Extremely multi-quark system: Color superconductivity*

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Theoretical studies of color superconducting phases in cold dense quark matter are reviewed. After an introduction to the bulk properties of color superconducting phases at high density, we give a review on the color superconductivity at moderate density paying a special attention to the role of the strange quark mass and the spatial structure of Cooper pairs. We also survey some of attempts to deal with the diquark condensations in the finite systems. These might have a relevance to the description of possible strangelets as well as the deeply bound kaonic nuclei reported recently.

I. COLOR SUPERCONDUCTIVITIES

There has been great developments in our understanding of the dynamics of QCD at the high baryon chemical potential for last few years [1]. We now believe some kind of so called color superconducting phases sets in as the ground state of QCD at some critical chemical potential. The purpose of my talk is to present the essence of these color superconducting phases paying an attention to the analogy with cold He^3 pairing system, and to survey the recent challenges to study the quark pairing in relatively lower densities, and in finite size systems.

In the high density quark matter, we naively expect a large Fermi sphere, where the most of the degrees of freedom are frozen in the deep Fermi sea by Pauli blocking. Only the surface degrees of freedom are weakly perturbed by the small quark-gluon coupling constant [2]. Although the coupling is weak, the degrees of freedom near the Fermi surface is highly degenerated, which leads to a nonperturbative infrared (IR) dynamics. In particular, in the case that the attractive force works between two quarks, an IR instability occurs in the on-shell 2-body T-matrix element: Let $T(\vec{p}, \vec{k}; E)$ be a T-matrix, in which the incoming momenta $(\vec{p}, -\vec{p})$ and the outgoing momenta $(\vec{k}, -\vec{k})$ are both located on the Fermi surface and E is an energy scale below which the quantum correction cannot be neglected. The quantum correction to this T-matrix glows as one integrates out higher energy modes towards Fermi surface, and we finally encounter a Landau pole at some point $E = E_{\rm LP}$, where T diverges logarithmically. This signals a breakdown of Fermi liquid, and is known as the Cooper instability [3]. A theory to overcome this instability is presented by Bardeen, Cooper and Schriefer [4, 5], and is now called the BCS theory. According to it, the instability is cured by the rearrangement of the Fermi sphere into the state with the condensed coherent pairs, and above which, the new quasi-fermion acquires an energy gap with almost the scale of $E_{\rm LP}$. The situation is the same in the case of the dense quark matter, though the primary attraction is provided directly by gluon-exchange between quarks in the color anti-triplet channel, while in metals the attraction between the electrons is supplied via atomic lattice vibration characterized by the Debye frequency ω_c. Historically, according to this observation, Bailin and Love first gave an extensive analysis on the color superconductivity in the early 1980's [6]. They took the simplest pairing ansatz, and used one gluon exchange interaction with the IR singularity in the magnetic sector tamed by a phenomenological magnetic mass.

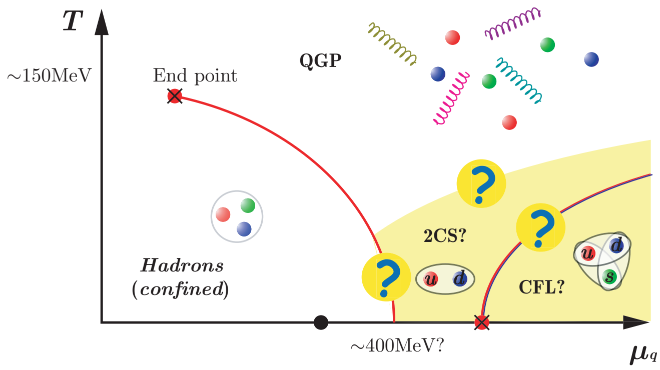

FIG. 1: A cartoon QCD phase diagram.

Unlike electrons in metal, quarks have several intrinsic quantum charges, say, the color and the flavor in addition to the spin. Accordingly, the several pairing patterns are proposed as the candidates of the QCD ground state at finite μ. In particular, the color-flavor locked (CFL) phase [7] and the two flavor pairing phase (2SC) [6, 8] have been studied extensively so far. Before describing the detail of these phases, we first show a schematic QCD phase diagram in Fig. 1. We expect that this simple phase diagram extracts the essential aspects of QCD phase diagram, although we now know it becomes more complex when the charge neutrality constraints under the $\beta-$equilibrium are imposed [9–14] and/or the possible meson condensation is taken into account [15]. In particular, we now believe that the ground state at the extremely high density is the CFL phase, and also expect a window for a

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realization of the 2SC pairing in relatively low densities.

a. The CFL pairing: The CFL state is characterized by the following form of the quark condensate [7]

$$\langle q_i^a(t,\boldsymbol{p}) q_j^b(t,-\boldsymbol{p})\rangle_{\rm CFL} \cong \sum_{L=1}^{3} \epsilon^{abL}\epsilon_{Lij}\Delta_{\rm CFL}(\boldsymbol{p}). \quad (1)$$

Here a,b and i,j are the color and flavor indices, respectively, We have restricted ourselves to the color and flavor structure of the condensate assuming the Dirac structure simply in the $J^P = 0^+$ and aligned chirality channel. This 9×9 matrix has one to one correspondence to the diagonal $\bar{\mathbf{3}}_c \times \bar{\mathbf{3}}_f$ matrix $\Delta_{\rm CFL}\delta_{\bar{a}\bar{i}}$. In this phase, all quarks in the color and flavor space equally participate in the pairing.

b. The 2SC pairing: The 2SC state is defined by [6]

$$\langle q_i^a(t,\boldsymbol{p}) q_j^b(t,-\boldsymbol{p})\rangle_{\rm 2SC} = \epsilon^{ab\boldsymbol{g}}\epsilon_{ij\boldsymbol{s}}\Delta_{\rm 2SC}(\boldsymbol{p}). \quad (2)$$

$\boldsymbol{s}$ and $\boldsymbol{g}$ represent the strange and green quark, respectively. This matrix can be expressed as $\Delta_{\rm 2SC}\delta_{\bar{a}\bar{\boldsymbol{g}}}\delta_{\bar{i}\bar{\boldsymbol{s}}}$ with $\bar{a}$ and $\bar{i}$ being the color and flavor anti-triplet indices. In this pairing ansatz, we have five quarks with g or s charges remain gapless on the Fermi surface.

c. Analogy with He^3 system: The color-flavor locked pairing looks somewhat peculiar because the independent quanta, the color and flavor are locked each other via Kronecker's δ. However, we can find a CFL analogue in the He^3 superfluid system at low temperature. In He^3 system, the attraction via the spin fluctuation causes a pairing in the $^3P\,(L=S=1)$ channel [17]. The general ^3P state can be written using 3×3 $\boldsymbol{d}$ matrix as

$$\langle \psi_\alpha(t,\boldsymbol{p})\psi_\beta(t,-\boldsymbol{p})\rangle = i\boldsymbol{d}_n^i(\boldsymbol{p})(\sigma_i\sigma_2)_{\alpha\beta}\,\hat{p}^n, \quad (3)$$

where σ_i are the Pauli matrices, α,β are the spin indices $\alpha = (\uparrow,\downarrow)$, and $\hat{p}$ denotes the unit vector of the relative momentum. Among all ^3P states, Balian-Wertharmer (BW) state defined by [18]

$$\boldsymbol{d}_n^i = \Delta_{\rm BW}\delta_n^i = \begin{pmatrix} \Delta_{\rm BW} & 0 & 0 \\ 0 & \Delta_{\rm BW} & 0 \\ 0 & 0 & \Delta_{\rm BW} \end{pmatrix}, \quad (4)$$

is known to be the ground state in the low T and low pressure region. (see Fig. 2.) This is an isotropic state and the quasi-particles on the BW state have the finite and equal gap on the entire Fermi surface. Thus, the BW state can gain the pairing energy on the whole Fermi surface. This is the reason why the BW state dominates over the other ^3P states in the low T/pressure region.

On the other hand, Anderson-Morel (ABM) state [19] defined by

$$\boldsymbol{d}_n^i = \begin{pmatrix} 0 & 0 & \Delta_{\rm ABM} \\ 0 & 0 & i\Delta_{\rm ABM} \\ 0 & 0 & 0 \end{pmatrix}, \quad (5)$$

appears at high T and high pressure region: The attraction via spin fluctuation is proportional to the spin magnetic susceptibility χ_{ij}, and the gap on the Fermi surface suppresses this susceptibility. Accordingly, the attraction gets weaken in the isotropic BW state, while in the anisotropic ABM state, the attraction does not so much suffer from the suppression because of the gapless region on the Fermi surface. Thus the ABM state takes over the BW state in the high T/pressure region where the spin fluctuation, a driving force to a pairing, gets stronger than that in the low T/pressure region [20]. The polar state is defined by

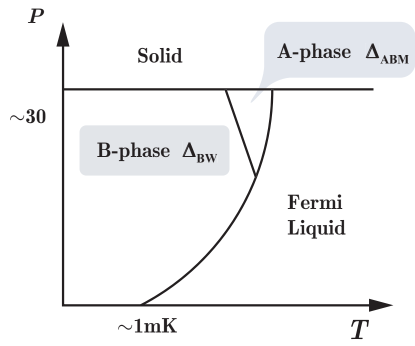

FIG. 2: The schematic phase diagram of He^3 system: The vertical axis shows the pressure in the unit of the atmospheric one. The A-phase and B-phase are the superfluid states, characterized by the gap $\Delta_{\rm BW}$ and $\Delta_{\rm ABM}$, respectively.

$$\boldsymbol{d}_n^i = \delta_3^i\delta_n^3\Delta_{\rm polar} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \Delta_{\rm polar} \end{pmatrix}, \quad (6)$$

but is known not to be realized in the homogeneous system. We can see the similarity between the CFL phase and the BW phase, and that between the 2SC state and the polar or the ABM state. The 2SC order parameter is anisotropic in the color and flavor space just as the ABM state is in the spin and angular space.

We remark that a similarity is found not only in the forms of the gap matrices, but also in the degeneracy of the gapped modes. The quasi-quark modes and excitation energy gap in the CFL and 2SC phase are summarized in Table. I. In the BW phase, all fermions on the Fermi surface obtain the finite energy gap, while in the ABM (polar) state, the gap is zero at the south and north poles (along the equator) of the Fermi sphere. Again, the BW state is similar to the CFL state in the sense that the all atoms on the Fermi surface obtain the gap in the former, while in the latter, all quarks in color-flavor space acquire the finite gap. Similarly, the 2SC state has some gapless quarks in color-flavor space as the ABM/polar state has some gapless points/line on the Fermi surface.

	2SC	CFL	
$\langle q_i^a q_j^b \rangle$	$\delta^{ab\boldsymbol{g}}\delta_{ij\boldsymbol{s}}\Delta_{\rm 2SC}$	$\delta^{abL}\delta_{Lij}\Delta_{\rm CFL}$	
gapless modes	5 (gu, gd, gs, rs, bs)	0	
unbroken symmetry	$SU(2)_c \times SU(2)_f$	$SU(3)_{C+V}$	
gaps	$\Delta_{\rm 2SC}$	$\Delta_1 = 2\Delta_{\rm CFL}$	$\Delta_8 = -\Delta_{\rm CFL}$
quasi-quarks	$SU(2)_c \times SU(2)_f$ doublets $\left[\mathbf{2}_{(r,b)} \times \mathbf{2}_{(u,d)}\right]$ $(\mathbf{4})$	$SU(3)_{C+V}$ singlet $(\mathbf{1})$	$SU(3)_{C+V}$ octet $(\mathbf{8})$

TABLE I: The gap matrices in the CFL and 2SC phases, and the classification of the quasi-quark modes on those states. $SU(3)_{C+V}$ denotes the simultaneous inverse color-flavor rotation under which the locking δ_i^a remains invariant.

d. Symmetries in more details If we confine ourselves to the pairing in the aligned chirality channel [1] and in the positive parity channel [2], then we have

$$\langle {q_L}_i^a {q_L}_j^b \rangle = \epsilon_{ab\boldsymbol{g}}\epsilon^{ij\boldsymbol{s}}\Delta_{\rm 2SC} = -\langle {q_R}_i^a {q_R}_j^b \rangle. \tag{7}$$

The condition of the parity (+) fixes the relative phase between the $q_L q_L$ and $q_R q_R$ condensates to be π. This ansatz does not break the chiral symmetry, because the form of condensate is unaffected both by $SU(2)_c \times SU(2)_L$ and $SU(2)_c \times SU(2)_R$ rotations. The symmetry breaking pattern in the 2SC phase can be specified as

$$\begin{aligned} & SU(3)_c \times \underbrace{U(1)_B \times SU(2)_V}_{\supset U(1)_{\rm EM}} \times SU(2)_A \, (\times U_A(1)) \\ & \to SU(2)_c \times \underbrace{\tilde{U}(1)_B \times SU(2)_V}_{\supset \tilde{U}(1)_{\rm EM}} \times SU(2)_A \, (\times Z_2^A). \end{aligned} \tag{8}$$

The baryon number $B = \mathbf{1}_f/3$ is broken, but the rotated baryon number

$$\tilde{B} = B \times \mathbf{1}_c - \frac{2}{\sqrt{3}}\mathbf{1}_f \times T_8, \tag{9}$$

is conserved in the 2SC state. Similarly, the rotated electro-magnetism

$$\tilde{Q} = \frac{1}{2}\tilde{B} + I_3, \tag{10}$$

with I_3 being the isospin charge, is the unbroken $U(1)$ charge in the 2SC phase. In the 2SC phase, no global symmetry other than axial baryon number is broken spontaneously, and only the color $SU(3)_c$ symmetry is broken down to $SU(2)_c$. Accordingly, five gluons out of eight acquire the Meissner mass in the 2SC state.

Similarly in the CFL phase, we have

$$\langle {q_L}_i^a {q_L}_j^b \rangle = \epsilon_{abL}\epsilon^{Lij}\Delta_{\rm CFL} = -\langle {q_R}_i^a {q_R}_j^b \rangle. \tag{11}$$

In this case, the equation in the left hand side (LHS) no longer decouples from that in the right hand side (RHS), because the color and the flavor rotation are related. To demonstrate this, we examine the color and the left hand chiral transformation, ${q_L}_i^a = (U_L)_i^j (U_c)_b^a {q_L}_j^b$. Then the condensate transforms as

$$\epsilon_{abL}\epsilon^{Lij}\Delta_{\rm CFL} \to \epsilon^{ijM}(U_L^{-1})_M^L (U_c^{-1})_L^N \epsilon_{Nab}. \tag{12}$$

Similarly, the right hand chiral transformation and the color rotation act on the condensate as

$$\epsilon_{abL}\epsilon^{Iij}\Delta_{\rm CFL} \to \epsilon^{ijM}(U_R^{-1})_M^L (U_c^{-1})_L^N \epsilon_{Nab}. \tag{13}$$

From these, we notice the arbitrary color rotation U_c can be compensated by the following simultaneous transformation $U_L = U_R = U_c^{-1}$. Under this vector-like transformation (combined with color rotation), the condensate remains invariant, while the axial-like $U_L = (U_R)^\dagger$ transformation does not conserve the form of the condensate. In this way, the chiral symmetry is spontaneously broken down to the vector like flavor symmetry. The axial-like vibrations of the relative phase of the $\langle q_L q_L \rangle$ and $-\langle q_R q_R \rangle$ in the chiral space propagate in the CFL medium as the Nambu-Goldstone (NG) modes. The gauge invariant chiral order parameter can be made by contracting the color indices, leading to

$$\begin{aligned} & \langle {q_R}_i^a(0,\boldsymbol{x}) {q_R}_j^b(0,\boldsymbol{x}) \bar{q}_{Lk}^{\,b}(0,\mathbf{0}) \bar{q}_{Ll}^{\,a}(0,\mathbf{0}) \rangle \\ & \overset{|\boldsymbol{x}|\to\infty}{\to} 2(\delta_{ik}\delta_{jl} - \delta_{il}\delta_{kj})|\Delta_{\rm CFL}|^2. \end{aligned}$$

This quartic nature of the chiral order parameter turns out to result in the linear relation between quark mass and the pion mass $m_\pi \sim m_q$ [21–23]. In short, the symmetry breaking pattern in the CFL is specified as

$$\begin{aligned} & SU(3)_c \times \underbrace{SU(3)_L \times SU(3)_R}_{\supset U(1)_{\rm EM}} \times U(1)_B \, (\times U(1)_A) \\ & \to \underbrace{SU(3)_{C+V}}_{\supset \tilde{U}(1)_{\rm EM}} (\times Z_2^A). \end{aligned}$$

Thus in the CFL phase the global symmetries, the chiral and the baryon number is broken. The broken baryon number means that the CFL state is not the eigenstate of the baryon number, and have an explicit phase. The expectation value of the baryon number in some finite volume is always changing, but is compensated by the

[1] $q_L q_R$ mixed condensate is suppressed by a factor m/μ compared to the $q_L q_L$ or the $q_R q_R$ condensate.
[2] This parity (+) channel is favored by the instantons [1].

supercurrent from the surface, which is expressed by the spatial derivative of the phase of the order parameter.

In the CFL phase, the color symmetry is completely broken. As a consequence, all eight gluon acquire the Meissner mass. Also, the massless H particle (phonon) associated with the baryon number breaking as the collective modes. In addition, if the stiffness is weak enough, $U(1)_B$ topological vortices can be excited by the rotation [24]. All these elementary modes are shown to have integer charge under the unbroken $\tilde{U}_{\rm EM}(1)$ gauge symmetry, and all the colored excitations are fully gapped in the CFL phase [1]. These features in the CFL phase look similar to the physics of confined phase. Based on this picture, the conjecture of the quark-hadron continuity is proposed [25] and is examined in [26, 27].

e. The CFL phase at extremely high density and the estimation of the gap: A number of efforts estimating the gap and the condensation energy have revealed that the phase realized at extremely high density is the CFL [28–31]. This is because the strange quark mass is negligible at the high density, and the all quarks can participate in pairing in the CFL phase. We can work with the perturbation technique to estimate the CFL gap in the high density regime, and the coupling dependence of the gap parameter is turned out to be different from that in the usual BCS result [32]:

$$\Delta_{\rm CFL} = 2^{-1/3} 256 \pi^4 (2/N_f)^{5/2} \mu g^{-5} \exp\left(-\frac{3\pi^2}{\sqrt{2}g}\right).$$

The anomalous exponent $\sim 1/g$ comes from the long ranged color magnetic attraction screened only by dynamical Landau damping. This was first obtained in [28] on the basis of the renormalization group [28]. If the coupling dependence of Δ was BCS like, $e^{-c/g^2} \sim (\Lambda_{\rm QCD}/\mu)^c$ with c being $3\pi^2/\sqrt{2}$, the gap would become smaller to larger chemical potential as $\Lambda_{\rm QCD}(\Lambda_{\rm QCD}/\mu)^{c-1} \to 0$. In contrast, Son's result of $e^{-c/g}$ predicts the arbitrary large gap at extremely high density. In contrast to the exponent and g^{-5} in the prefactor, the overall coefficient of the gap has not been fixed fully yet and determining this remains a challenging problem [33, 34].

II. DIQUARK CONDENSATION AT MODERATE DENSITY

In the high density (and chiral) limit, the ground state is the symmetric CFL. As the density is decreased, the CFL phase suffers from a distortion in gaps due to the stress owing to the gradual increase of $M_s^2/\mu\Delta_{\rm CFL}$. We naively expect at some critical point, the 2SC takes over the CFL. The criterion for this unlocking transition is first given in literature [26, 27], and in which it has been shown that the CFL state turns into the 2SC state at the point $M_s^2/\mu\Delta_{\rm CFL} \cong 4$ in the explicit NJL model calculation. This transition is first order because the quasi-quark pole disappears for $M_s^2/\mu\Delta_{\rm CFL} \gtrsim 4$ from the integrand in the gap equation. FIG. 3 shows the phase diagram calculated with the Schwinger-Dyson equation in the improved ladder approximation [35]. This model reproduces the rigorous result of gap at extremely high density, as well as the physics at $\mu = 0$ associated with the chiral symmetry breaking in the QCD vacuum. In this sense, this model provides the unique description of the pairing phenomena from high density to low density, covering a wide region of the quark chemical potential. Four unlocking lines in the figure are determined with the aide of simple kinematical criterion for the in-medium strange quark mass, $m_s = 0, 150, 200, 250\,\text{MeV}$, from up to down. The 2SC $\leftrightarrow$ QGP transition is second order and is unaffected by the change in the in-medium strange quark mass. This 2SC $\leftrightarrow$ QGP line coincides with the actual unlocking line in the chiral limit.

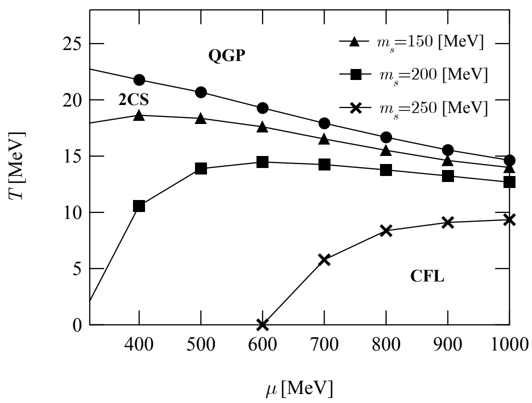

FIG. 3: The phase diagram calculated in the improved Schwinger-Dyson approach.

As the density is decreased, not only a phase transition due to kinematical origin, but also the intrinsic character change of Cooper pairing is also caused by the strong coupling effects [36, 37]. FIG. 4 shows the coherence length ξ_c divided by the averaged inter-quark distance d_q as a function of μ. ξ_c measures the *size* of Cooper pair, which must be much larger than the scale of d_q for the justification of mean field approximation. In fact, this quantity ξ_c/d_q is of the order 10^5 in the usual metallic superconductor. As the density is decreased, this quantity becomes the order of 10, and a simple extrapolation to the physically relevant density $\sim$ 500 MeV indicates that it could be the order of unity. This signals the breakdown of mean field approximation, and possible crossover from BCS to the Bose-Einstein condensed phase with tightly bound Cooper pair bosons [36, 37]. It would be interesting to study the possibility of BCS-BEC crossover in the relativistic superfluid beyond the mean field approximation, though it cannot be realized in the massless limit in a rigorous sense [38]. Another interesting phenomenon associated with the strong coupling nature of color superconductivity is the existence of the precursory collective

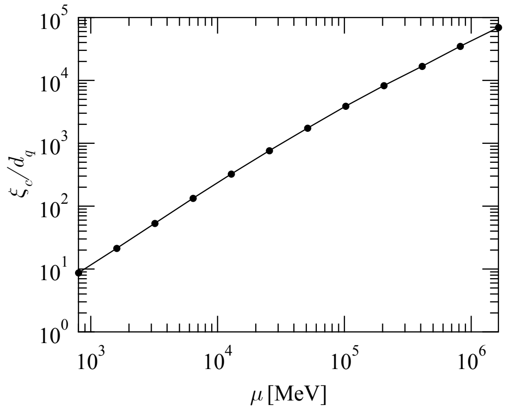

FIG. 4: The coherence length divided by the inter-quark distance as a function of the quark chemical potential.

mode above the critical temperature [39, 40].

III. DIQUARK CONDENSATIONS IN THE FINITE SIZE SYSTEM

Although a number of studies have revealed many rich physics of color superconducting phases in bulk, few are known for those in the finite size systems. It is, however, interesting to study the possibility of diquark condensation in finite systems, because it must be confined in small sizes even if such state of matter is created in the heavy ion collisions or is found in the galactic cosmic rays. It is still controversial even in bulk, whether the strange quark matter can exit absolutely stably or not, depending on the model parameters [41–47]. However, the discovery of the CFL pairing at high density has significantly broaden a chance for the realization of (meta-)stable quark matter even in small chunks (strangelets) [48, 49]. The CFL pairing not only suppresses the energy per baryon E/A for all region of A, but is also shown to lead the unusual charge to mass ratio $Z/A \sim 0.3A^{2/3}$ [48] rather than the ordinary result $Z/A \cong 0.1A$ [43, 44]. This is originated in the bulk property of the CFL phase; the constraints of color and electric charge neutralities pull up or down the Fermi momenta of each flavor at equal for a small strange quark mass [10, 11, 50, 51]. Because of this, the net charge of the strangelets come mainly from the that distributed at surface [48]. In [48], the MIT bag pressure in addition to the pressure due to the diquark condensed background up to second order in $\Delta_{\rm CFL}$ is taken into account. Thus, the gap $\Delta_{\rm CFL}$ is treated only as a parameter. In [49], this point is improved; the NJL model with two couplings, one in the scaler channel G_1, and the other in the diquark channel G_2 is used, and the pressure due to the diquark condensation is determined without any other ambiguity after the mean field approximation. The results also show that the CFL pairing in the strangelets greatly affects the E/A. Nevertheless, the diquark coupling should be somewhat larger than the typically adopted value for the realization of absolutely stable strangelets, as long as scaler coupling G_1 is fixed to reasonable value.

In [48, 49], the finite size effects are incorporated only with the help of smoothed density of state estimated in the multiple reflection expansion [44, 52]. In such treatment, we cannot obtain the energy shell structure which appears in the explicit mode-filling in the MIT bag model [44]. To improve this point in the easy way, we can follow the literature [53], in which the 2SC pairing in the finite size box is considered. In the reference, the discreteness of momentum level due to the finite size as well as the projections onto the definite quantum numbers (baryon number and zero color charge) is taken into account. The results show that the former effect of discretized energy level brings the shell structure in the gap and E/A. In fact some windows where the gap becomes zero open in the case of small box size. Nevertheless, owing to the latter effect of the projection, the system recovers a sizable gap [53]. As a results, the bulk estimation of the gap and E/A is not so bad even when the superconductivity in finite systems is considered. In this work, however, the size of the system is treated as a simple parameter. In reality, the size of strangelet (if exist) is determined by the pressure balance equation. The gap equation and the pressure balance equation must be solved self-consistently for only given parameter A. This may be needed for the realistic treatment of the color superconducting quark nuggets.

These theoretical efforts to describe the superconducting quark nuggets might possibly have a relevance to the physics of the deeply bound kaonic nuclei recently found in the stopped K^- absorption experiment at KEK-PS [54], predicted firstly in [55] based on the assumption of strong $\bar{K}$-N interaction.

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[1] For reviews, see K. Rajagopal and F. Wilczek, arXiv:hep-ph/0011333; M. G. Alford, Ann. Rev. Nucl. Part. Sci. **51**, 131 (2001); G. Nardulli, Riv. Nuovo Cim. **25N3**, 1 (2002); S. Reddy, Acta Phys. Polon. B **33**, 4101 (2002); D.H. Rischke, Prog. Part. Nucl. Phys. **52**, 197 (2004); T. Schäfer, arXiv:hep-ph/0304281; M. Alford, arXiv:nucl-th/0312007.
[2] J.C. Collins and M.J. Perry, Phys. Rev. Lett. **34**, 1353 (1975).
[3] L.N. Cooper, Phys. Rev. **104**, 1189 (1956).
[4] J. Bardeen, L.N. Cooper and J.R. Schrieffer, Phys. Rev. **108**, 1175 (1957).

[5] J.R. Schrieffer, "*Theory of Superconductivity*", (Benjamin, New York, 1964); L. Leplae, H. Umezawa and F. Mancini, Phys. Rept. **10**, 152 (1974).
[6] D. Bailin and A. Love, Phys. Rept. **107**, 325 (1984).
[7] M. G. Alford, K. Rajagopal and F. Wilczek, Nucl. Phys. B **537**, 443 (1999).
[8] M. Iwasaki and T. Iwado, Phys. Lett. B **350**, 163 (1995).
[9] K. Iida, T. Matsuura, M. Tachibana and T. Hatsuda, Phys. Rev. Lett. **93**, 132001 (2004); for a further development, see, K. Iida, T. Matsuura, M. Tachibana and T. Hatsuda, arXiv: hep-ph/0411356.
[10] M. Alford, C. Kouvaris and K. Rajagopal, Phys. Rev. Lett. **92**, 222001 (2004).
[11] M. Alford, C. Kouvaris and K. Rajagopal, arXiv:hep-ph/0406137.
[12] S. B. Ruster, I. Shovkovy and D. Rischke, Nucl. Phys. A **743**, 127 (2004).
[13] K. Fukushima, C. Kouvaris and K. Rajagopal, arXiv:hep-ph/0408322.
[14] H. Abuki, M. Kitazawa and T. Kunihiro, arXiv:hep-ph/0412382.
[15] P. F. Bedaque and T. Schäfer, Nucl. Phys. A **697**, 802 (2002); D. B. Kaplan and S. Reddy, Phys. Rev. D **65**, 054042 (2002).
[16] R.D. Pisarski and D.H. Rischke, Phys. Rev. D**60**, 094013 (1999).
[17] A.J. Leggett, Rev. Mod. Phys. **47**, 331 (1975).
[18] R. Balian and N.R. Werthaner, Phys. Rev. **131**, 1553 (1963).
[19] P.W. Anderson and P. Morel, Physica **26**, 671 (1960); P.W. Anderson and P. Morel, Phys. Rev. **123**, 1911 (1961).
[20] P.W. Anderson and W.F. Brinkman, Phys. Rev. Lett. **30**, 1108 (1973).
[21] D.T. Son and M.A. Stephanov, Phys. Rev. D **61**, 074012 (2000).
[22] R. Casalbuoni and R. Gatto, Phys. Lett. B **464**, 111 (1999).
[23] D. K. Hong, T. Lee and D. Min, Phys. Lett. B **477**, 137 (2000).
[24] K. Iida and G. Baym, Phys. Rev. D **66**, 014015 (2002).
[25] T. Schäfer and F. Wilczek,Phys. Rev. Lett. **82**, 3956 (1999).
[26] T. Schäfer and F. Wilczek, Phys. Rev. D **60**, 074014 (1999).
[27] M. Alford, J. Berges and K. Rajagopal, Nucl. Phys. B **558**, 219 (1999).
[28] D.T. Son, Phys. Rev. D **59**, 094019 (1999).
[29] T. Schäfer and F. Wilczek, Phys. Rev. D **60**, 114033 (1999).
[30] R.D. Pisarski and D.H. Rischke, Phys. Rev. D **61**, 074017 (2000).
[31] D.K. Hong, V.A. Miransky, I.A. Shovkovy and L.C.R. Wijewardhana, Phys. Rev. D **61**, 056001 (2000), Erratum-ibid. D **62**, 059903 (2000).
[32] T. Schäfer, Nucl. Phys. B **575**, 269 (2000).
[33] W.E. Brown, J.T. Liu and H.C. Ren, Phys. Rev. D **61**, 114012 (2000); *ibid.* D **62**, 054013 (2000); *ibid.* D **62**, 054016 (2000).
[34] D.K. Hong, T. Lee, D. Min, D. Seo and C. Song, Phys. Lett. B **565**, 153 (2003).
[35] H. Abuki, Prog. Theor. Phys. **110**, 937 (2003).
[36] M. Matsuzaki, Phys. Rev. D **62**, 017501 (2000).
[37] H. Abuki, T. Hatsuda and K. Itakura, Phys. Rev. D **65**, 074014 (2002).
[38] Y. Nishida and H. Abuki, in preparation.
[39] M. Kitazawa, T. Koide, T. Kunihiro and Y. Nemoto, Phys. Rev. D **65**, 091504 (2002).
[40] M. Kitazawa, T. Koide, T. Kunihiro and Y. Nemoto, Phys. Rev. D **70**, 056003 (2004).
[41] A.R. Bodmer, Phys. Rev. D **4**, 1601 (1971); E. Witten, *ibid.* **30**, 272 (1984).
[42] For a general review, see, J. Madsen, Lect. Notes Phys. **516**, 162 (1999) [arXiv:astro-ph/9809032].
[43] E. Farhi and R.L. Jaffe, Phys. Rev. D **30**, 2379 (1984).
[44] M.S. Berger and R.L. Jaffe, Phys. Rev. C **35**, 213 (1987).
[45] E.P. Gilson and R.L. Jaffe, Phys. Rev. Lett. **71**, 332 (1993).
[46] R. Tamagaki, Prog. Theor. Phys. **85**, 321 (1991).
[47] J. Madsen, Phys. Rev. Lett. **70**, 391 (1993); Phys. Rev. D **47**, 5156 (1993); Phys. Rev. D **50**, 3328 (1994).
[48] J. Madsen, Phys. Rev. Lett. **87**, 172003 (2001).
[49] O. Kiriyama, arXive:hep-ph/0401075.
[50] K. Rajagopal and F. Wilczek, Phys. Rev. Lett. **86**, 3492 (2001).
[51] M. Alford and K. Rajagopal, JHEP **0206**, 031 (2002).
[52] R. Balian and C. Bloch, Ann. Phys. **60**, 401 (1970).
[53] P. Amore, M.C. Birse, J.A. McGovern and N.R. Walet, Phys. Rev. D **65**, 074005 (2002).
[54] T. Suzuki et al., Phys. Lett. B **597**, 263 (2004).
[55] Y. Akaishi and T. Yamazaki, Phys. Rev. C **65**, 044005 (2002).