Supernova 1987A Neutrino Signal: Statistical Power of a Small Sample

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Of the 20 neutrino events associated with SN1987A, all eight from IMB and four from Kamiokande II have measured energies lying at or above 20 MeV. Unlike the eight lower-energy events, this sample shows strong forward peaking, so that the *a priori* probability of obtaining a distribution as forward as this from an isotropic mechanism is less than one part in a million. Standard theory expects a nearly isotropic distribution. Previous analysis of these events are reviewed. More sophisticated statistical approaches (which should give a higher probability than quoted above, but lower than in previous analysis) are invited.

1. INTRODUCTION

At the beginning of 1987 there appeared by far the nearest supernova in living memory. Without very much conscious planning, it happened that there were not one but two underground Čerenkov water detectors in operation, able to detect neutrinos emitted by the supernova [1–4]. The great fact about these observations is that, within an order of magnitude, they confirmed the evolving picture of supernova development, which implies that most of the energy gained from gravity during the collapse phase should be radiated in the form of neutrinos.

However, from the very beginning it was clear that at the next stage of detail these events present a puzzle: For events with more than 20 MeV visible electron or positron energy, an expected nearly isotropic distribution is observed to be rather forward peaked (see Figure 1). Except for small impurities [5], there are only three types of target in the water, electrons, protons, and ^{16}O nuclei. For energetic (20 MeV or more) incident neutrinos, center of mass motion guarantees an extremely forward-peaked ($\cos\theta > 0.98$), angular distribution on the electron target for final electrons or positrons, and the cross section is quite small because of the small target mass. For proton targets, the interaction due to the charged weak current is very well understood, being close to isotropic for incident electron antineutrinos, and vanishing for all other types in the energy range of interest.

The oxygen nuclei have a substantial threshold for charged current interactions by electron neutrinos or antineutrinos, and standard nuclear wave functions imply if anything a backward bias in the angular distribution. Thus the perfectly known cross sections are either too small and much too forward peaked (electrons), or too nearly isotropic to fit well with the data. Even the oxygen nuclei do not seem to give much room for understanding the results. One concludes that either there is a substantial statistical fluctuation or there is some physics that has not yet been identified.

The aim of the present work is to present a case, using a simple probabilistic analysis, that the significance of this discrepancy is large enough to justify further attention, even if it is beyond our control to repeat the experiment on demand by ordering up another nearby supernova!

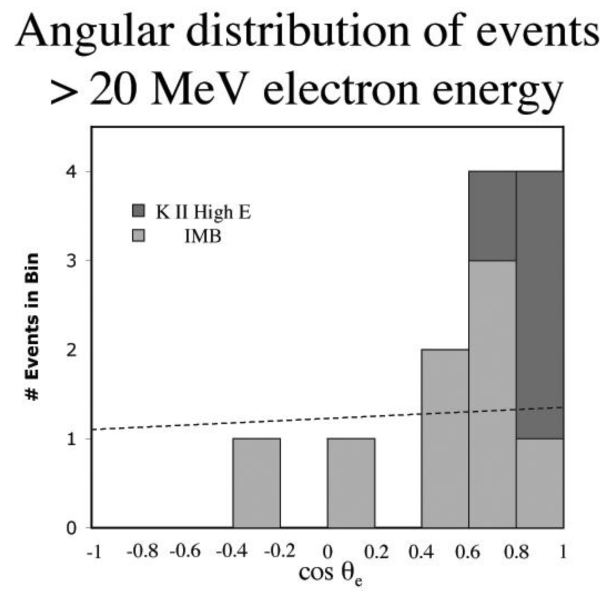

Figure 1: This histogram shows the distribution in direction cosine with respect to a line from SN 1987A to the detector for the eight events of IMB [3], all at or above 20 MeV in energy, and the four events in that energy range out of 12 events altogether of Kamiokande II [K II] [4]. The dashed line illustrates the expected angular distribution for the final positrons in the reaction $\bar{\nu}_e p \to e^+ n$. Evidently there is a substantial forward fluctuation in the samples from both detectors.

2. PROBABILITY CALCULATION

Let us start by focusing on the four events at or above 20 MeV in the data from K II [1]. The 8 events at or below 13 MeV arguably may be excluded on the basis of two considerations having nothing to do with angular distribution. First, 20 MeV was the threshold for detection at IMB, and thus this sample is directly comparable to the entire sample of 8 events from IMB. Secondly, there is a notable gap of about 7 MeV between the lower- and higher-energy events at K II, giving at least the possibility that different mechanisms are responsible for the signals in the two energy ranges. If one now does look at the angular distribution of the lower-energy events, it is quite consistent with isotropy, appropriate to absorption of electron antineutrinos on protons [4].

Of the four higher-energy events at K II, three are in the most forward decile in $\cos\theta$, and the remaining one is in the next most forward decile. If we ask the *a priori* probability that this distribution or a more forward one would have resulted by chance from an isotropic distribution, it is easy to compute the answer. The most forward distribution would be all four events in the top decile, with probability 10^{-4}. The next most forward is the observed signal, with exactly one of the four events in the second decile. This from simple combinatorics has probability 4×10^{-4}, yielding finally a total probability 5×10^{-4}.

The general method of computation exemplified by the above exploits the probability of any particular distribution of events among bins. If there are N events altogether, this probability is

$$P(\{n_i\}) = N! \prod p_i^{n_i}/n_i! \quad , \tag{1}$$

where p_i is the expected probability for an event to appear in bin i. The total *a priori* probability P_T for a distribution at least as forward as a given distribution is obtained by summing the above probabilities over all distributions in which any number of events are moved forward by any allowed number of bins from their places in the given distribution.

The result for the 8 events seen at IMB, again assuming an isotropic mechanism, is approximately $P_{IMB} = 1.75 \times 10^{-3}$, so that the product of the two *a priori* probabilities is smaller than one part in a million. This seems more than enough to justify serious attention to the possibility of a genuine physical mechanism for the forward distribution, but we still need to look at this calculation more closely.

3. CRITIQUE AND ALTERNATE APPROACHES

There are a number of considerations needed to obtain a full perspective on the likelihood that the observed distribution could have arisen by chance.

3.1. Bin Size

An attractive feature of the probability defined above is that it decreases monotonically as bins are subdivided, going to a nonzero asymptotic value in the limit of infinite subdivision, with little change once the bin size becomes small compared to event spacings. Thus the binning adopted here should be quite close to optimum for efficiency and accuracy together, especially in view of the angular uncertainties of the observations.

3.2. Anisotropy in Prediction

Although the angular distribution for $\bar{\nu}_e p \rightarrow e^+ n$ in the relevant energy range is roughly isotropic, there is a mild forward distortion linear in $\cos\theta$ [6], with amplitude about 0.1 in the energy range 20-40 MeV. Evidently this increases somewhat the probability of forward-peaked distributions for higher-energy events as observed in the two experiments. Further, for IMB, the efficiency in the most backward bin is reduced relative to the average by about 10 %, a small additional contribution to expectation of a forward tendency in the data [3].

3.3. General Fluctuation versus Forward Fluctuation

The conventional method of estimating the probability of a large fluctuation from isotropy is to allow that fluctuation to be in any direction, not just forward along the beam direction. Clearly this would give a substantially larger probability for the fluctuations of each of the two distributions. A Monte Carlo estimate of this probability using the Smirnov-Cramer-von Mises statistic was given for the IMB events [3], and not surprisingly was almost one order of magnitude bigger than the estimate above. Clearly a similar statement would apply for K II. However, multiplying these two numbers together surely is too conservative, because the fluctuations in both samples actually are forward with respect to the same (beam) direction.

3.4. Towards a More Appropriate Measure of Probability

To make a precise statement about the relevant probability in view of the above considerations could require a more sophisticated analysis, but the following might be a reasonable first approximation: Use the IMB calculation for the probability of such a large fluctuation in their data. Then keep the 'as forward as' calculation for K II, because that happens to be the direction of the actual IMB fluctuation. This would imply a probability of the combined fluctuations in the two samples in the range of a part in 100 thousand. That still is enough according to usual thinking to impel careful searching for some physics underlying the observation. A more conventional alternative worth pursuing would be to repeat the Monte Carlo estimation approach for the combined sample, which also might give a smaller probability than the product of separate probabilities for the two samples.

3.5. Keeping the Low-Energy Events

Another estimation was given by Vogel and Beacom [VB] [7], who lumped together the high- and low-energy data from K II (consistent with the hypothesis that all events are on a proton target). Evidently this assumption dilutes the signal as portrayed in Figure 1. To estimate the probability of such a fluctuation, they considered only values of the first and second moments of the $\cos\theta$ distribution. A glance at the figure makes it obvious that with a fairly uniform distribution added in for the eight low-energy events one needs more than these two moments to characterize the fluctuation. Thus both the high-energy and the inclusive data selections clearly call for more sophisticated statistical treatment.

3.6. Electron Target Hypothesis

As already mentioned, the forward events (actually with one possible exception) are not forward enough to be associated easily with electron targets, given the claimed angular resolution [3, 4]. Kiełczewska [8] investigated the flux of μ and τ neutrinos needed to account for the forward events (ignoring the implications of the stated angular resolution). [The restriction to these flavors was to avoid feeding the reaction on proton targets.] She found that rather extreme assumptions, including a much harder spectrum for these neutrinos than for e neutrinos, were needed to make the total radiated neutrino energy less than the energy available from gravitational collapse. Taking into account mixing of neutrino species, something not established at that time, would require revision of that analysis, using considerations found, e.g., in [9].

4. ANOTHER PUZZLE

So far we have been exploring an anomaly in observation of a single astrophysical event. However, there is a generic anomaly in the theory of supernova formation. Even the most advanced computer simulations, including as best is feasible all recognized physics, fail to predict supernova explosions [10]. A possible reason for this failure may lie in what has not yet been possible to include in the calculations, in particular full three-dimensionality for the spatial evolution, which up to now has been mimicked using only two space dimensions. However, it is at least conceivable that some physics of neutrino interactions with matter has yet to be included. [It is generally accepted that momentum transfer from outgoing neutrinos to infalling matter is crucial to detonation.] If so, then one has the appealing possibility that a single mechanism could explain the angular anomaly in the SN1987A data, and also supply the additional 'punch' necessary to detonate supernovae. As explained already, neither the electron nor the proton targets are promising; neutrino interactions with these targets are well determined by existing knowledge. While nuclear targets are sufficiently complex to make some new mechanism conceivable, there is as yet no explicit demonstration of such a mechanism either theoretically or by experiments other than the SN1987A observations. Given extensive analysis using reliable nuclear wave functions, and including, e.g., virtual pion coupling simultaneously to two nucleons, a new mechanism most likely would involve a collective interaction with the nucleus as a whole.

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