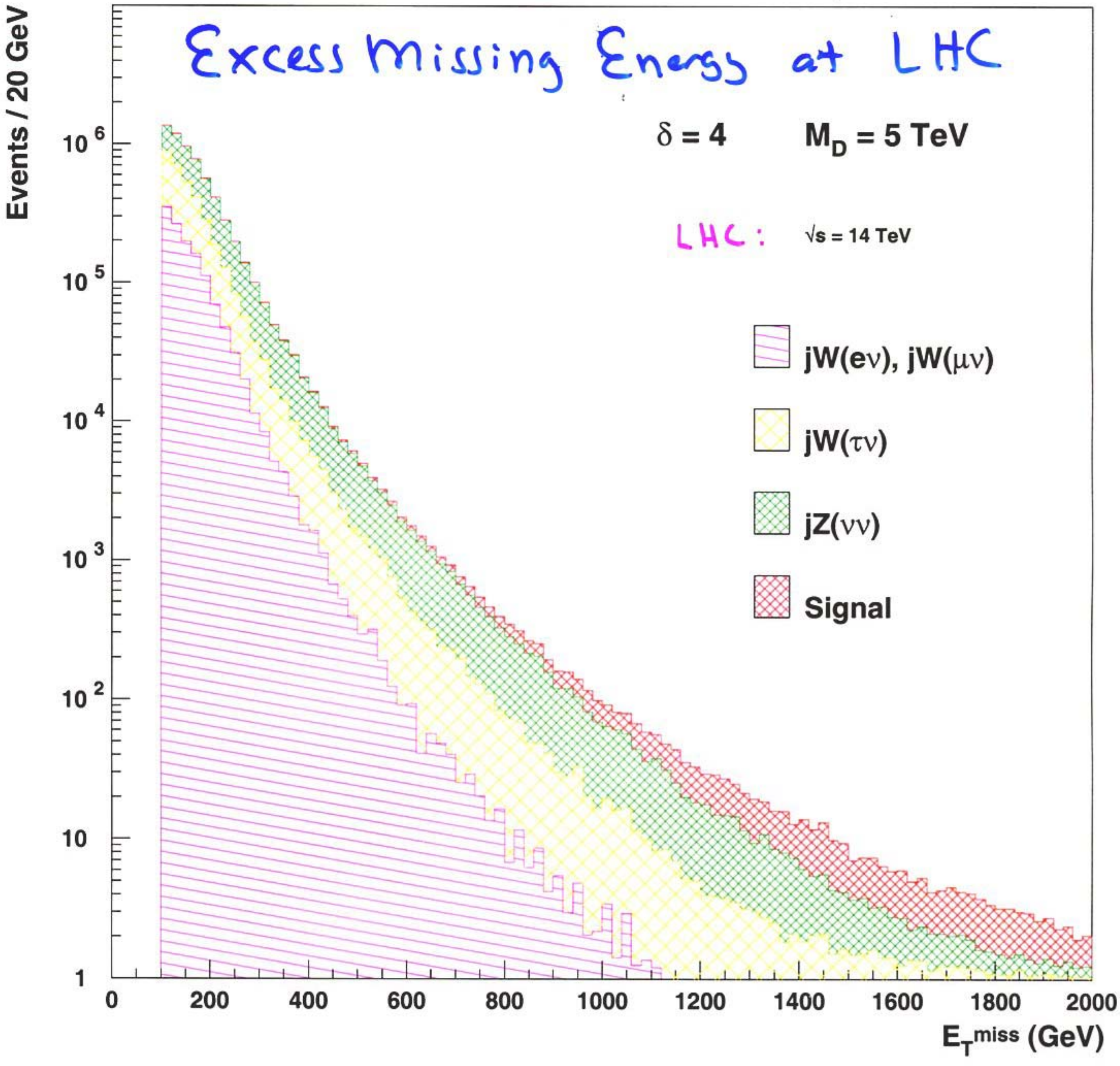
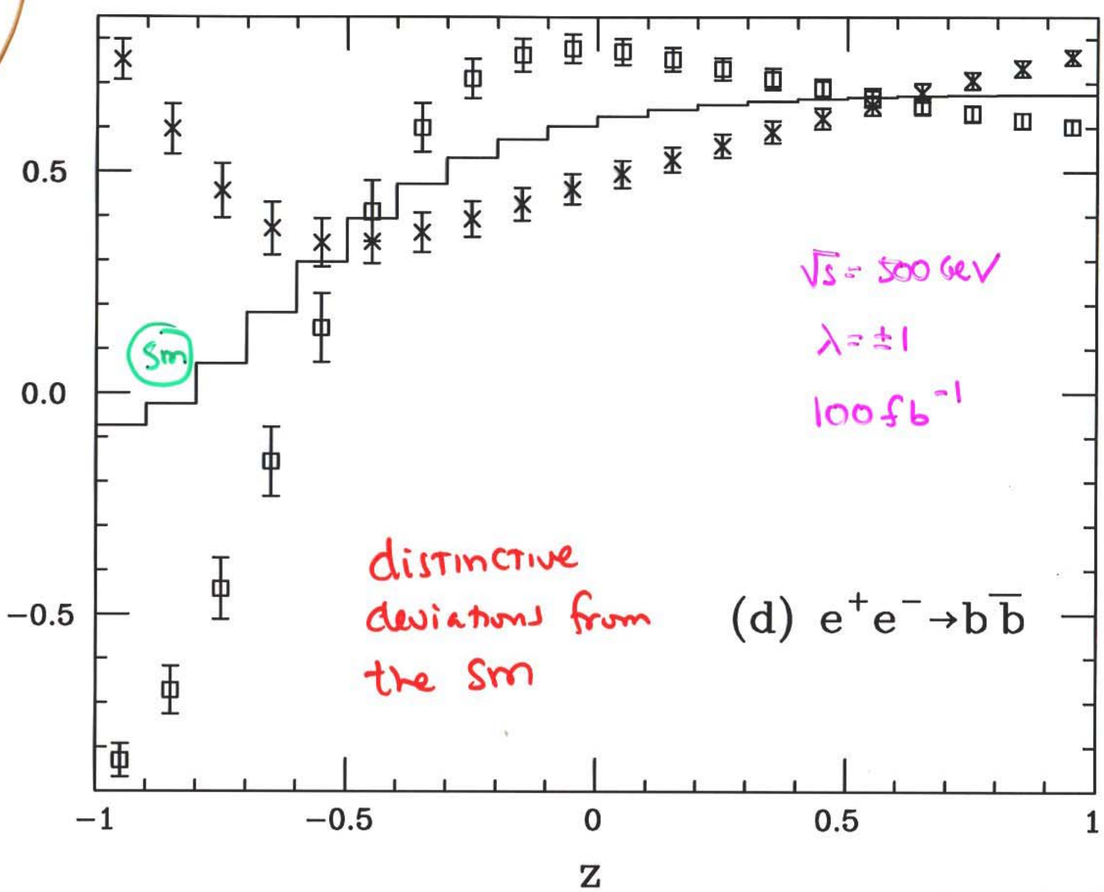


Figure 1: Missing energy spectrum at the LHC.

ADD at the Linear Collider

graviton KK exchange

Hewett



LR Polarization Asymmetry

$\cos\theta$

$\Lambda_H = 3\sqrt{s}$

Warped Extra Dimensions -
Randall - Sundrum Model

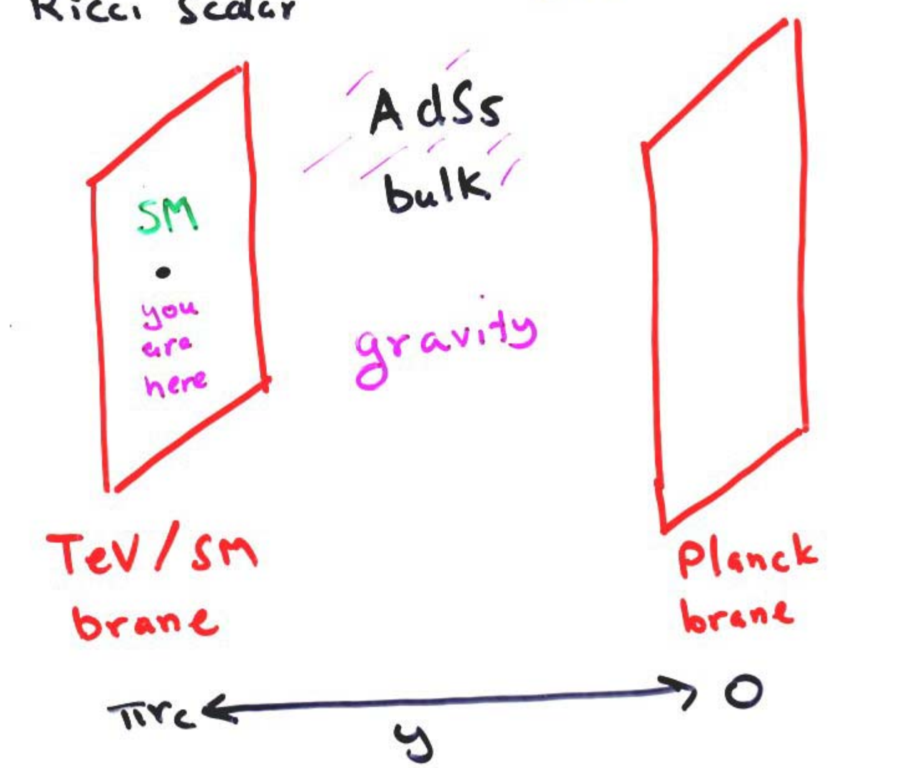
- One extra 'non-factorizable' dimension

$$ds^2 = \underbrace{e^{-2ky}}_{\text{Warp factor!}} \underbrace{\eta_{\mu\nu} dx^\mu dx^\nu}_{\text{usual Poincare invariant}} - dy^2$$

$\boxed{k \sim M_{pl}}$

$\Rightarrow$ non-trivial metric tensor - space is $\boxed{\text{curved}}$ - $\boxed{R_5 = -20k^2}$ $\longleftrightarrow$ constant negative curvature

5D Ricci scalar $\Longrightarrow$ Anti-de Sitter Space



- Idea: all mass parameters are $\sim M_{pl} \Leftrightarrow$ no tuning

- The warp factor rescales them appropriately.

... In fact, ...

How does 'warping' work?

- imagine the Higgs field on the TeV brane....

$$S = \int d^4x\, dy\, \sqrt{-g}\, \left\{ g^{\mu\nu} \partial_\mu \hat{H}^\dagger \partial_\nu \hat{H} - \lambda \left(\hat{H}^2 - v_0^2 \right)^2 \right\} \delta(y - \pi r_c)$$

$\left\{ \left[e^{-2ky} \right]^4 \right\}^{1/2}$ $\quad$ $e^{2ky}\, \delta^{\mu\nu}$

Higgs vev $\sim M_{pl}$

$0 \le y \le \pi r_c$

$$S = \int d^4x \left\{ e^{-2kr_c\pi} \partial_\mu \hat{H}^\dagger \partial^\mu \hat{H} - e^{-4kr_c\pi} \lambda \left(\hat{H}^2 - v_0^2 \right)^2 \right\}$$

now rescale $\hat{H} \to e^{kr_c\pi} H$

$$S = \int d^4x \left\{ \partial_\mu H^\dagger \partial^\mu H - \lambda \left(H^2 - \underbrace{v_0^2\, e^{-2kr_c\pi}}_{v} \right)^2 \right\}$$

"Canonically" normalized!

v is TeV scale now

The Higgs on the TeV brane gets a TeV scale vev ... even though we started at $\sim M_{pl}$!

- Warping modifies all energy scales.

- Any Lagrangian mass parameter, $\boxed{m}$, which is naturally $\sim M_{pl}$ will appear to be $\boxed{\sim \text{TeV}}$ on the TeV scale – $\boxed{\text{to US}}$

ie., on the TeV m will appear as $m e^{-\pi k r_c}$

If $\boxed{k r_c \sim 10} \Rightarrow m e^{-k\pi r_c} \sim \text{TeV}$ not $\sim M_{pl}$

a tiny hierarchy

The ratio of M_{pl}/M_{weak} is explained as an exponential of a number of $O(10)$!

- Goldberger + Wise
  $\hookrightarrow$ $k r_c \sim 10$ is stable + natural !

What are KK's like in this model ? return to Klein-Gordon

$$(\sqrt{-g})^{-1}\,\partial_A \left\{ \sqrt{-g}\, g^{AB}\, \partial_B \Phi \right\} = 0 \qquad [\text{non-trivial metric!}]$$

$$\hookrightarrow -e^{2ky}\,\partial_y \left\{ e^{-2ky}\,\partial_y\, x_n \right\} = m_n^2\, x_n$$

$\rightarrow$ Bessel functions not sines + cosines !

$$\Rightarrow \boxed{m_n = x_n \underbrace{k e^{-\pi k r_c}}_{\sim \text{TeV}}}\ , \quad \boxed{J_1(x_n) = 0}$$

$x_n = 0,\ 3.83, \ldots$

KK's are ~ TeV with ~ TeV spacing!

(not closely packed as in ADD ...)

$\boxed{\text{Same}}$ result for gravitons! But also ...

$$\mathcal{L} = -\left\{ \frac{G^{(0)}_{\mu\nu}}{\overline{M}_{Pl}} + \sum_n \frac{G^{(n)}_{\mu\nu}}{\Lambda_\pi} \right\} T^{\mu\nu}$$

ordinary GR

$\boxed{\overline{M}_{Pl}\, e^{-k\pi r_c}}$ ~ TeV!

→ Weak scale graviton KK's w/ weak scale couplings ⟹ resonances at colliders!

...... $\boxed{\text{spin - 2}}$ resonances

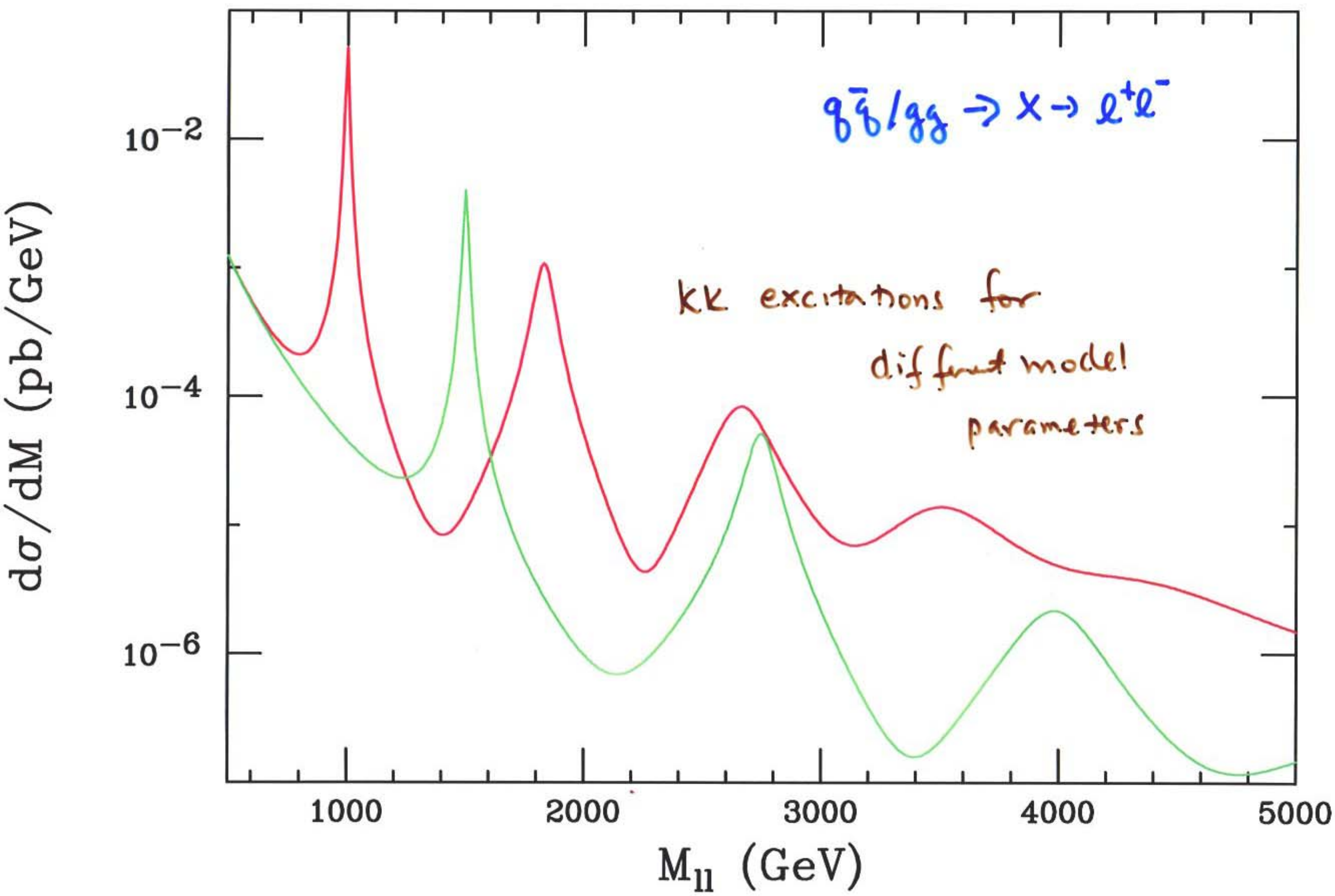
- And you will see them in many, many processes

⟹ Determine that these ARE gravitons ...

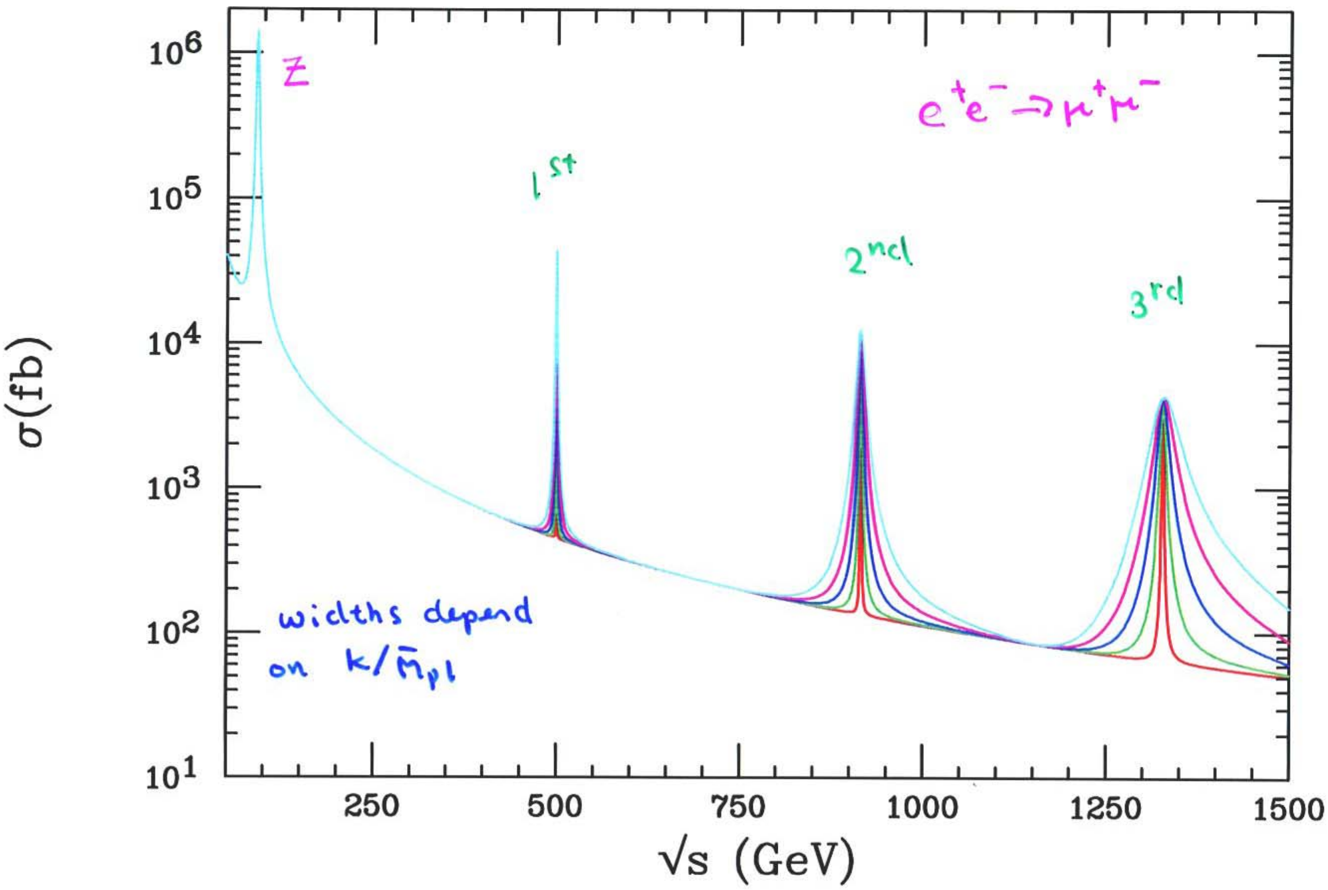
- Spin -2 & distributions
- branching fractions

both easy at e^+e^- colliders

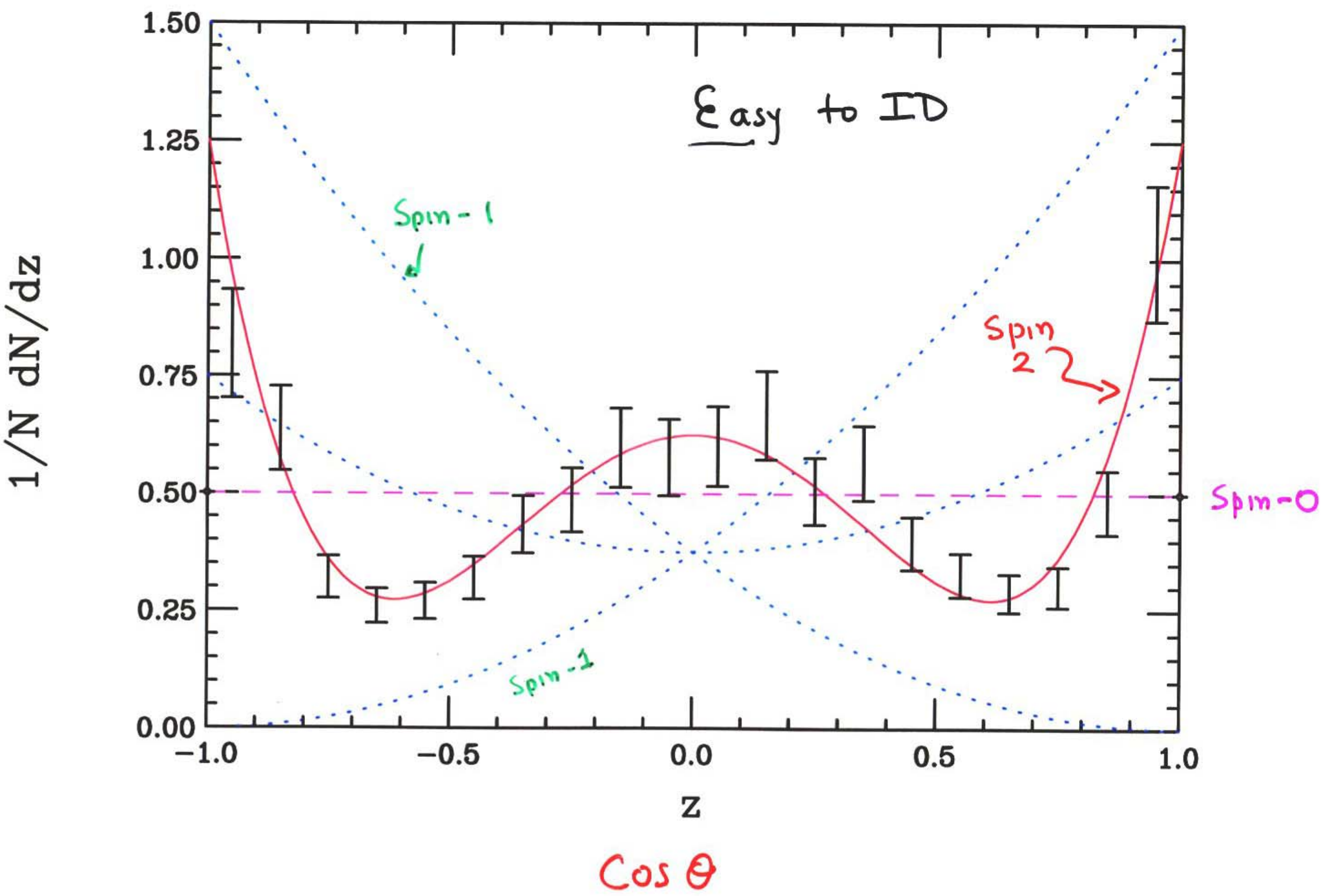
graviton resonance production at LHC



RS graviton KK resonances at the Linear Collider

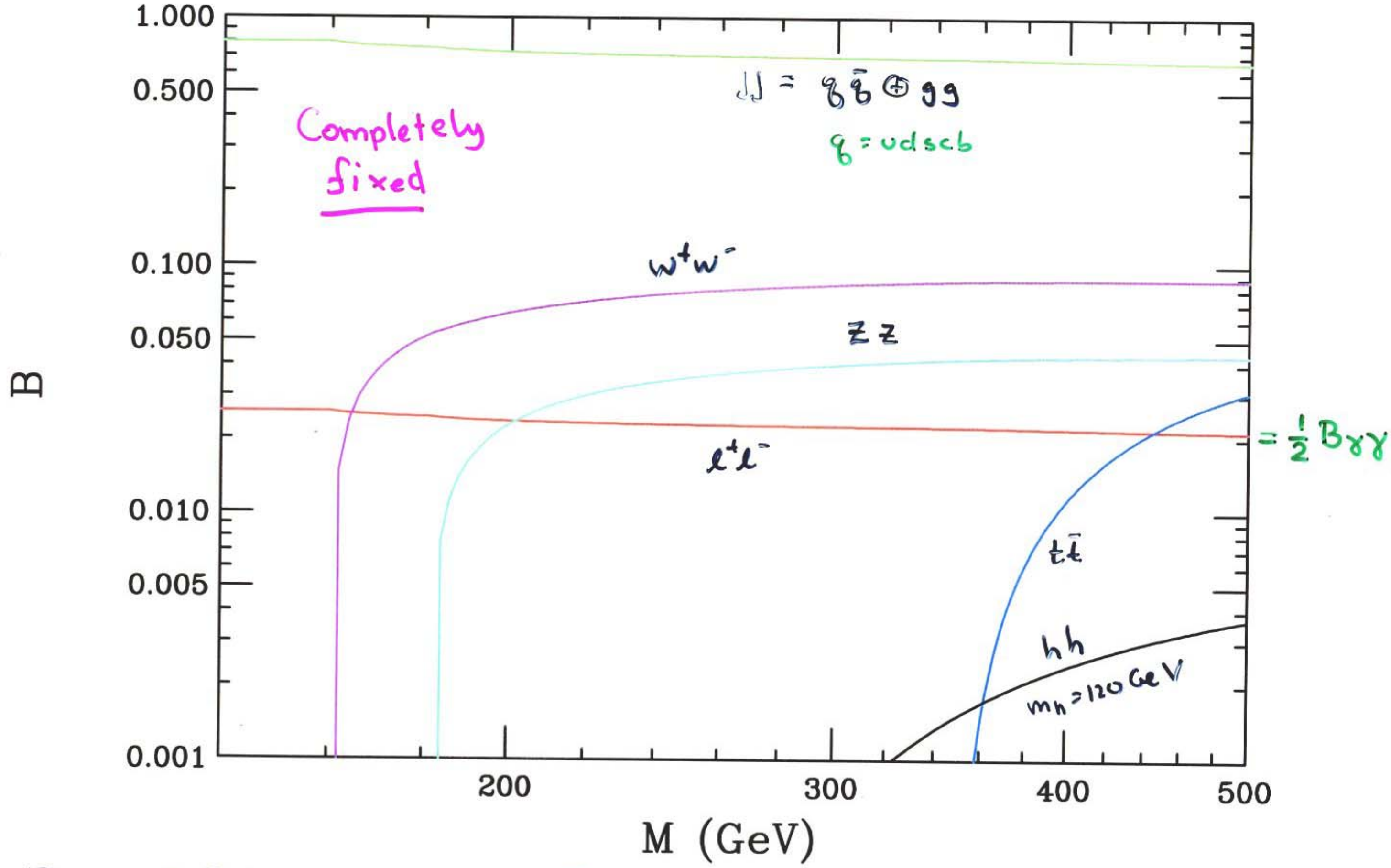


$f\bar{f} \to \bar{f}'f'$ on a graviton resonance



SM on wall

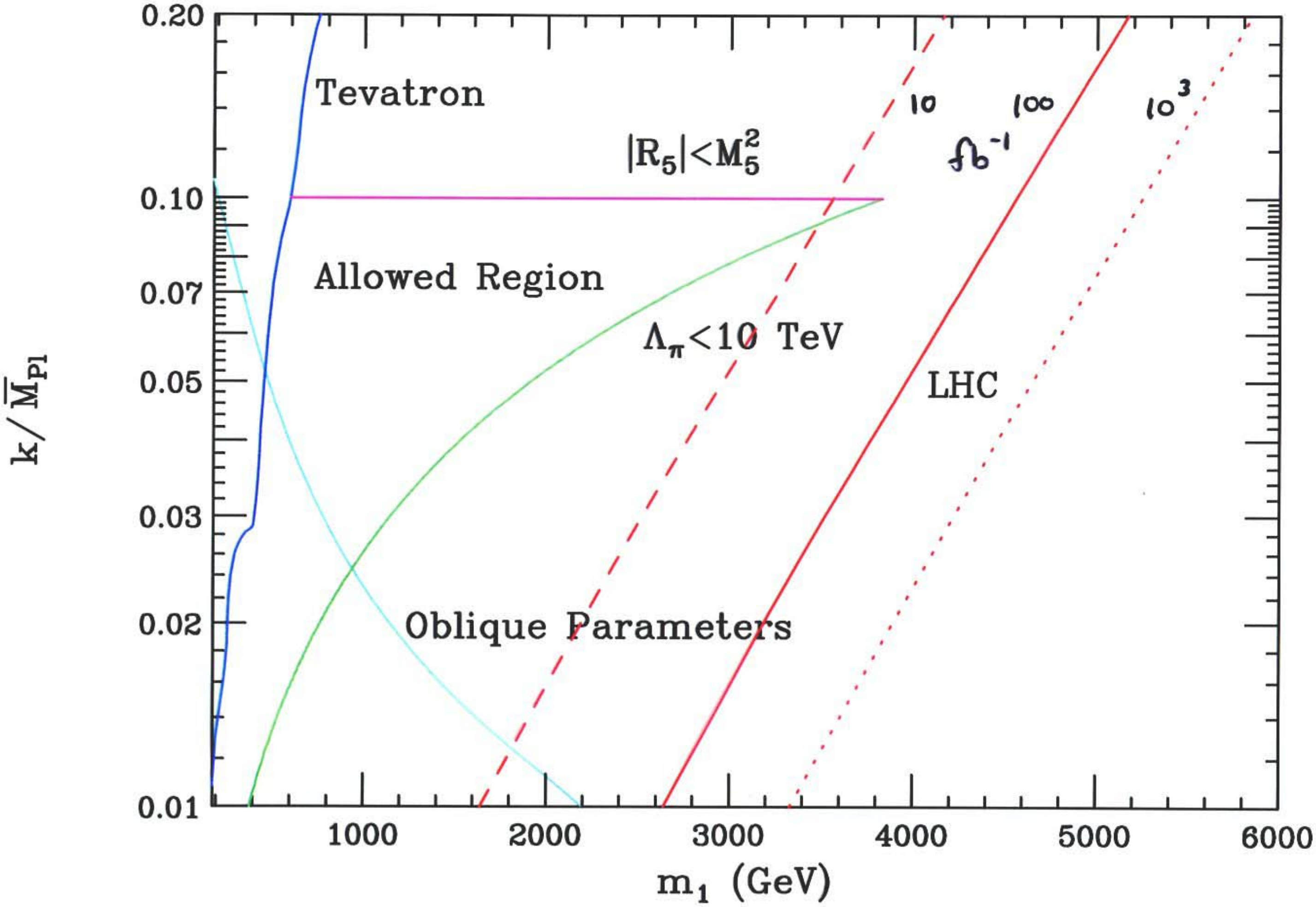
RS Graviton branching fractions



$\rightarrow B_{\gamma\gamma} = 2 B_{\ell^+\ell^-}$

1st graviton mass

The LHC has good RS coverage



Universal Extra Dimensions

Appelquist
Cheng +
Dobrescu

→ Unique phenomenlogy!!

Idea: ① Put all SM fields in the bulk

② Assume one flat Extra Dim on S^1/Z_2 with $R^{-1} \sim \text{TeV}$

⇒ All SM fields will have KK excitations

$$\boxed{m_{KK}^2 = m_{SM}^2 + n^2/R^2}$$

Note that even if $R^{-1} \sim \frac{1}{2}$ TeV there will be a lot of degeneracy in the KK spectrum

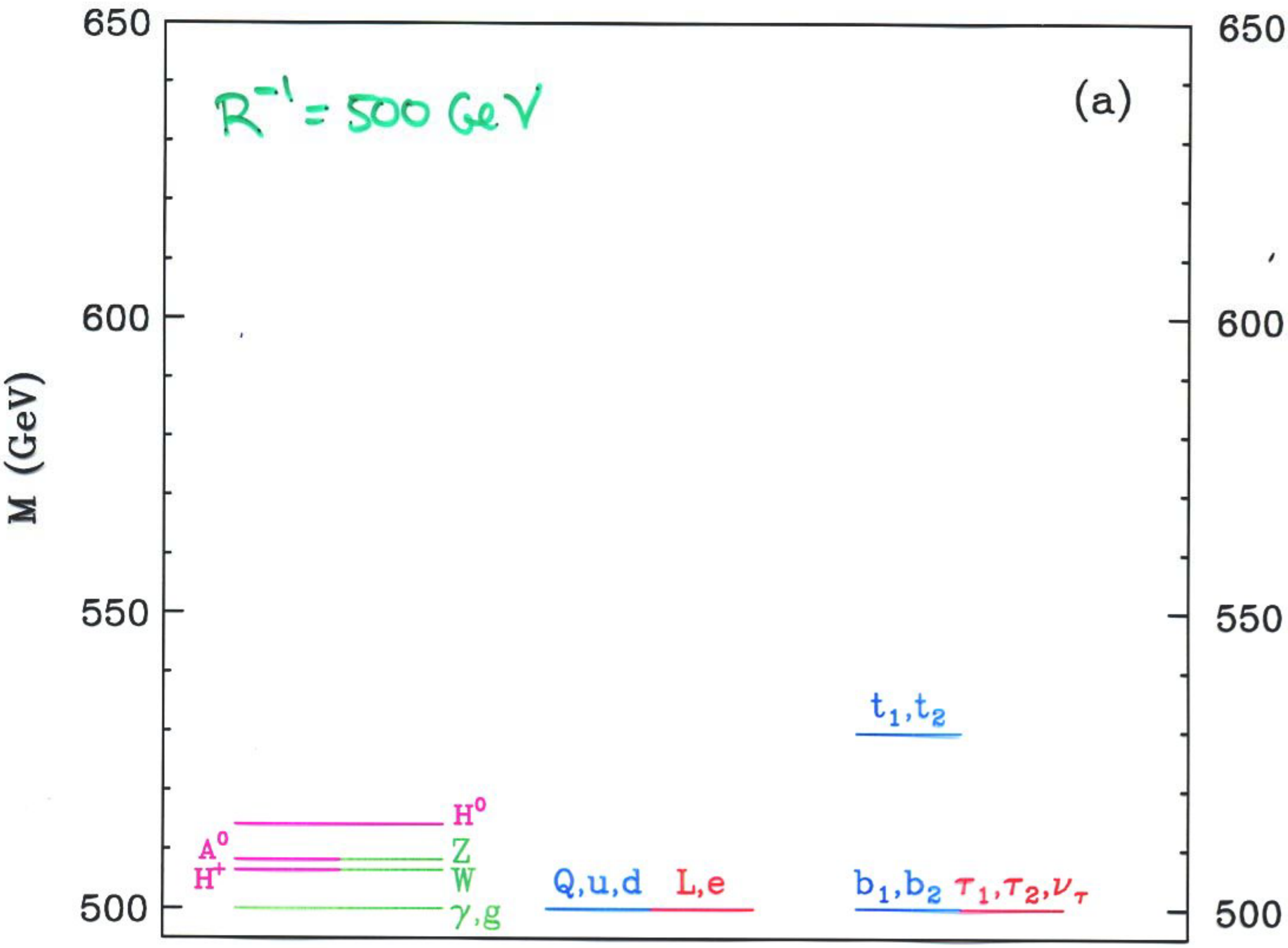
$(m_{SM} R)^2 \ll 1$, generally ...

⇒ details of mass spectrum are very important for understanding phenomenology

e.g. ⇒ $\Gamma(e_1 \to e_0 + \gamma_1)$ is very sensitive to detailed masses

⇒ Radiative Corrections Important!

Cheng, Matchev + Schmaltz



$\rightarrow$ a highly degenerate spectrum

$L = \begin{pmatrix} \nu \\ e \end{pmatrix}$ Dirac field etc

$t_{1,2} = \{t_L, t_R\}$ linear combination of Dirac fields

Fast + Dirty Fermions in One Flat E.D.

$$S = \int d^4x \int dx_5 \;\; i\, \bar{\Psi}\, \gamma^A \partial_A \Psi \qquad \boxed{\gamma^A \equiv \{\gamma^\mu, i\gamma_5\}}$$

$$\boxed{\Psi = P_L \Psi_L + P_R \Psi_R} \qquad \text{expand}$$

$$S = \int d^4x \int dx_5 \Big\{ i\, \bar{\Psi}_L \gamma^\mu \partial_\mu P_L \Psi_L + (L \to R)$$

$$- \bar{\Psi}_L P_R \partial_5 \Psi_R + \bar{\Psi}_R P_L \partial_5 \Psi_L \Big\}$$

let $\quad \Psi_{L,R} = \sum_n f^{(n)}_{L,R}(x_5)\, \psi^{(n)}_{L,R}(x_\mu) \quad$ with

$$\int dx_5 \; f^{(n)}_{L,R}\, f^{(m)}_{L,R} = \delta_{nm} \qquad \text{as usual}$$

→ insert and integrate over $\int dx_5$ ...

$$\rightarrow \sum_n \Big\{ i\, \bar{\psi}^{(n)}_L \gamma^\mu \partial_\mu \psi^{(n)}_L + (L \to R) \Big\}$$

2 separate KK towers

$$+ \sum_{nm} \int dx_5 \Big\{ - \bar{\psi}^{(n)}_L \psi^{(m)}_R\, f^{(n)}_L \partial_5 f^{(m)}_R$$

$$+ \bar{\psi}^{(m)}_R \psi^{(n)}_L\, f^{(m)}_R \partial_5 f^{(n)}_L \Big\}$$

? compactification ?

Choose S^1/Z_2 orbifold ... why?

- Terms in the action MUST be Z_2-even or they vanish $\therefore$

$$f_L \partial_5 f_R \quad \text{or} \quad f_R \partial_5 f_L \quad \text{is even}$$

$$\therefore \quad f_L \cdot f_R \text{ is odd} \quad \Rightarrow \begin{cases} f_L \text{ even} & f_R \text{ odd} \\ f_L \text{ odd} & f_R \text{ even} \end{cases}$$

$\Rightarrow$ {Only 1 of the f_i can have a zero mode

The choice is up to you! Let's pick f_R...

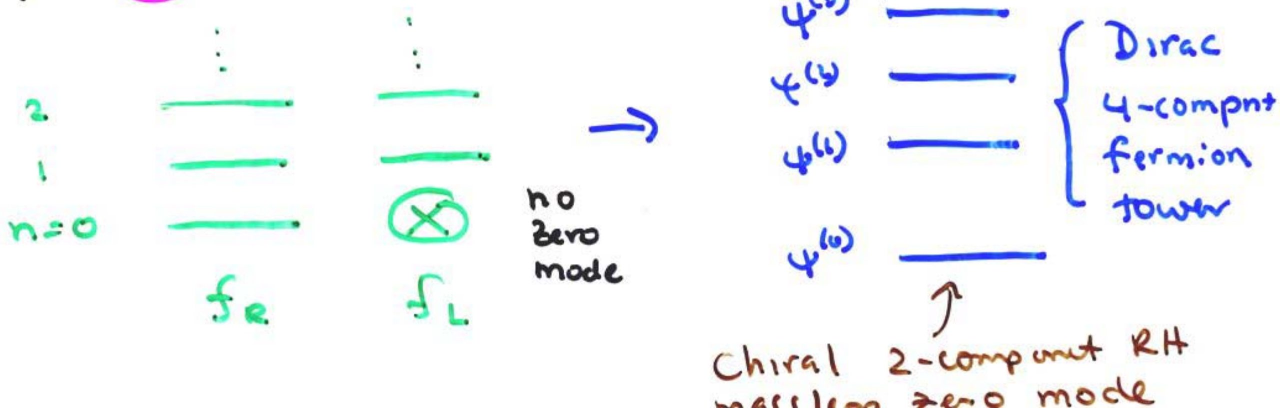
We then observe that if:

$$\begin{cases} \partial_5 f_R^{(n)} = m_n f_L^{(n)} \\ \partial_5 f_L^{(n)} = -m_n f_R^{(n)} \end{cases} \xrightarrow{BC's} \boxed{m_n^2 = n^2/R^2}$$

and then

$$S = \sum_n \int d^4x \left\{ i\, \bar{\psi}_L^{(n)} \gamma^\mu \partial_\mu \psi_L^{(n)} - m_n \bar{\psi}_L^{(n)} \psi_R^{(n)} + (L \to R) \right\}$$

$\rightarrow$ (2) "massive" chiral 2-component fermion towers!



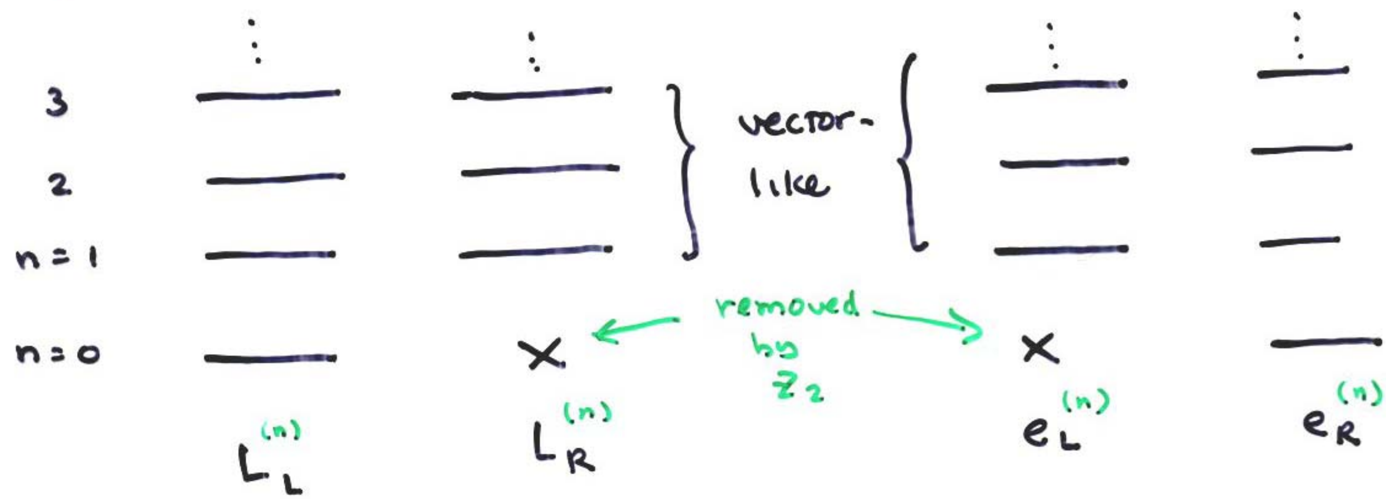
$\therefore$ Orbifolding yields a chiral zero mode plus a Dirac KK tower for fermions

f_L vs f_R is a BC choice

$\boxed{\text{Implications}}$: SM-type fermions are $\boxed{\text{chiral}}$

eg, $L_L = \begin{pmatrix} \nu \\ e \end{pmatrix}_L$ + e_R : $\longrightarrow$ 1 extra dim, S^1/Z_2

$\Rightarrow$ which leads to a KK structure

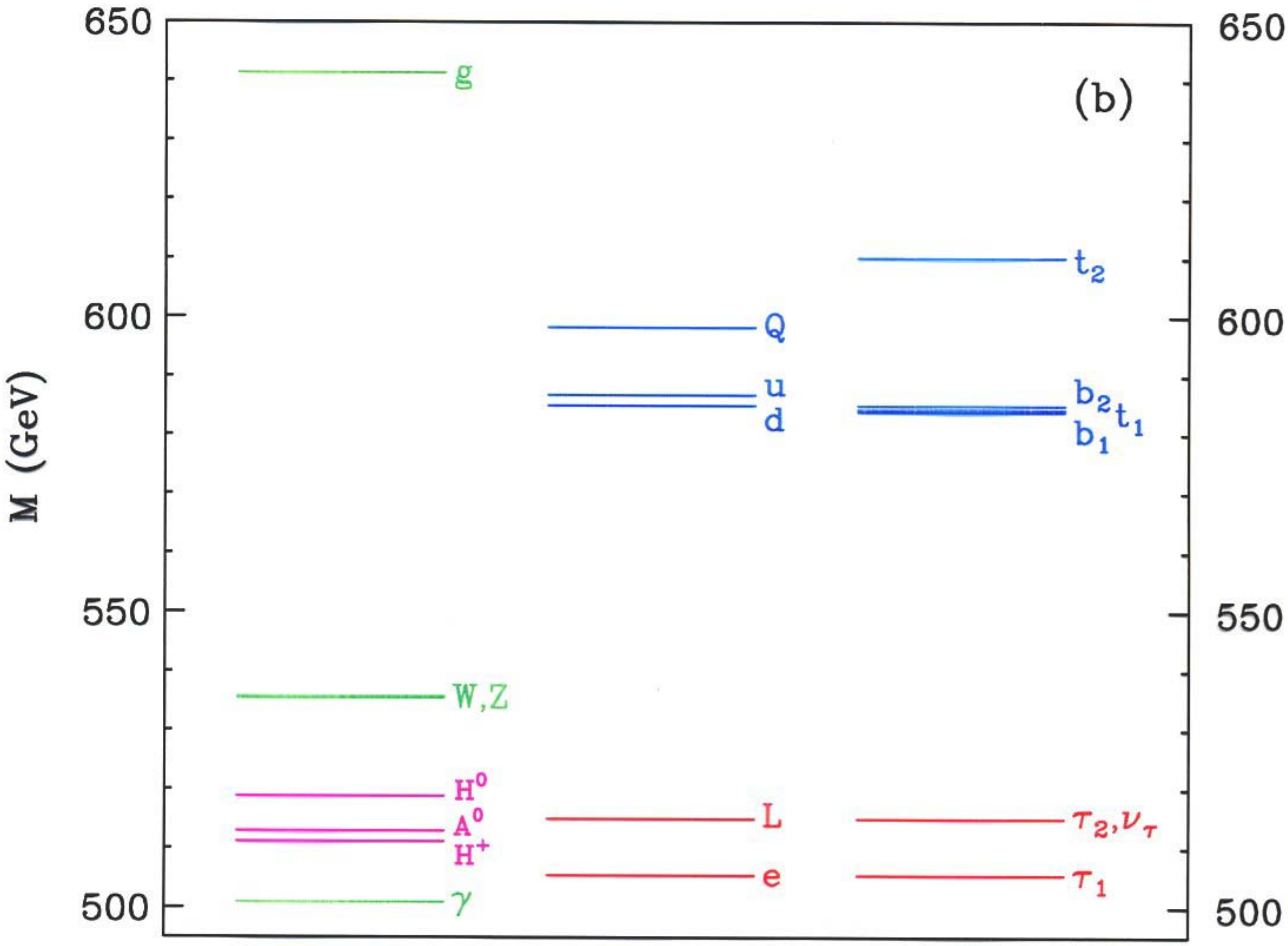


- $\boxed{L_L^{(0)}, e_R^{(0)}}$ = usual SM fermions + chiral
- $L^{(i)}$ + $e^{(i)}$ are different VECTOR-LIKE FERMIONS i.e., not-chiral

$\rightarrow$ Rich KK structure

After Radiative Corrections:

Cheng Matchev Schmaltz



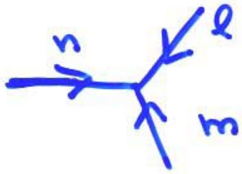
Splittings become apparent...

Looks similar to a SUSY spectrum.

 etc

$\rightarrow$ moves the spectrum around quite a bit **AND** generates new interactions at the orbifold fixed points

$\boxed{\text{KK parity}}$: Consider the interaction of 3 KK states



on S^1 [not S^1/Z_2]

Int $\sim \int dy \; e^{i(n+m+l)y/R} \sim \delta(n+m+l)$

$\Rightarrow$ P_5 is conserved $\Leftrightarrow$ KK number conserved

However : on S^1/Z_2 w/ Fixed point int s , this law is violated + only $\boxed{\text{KK parity } (-1)^n}$ is conserved. orbifolding breaks 5D translat. invariance

$\Rightarrow$ $\left\{ \begin{array}{l} \text{lightest KK state with negative KK parity} \\ \text{is STABLE, ie, } \gamma_1 \text{ is the } \boxed{\text{LKP}} \end{array} \right.$

The LKP is a DM candidate !!

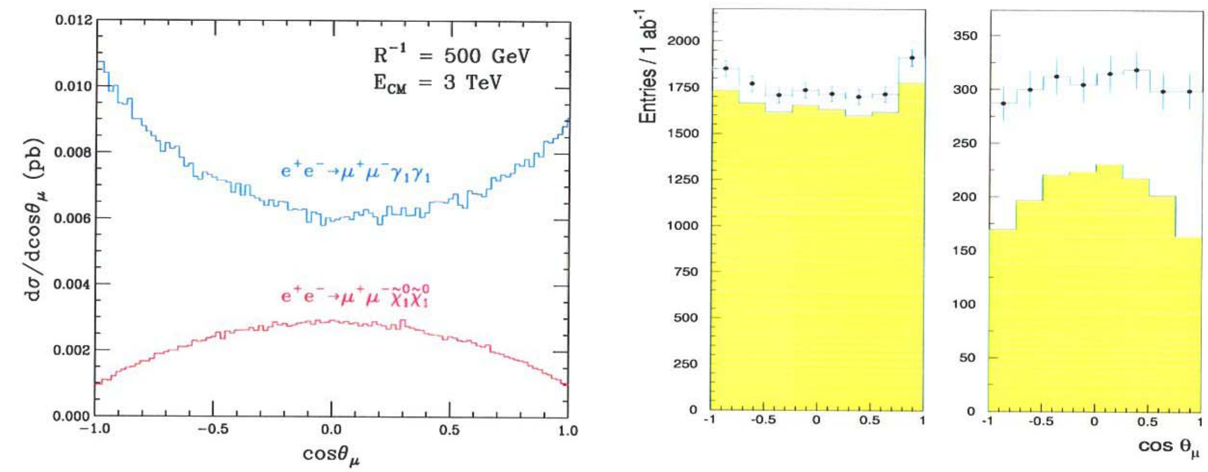
The $n=1$ level of UED looks a lot like SUSY (state for state) w/ KK parity $\Leftrightarrow$ R-parity and LSP $\Leftrightarrow$ LKP ! • but the spins are different ...

- Can we distinguish SUSY /UED at Colliders ?
  - Difficult at LHC ... see next KK level ?
  - measuse spins at the LC !

"Bosonic Supersymmetry"

Battaglia etal

$$\left(\frac{d\sigma}{d\cos\theta}\right)_{UED} \sim 1 + \cos^2\theta \qquad \left(\frac{d\sigma}{d\cos\theta}\right)_{SUSY} \sim 1 - \cos^2\theta$$



By performing a χ^2 fit to the normalised polar angle distribution, the UED scenario considered here could be distinguished from the MSSM, on the sole basis of the distribution shape, with 350 fb^{-1} of data at $\sqrt{s} = 3$ TeV.

Summary

- The subject of Extra Dimensions is a huge area of research + we have barely scratched the surface here – immense variety of models

- E.D. can lead to a wide range of new phenomena (DM, collider signatures, ...) that will be sought over the next decade...

- Expect many new ideas in the future

- Only Experiment will tell us if ED's really exist !