X-RAYS FROM GALAXY CLUSTERS AND COSMOLOGY

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ABSTRACT

The properties of clusters of galaxies, and the evolution of those properties over cosmic time, are exquisitely sensitive to several parameters that describe the cosmology of the universe. X-ray observations are a powerful tool to extract this cosmological information. In this class we describe clusters, the techniques of X-ray astronomy relevant to their observation and give examples of these observations. Then we review current methods used to do cosmology with the observations, and summarize the resuls. Finally, we indicate possible future directions for this field.

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1 X-rays from Clusters*

We start with an introduction to clusters of galaxies, describe how they are observed at X-ray wavelengths and give some illustrative examples of X-ray observations of clusters.

1.1 Short History of Our Understanding of Clusters

Charles Messier (1730 - 1817) was a French astronomer who worked in Paris. He was very interested in discovering and tracking comets, but he was continually diverted by pesky faint diffuse non-moving objects. Of course these objects appeared to be comets, but they were not since they were stationary. Messier had to observe them over many nights before concluding they were not moving. In order to save time, Messier compiled and published[1] a list of 109 diffuse objects now known by their Messier or M numbers. These objects are some of the most beautiful in the sky. They are galaxies or supernova remnants. There are 13 Messier objects in a small region at the Virgo-Coma border. Although he did not know it, Messier discovered something entirely different from what he was searching for; he found the Virgo cluster of galaxies, the nearest of the great clusters. The central galaxy in the Virgo cluster is M87. It is common in astronomy to find something completely different from what one is searching for.

Finding more clusters was difficult because the human eye is not a very sensitive detector. In 1933, Harlow Shapley cataloged only 25 clusters[2] and by 1948 only a few dozen were known. The advent of photographic surveys of the entire sky has now yielded $\sim 10,000$ clusters. The Abell catalog was the first very large compilation.[3,4]

Now we arrive at the first example of “What you see is not what you have”. The line of sight velocity of the cluster galaxies may be measured by the Doppler shift of their spectral lines. The first such measurements were of the great clusters in Coma and Virgo in the 1930s.[5,6] The velocities are so fast that either a cluster is a transient phenomenon, lasting a small fraction of the age of the universe, or the cluster contains $\sim 100\times$ more mass than contained by the galaxies. Clusters are not transient since their size/(speed of the galaxies) $<<$ the age of the universe. That is, they could have dispersed long ago, yet they are still there. The conclusion was that clusters must contain a large amount of dark matter binding their galaxies.

By the early 1970s astronomers thought they knew what was in a cluster of galaxies:

*“Baryons and baryonic matter" used in this lecture is synonymous to “the normal matter" consisting of nuclei, electrons and positrons.

dark matter and galaxies. Then in 1971 astronomers discovered X-rays coming from clusters,[7,8] the second example of "What you see is not what you have". In fairly short order the basic properties of this newly discovered component were elucidated. "... most, if not all, rich clusters include an X-ray emission region of large size and of net luminosity $10^{43-44}\mathrm{erg\,s^{-1}}$".[9] "... observations of the X-ray flux ... from the Virgo, Perseus and Coma clusters provide strong evidence for the thermal origin of the radiation, including iron line emission."[10] "We report the first extensive detection of X-ray emission from clusters of galaxies at cosmological distances."[11]

The implications of these new cluster observations are another example of what you see is not what you have. The existence of dark matter is confirmed since the gas is too hot ($\sim 10^8$ K) to be held by the galaxies alone. Some of the dark matter became suddenly visible. The size, luminosity and emission mechanism give the amount of X-ray emitting matter, which is $\sim 20\%$ of the dark matter and much much larger than the mass of the galaxies. Clusters of galaxies are actually fuzz balls of hot gas. They can be used for cosmology studies, if we only knew how.

1.2 Overview of the Techniques of X-ray Astronomy

The wavelength range of X-ray astronomy is roughly 100Å- 0.6Å, which corresponds to an energy range of about 0.1 keV - 20 keV where 1 keV = 1.6×10^{-9} erg. The energy range also corresponds to a temperature range of about 10^{6-8} K, using E/k where k is Planck's constant. The last three sentences illustrate that astronomers are fond of using a wide diversity of unit systems.

The photoelectric effect is the dominant interaction process between matter and photons in the above energy range for matter composed of anything other than pure Hydrogen. This has a number of consequences. First and foremost, from the photoelectric cross sections and constituents of the atmosphere as a function of altitude, X-rays do not penetrate to the ground. Rather, X-ray instrumentation must be lifted above $\sim 100\ \mathrm{km}$. Second, we may define an effective absorption cross section of the interstellar medium of the Milky Way by weighting the photoelectric cross sections of each element by the abundance of that element relative to Hydrogen.

$$\sigma_{\mathrm{eff}}(\mathrm{E}) \equiv \sum \frac{\mathrm{n_i}}{\mathrm{n_H}} \sigma_{\mathrm{i}}(\mathrm{E}) \tag{1}$$

The transmission of the interstellar medium is then

$$\mathrm{e}^{-\mathrm{n_H l}\sigma_{\mathrm{eff}}} = \mathrm{e}^{-\mathrm{N_H}\sigma_{\mathrm{eff}}} \tag{2}$$

Table 1. Telescope Parameters of X-ray Astronomy Imaging Missions

	Einstein	ROSAT	ASCA	Chandra	XMM	ASTRO-E2
Area (cm^2)	171	450	1300/4	800	4500/3	2250/3
Field (°)	1	2	0.6	0.5	0.5	0.6
Resolution ($''$)	4	5	40	0.5	6	30
Focus (m)	3.4	2.4	3.5	10	7.5	4.5
# Nest	4	4	120	4	58	175
Range (keV)	0.1-4.5	0.1-2.4	0.5-12	0.1-8	0.1-15	0.5-12

parameterized by the Hydrogen column density $\mathrm{N_H}$, which is measured by radio astronomy techniques. The interstellar absorption removes low energy photons. At the galactic poles, where the Hydrogen column density is $3 \times 10^{20}\ \mathrm{cm}^{-2}$, only photons with energies above 0.3 keV get through. Toward the galactic center, where the Hydrogen column density is $2 \times 10^{22}\ \mathrm{cm}^{-2}$, only photons with energies above 2 keV reach us. It is important to realize that, although the amount of absorption is parameterized by the amount of Hydrogen, Hydrogen is not the absorber. It is Helium for $\mathrm{E} \leq 0.53\ \mathrm{keV}$ and Oxygen for $\mathrm{E} > 0.53\ \mathrm{keV}$. Beware of "excess" absorption found with cyrogenic detectors because $\mathrm{H_2O}$ freezes on them!

It is very difficult to focus X-rays. For many years there were no images and X-ray astronomy was done with arrangements of slats and wires that mechanically limited the field of view. Now however most current and future missions use X-ray optics. X-rays will reflect off of some materials if they arrive at grazing incidence. The index of refraction at X-ray energies is $\mathrm{n(E)} = 1 - \delta(\mathrm{E}) + \mathrm{i}\beta(\mathrm{E})$. The imaginary part is related to the cross section described in the previous paragraph. Total external reflection, analogous to total internal reflection of light, occurs below a critical grazing angle $\cos\theta_\mathrm{c} = 1 - \delta(\mathrm{E})$ or $\theta_\mathrm{c} \approx \sqrt{2\delta}$, which is proportional to 1/E from many materials. In practical terms, the grazing angles are between about 1 and 5 degrees. The telescope designs are similar to Cassegrain optical telescopes, except with the parabolic and hyperbolic surfaces greatly extended and truncated so only material at grazing angles intercept the incoming radiation. Individual mirrors are usually nested in an effort to increase the frontal area that is, of course, always small if the photons have to arrive at grazing incidence. Table 1 provides a summary of the telescope parameters of X-ray astronomy missions. The area is quoted for an X-ray energy of 1 keV and the field for 0.28 keV.

X-ray astronomy detectors are photon counting. Entire courses may be devoted to

Table 2. Properties of X-ray Astronomy Imaging Detectors

Detector	Mission	Δx	$E/\Delta E$	Δt	QE
Proportional Counter	Einstein, ROSAT	1mm	12.5	10μs	100%
Micro Channel Plate	Einstein, ROSAT, Chandra	15μm	1	10μs	10%
CCD	ASCA, XMM, Chandra ASTRO-E2	15μm	30	1s	100%
Bolometer	ASTRO-E2	0.6mm	1000	10μs	100%

them, but we provide only Table 2. We define the following parameters used in the Table: Δx, ΔE and Δt are the spatial, energy and temporal resolutions respectively.

A big advance with the Chandra and XMM-Newton missions is the use of efficient objective gratings coupled to large-area telescopes. The effective area of the telescope + grating + detector is about 100 cm^2 at an energy of 1.5 keV and the resolution $E/\Delta E$ varies from 200 to 1000 at an energy of 1 keV. The resolution is given by the grating equation.

$$R = \frac{E}{\Delta E} = \frac{\lambda}{f}\frac{dl}{d\lambda}\frac{1}{\phi} \quad (3)$$

Here f is the optic focal length, dl/dλ is the dispersion (mm/Å) and ϕ is the angular size of the source. So only point-like sources have high spectral resolution using objective gratings. That is what makes the bolometer on ASTRO-E2 so powerful for clusters; it will have grating resolution for diffuse sources. The XMM-Newton gratings are always on; Chandra's need to be inserted into the light path.

1.3 Examples of Cluster X-ray Observations

In this section we will describe cluster X-ray morphology and spectra and then give two relations, one well-established and the other expected but not so well-observed. Finally, we will describe what is known empirically about the evolution of cluster X-ray luminosities and temperatures.

The morphology of cluster X-ray emission may be described as train wrecks or dart boards, in astronomer jargon mergers or relaxed. The Coma cluster has been extensively studied for over 75 years. Ever since Zwicky's work[5] it was considered the prototype for a rich relaxed cluster. It is very massive (rich) and was thought to have reached its final evolutionary state, no longer changing (relaxed). However, the X-ray image[12] is very lumpy and each major lump is associated with a bright galaxy. The lumps seem to be unassimilated groups of galaxies. The Coma cluster is in reality very far from relaxed. Another example of a train wreck is the cluster A754, the seven hundred fifty-fourth member of the Abell catalog.[3] This cluster is an example of a major merger; it is far from equilibrium. It has at least three highly elongated substructures and is very non-isothermal.[13,14] The gross surface brightness and temperature distribution of A754 agree well with numerical hydrodynamic calculations of a merger of a group of galaxies with a cluster.[15,16]

Dart boards, indicative of relaxed clusters in hydrostatic equilibrium, have symmetric, nearly circular morphologies and little temperature structure except for cool centers. An example of this type of morphology is A1795.[17] Chandra observations of these clusters often show non-symmetric morphologies.[18] One should remember that becasue of the high angular resolution of Chandra (Table 1), these features are usually small and associated only with the highest surface brightnesses, not with the overall structure of the cluster. The X-ray observations of these types of clusters may be used to measure the total mass needed to hold the hot X-ray gas under the assumption that the gas is in hydrostatic equilibrium.

$$\mathrm{M}(<\mathrm{r}) = -\frac{\mathrm{kT(r)r}}{\mathrm{G}\mu\mathrm{m_p}}\left[\frac{\mathrm{dln}\rho}{\mathrm{dlnr}} + \frac{\mathrm{dlnT}}{\mathrm{dlnr}}\right] \tag{4}$$

where M(<r) is the mass interior to r, T(r) and ρ(r) are the X-ray gas temperature and density at radius r. Something must be assumed to determine the 3D quantities T(r) and ρ(r) from the observations projected onto the plane of the sky. Usually it is spherical symmetry.

Spectral analysis is the major part of astrophysics. We mention only the beautiful XMM grating spectra of the cool centers of relaxed clusters, of which Reference 19 is an example. These spectra solve a long-standing problem in cluster astrophysics. The weak or nonexistent FeXVII and CVI Lyα lines show that the hot gas does not cool below about half of the ambient temperature in the cool centers, contrary to 25 years of accepted wisdom. Further, the cluster gas is enriched, that is it has elements heavier than Hydrogen and Helium. This result is not new. The abundance of these elements is about a third of the solar value. The only known source of heavy elements is supernovae in the cluster galaxies, so the cluster gas is a record of the supernovae history in the galaxies. It is not yet clear what is the mix of the two types of supernovae responsible for the observed elements.

The cluster luminosity-temperature relation is the oldest[20–22] and best studied[23–26] in cluster X-ray astrophysics. There is a definite relation, but there is also substantial scatter. The scatter appears to be Gaussian with $\sigma(\log(\mathrm{L})) \sim 0.3$ at constant kT.[27] The scatter is reduced if the cool centers are excised. We adopt the form L(bol) = C $[\mathrm{kT}]^{\alpha}$ $(1+\mathrm{z})^{\mathrm{A}}$, where L(bol) is the bolometric (total) cluster luminosity. Simple scaling relations predict $\alpha \sim 2$ and A $\sim$ 1.5, but $\sim$ 3 and $\sim$ 1 respectively are observed.[27,28] A and α are closely related to the thermodynamic history of the hot gas. This history is not simple since they do not agree with the simple scaling relations.

We will see in Sevtions 2.2 and 2.4 that mass is the fundamental variable of the theory by which cosmological information is extracted from cluster observations. Unfortunately, the mass of a cluster is one of the most difficult property to measure. Most analyses use equation (4) along with various simplifying assumptions. Analyses with less restrictive assumptions contain fewer clusters. So far the work with the least assumptions analyzes only apparently relaxed clusters coupled with a direct deprojection of the X-ray gas density and temperature profiles, yielding seventeen clusters.[29] A large fraction of these clusters have independent, confirming mass measurements from analysis of weak gravitational lensing. There are M - T and M - L relations. More massive clusters are hotter and more luminous like most stars. Thus it will be possible to use observations of cluster luminosities or temperatures as proxies for cluster masses and therby do cosmology. However, there is substantial scatter about the relations. This will make doing cosmology more difficult. It would be extremely useful if a way could be found to reduce the scatter in the ralations, but so far the sample sizes are too small to examine that scatter in a systematic way.

The properties of clusters of galaxies at a single epoch provide cosmological information. Further, the evolution of those properties provides additional, often complementary information. However, clusters are not standard candles or standard temperature baths. Therefore, the distribution of cluster luminosities or temperatures, the luminosity or temperature functions in astronomy jargon, needs to be determined at different redshifts (epochs) in order to measure luminosity or temperature evolution. This requirement implies that statistically complete samples need to be constructed.

Measuring luminosities is relatively easy; only 25 photons are needed in order to have a 5σ detection. Assembling large samples is possible and quite a few, about 10 so far, have been so assembled. Interpreting the luminosity evolution, or lack of it, is not so easy. The luminosity is proportional to the square of the gas density, so the luminosity evolution is a strong function of the thermodynamic history of the gas. The L - T relation shows that that history is non-trivial. It is much harder to measure cluster temperatures than luminosities since $\sim$1000 photons are required. There is only one high redshift sample with complete temperatures[30] so far and the low redshift samples all contain the same bright clusters.[25,31,32] The interpretation of cluster temperature evolution, or lack of it, should be easier since the cluster temperature is an independent thermodynamic variable.

The summary statement on cluster evolution is there are fewer high luminosity, high temperature (i.e. high mass) clusters in the past. The existence of luminosity evolution has been controversial since the first reports.[33,34] The most recent results are that six independent groups measure statistically significant luminosity evolution[34–39] and the consensus, but not unanimous, opinion is that it does exist. The amount of evolution is a factor of 2 for a luminosity of $2 \times 10^{44} \mathrm{erg\ s}^{-1}$ for redshifts between 0.11 and 0.45. The evolution is larger for larger luminosities and redshift baselines.

2 Using X-ray Observations of Clusters for Cosmology

2.1 Introduction

Why are clusters so useful for cosmology? The answer to this question entails several parts. First, clusters are the most recent objects to form in the universe. Their number density is on the tail of a distribution function. Thus the number density and its evolution are exquisitely sensitive to the parameters describing this distribution function, the cosmological parameters. Second, they are bright and so can be easily seen to redshifts of about unity. The redshift interval from 0 to 1 seems to be when the universal expansion changed from deceleration to acceleration. Clusters are visible and seem to have formed during this crucial interval. Third, they can be well modeled. Clusters are not as simple as the microwave background, but they are simplier than supernovae, galaxies or active galactic nuclei, the only other objects visible to redshifts of 1 and beyond. Fourth, clusters are excellent tracers of large scale structure. The distribution of clusters above random is much more straightforwardly computed from theory than that of galaxies. Fifth and finally, clusters are so massive that their composition is thought to accurately reflect all matter in the universe.

Clusters may be observed at many wavelengths. Why are X-ray observations of clusters so useful for cosmology? The answer is again multi-part. First, the volume searched to find each cluster is known if clusters are found by X-ray searches, but it is not so easily calculated for other searches to date. Second, there seems to be a good relation between X-ray observables and the cluster mass, the fundamental variable of the

Table 3. Standard Model Parameters Relevant to Clusters

Parameter	Value
h	$0.71^{+0.04}_{-0.03}$
Ω_{m0}	0.27 ± 0.04
$\Omega_{\Lambda 0}$	$\equiv 1 - \Omega_{m0}$
Ω_{b0}	0.044 ± 0.004
σ_8	0.84 ± 0.04
w	$\equiv -1$
$\mathrm{n_s(k = 0.05Mpc^{-1})}$	0.93 ± 0.03

theory. Third, there are small or nonexistent projection effects since the X-ray emission goes as the gas density squared, but the signal of all other tracers is linear with density. Essentially all diffuse X-ray sources away from the Milky Way are clusters. Fourth, galactic extinction effects are small at X-ray wavelengths. Fifth, the X-rays come from optically thin thermal radiation from a (nearly) fully ionized gas in collisional equilibrium. This situation aids in cluster modeling since it is about the simplest case imaginable.

The last of the introductory remarks is to give the standard model parameters relevant to clusters in Table 3.[40] The symbols in the Table are the following. The present value of the Hubble parameter is $\mathrm{H_0 = 100\ h\ km\ s^{-1}\ Mpc^{-1}}$. The present matter density in terms of the critical density needed to stop the expansion is Ω_{m0} and Ω_{b0} is defined similarly for the present baryon density. $\Omega_{\Lambda 0}$ parameterizes the cosmological constant or dark energy. The present root mean square matter fluctuation in spheres of 8 h^{-1} Mpc is σ_8. The dark energy equation of state is P = wρc^2. If w = -1, then the dark energy is the cosmological constant. Recall that w = 0 for cold dark matter and w = 1/3 for radiation, although neither of these is the dark energy. The power law index of the spatial Fourier transform of density fluctuations in the early universe is $\mathrm{n_s}$.

This is not the first standard cosmological model that astronomy has had. In the early 1990's astronomers "knew" that $\Omega_{m0} = 1$ and $\Omega_{\Lambda 0} = 0$. Since no theory gives the current standard model values in a natural way, we do not know how long this standard model will last. The whole business has touches of the epicycle description of planetary motion. All the data were described, but the underlying premise was wrong. So one goal of contemporary cosmology is to disprove the standard model in the hope of finding clues to an underlying theory. No cluster data were used to fit the parameters in Table 3, so clusters are an independent test of the standard model.

2.2 Number Density and Its Evolution

Cluster abundance and evolution as driven by gravity are strongly dependent on cosmology, which provides a way to measure Ω_{m0} and other parameters. The first person to realize this fact of whom we are aware was S. C. Perrenod in 1980: "I conclude that a relatively powerful cosmological test may be provided by number counts of X-ray [clusters] out to z ~ 1. If Ω is large, very few high-redshift sources will be seen ... If Ω is small, a moderate number should be detected."[41] There has been a huge amount of work in this field. Some representative references are (Ref. 31,32,34,37,42–55).

The theory used to convert cluster number density to cosmological constraints begins with the mass function, the number of objects per unit mass per unit volume. There are two widely used analytic functions[56,57] and another that is a fit to N-body simulations.[58] The mass function may be written in a universal form, applicable to all cosmologies, if expressed in certain variables and if the mass is measured in a certain way. The scatter is about $\pm 20\%$ for different cosmologies. Additional discussion of the mass function is in (Ref. 59–61).

The comoving (that is with the expansion of the universe removed so the volumes do not change from this effect) mass function of collapsed objects is

$$\mathrm{n}(\Omega_{\mathrm{m0}}, \Omega_{\Lambda 0}, \mathrm{w}, \mathrm{z}, \mathrm{M}) = \frac{\rho_{\mathrm{b0}}}{\mathrm{M}} \frac{\mathrm{d}\nu}{\mathrm{dM}} \mathrm{f}(\nu) \quad (5)$$

with

$$\nu(\Omega_{\mathrm{m0}}, \Omega_{\Lambda 0}, \mathrm{w}, \mathrm{z}, \mathrm{M}) = \frac{\delta_{\mathrm{c}}(\Omega_{\mathrm{m0}}, \Omega_{\Lambda 0}, \mathrm{w}, \mathrm{z})}{\sigma(\Omega_{\mathrm{m0}}, \Omega_{\Lambda 0}, \mathrm{z}, \mathrm{M})} \quad (6)$$

Here δ_c is the mass density fluctuation required for the gravity of a pertubation to overcome the expansion and collapse at redshift z, σ is the rms mass fluctuation in spheres containing mass M and

$$\mathrm{f}(\nu) = \mathrm{A}\sqrt{\frac{2\mathrm{a}}{\pi}}(1 + (\mathrm{a}\nu^2)^{-\mathrm{p}})\exp(-\mathrm{a}\nu^2/2) \quad (7)$$

Different authors calculated different values of the parameters A, a and p; A=0.5, a=1, p=0[56] or A=0.322, a=0.707, p=0.3.[57]

Now the mass of a cluster is difficult to observe, so in order to use the theory the mass must be converted to a more easily observable X-ray luminosity or temperature. Hot (> 3 keV) luminous ($> 10^{43}$ erg s^{-1}) clusters are best since their physics is dominated by gravity. There are two ways to do this. Either adopt the empirical M-L and M-T relations discussed in Section 1.3 or use the top hat (uniform initial perturbation inside a spherical region) collapse theory. This latter theory gives the following M-T relation:

$$\mathrm{kT} = \frac{1.42}{\beta_{\mathrm{TM}}}[\Omega_{\mathrm{m0}}\Delta(\Omega_{\mathrm{m0}}, \Omega_{\Lambda 0}, \mathrm{w}, \mathrm{z})]^{1/3}(\mathrm{hM}_{15})^{2/3}(1+\mathrm{z}) \qquad (8)$$

Here hM_{15} is the mass of the cluster in units of 10^{15} solar masses, $\Delta(\Omega_{\mathrm{m0}}, \Omega_{\Lambda 0}, \mathrm{w}, \mathrm{z})$ is the ratio of the average cluster mass density to the average mass density of the universe[62] and β_{TM} is a "fudge factor" to account for incomplete virialization of the cluster during its collapse. The actual value of β_{TM} can come either from empirical measurements or from hydrodynamic simulations. The former yield values around 0.7 while the latter give values around 1.[63] This uncertainity can induce systematic errors. Often the results are marginalized over the range $\beta_{\mathrm{TM}} = 0.7 - 1.0$. Finally the M-L relation comes from combining the top hat M-T formula with the observed L-T relation discussed in Section 1.3.

The program then is to convert the cosmology dependent mass function given above into a temperature of luminosity function (the number of objects per unit temperature or luminosity per unit volume) for comparison with the observed temperature or luminosity functions. A best fitting procedure then finds the cosmological parameters and their errors compatible with the observations. This procedure yields allowed regions in the hyperspace of cosmological parameters. Cluster data typically provide constraints in the $\Omega_{\Lambda 0} - \Omega_{m0}$, w-$\Omega_{m0}$ and σ_8-Ω_{m0} planes. References (30,32,37,49–55) report recent work in this area.

Constraints from clusters generally agree with but are complementary to those from other independent data sets such as supernovae and the microwave background. The allowed regions are oriented differently in the cosmological parameter hyperspace. The cluster results have different systematic uncertainities. For the current sample sizes of ~ 100 clusters, the statistical errors on Ω_{m0} and σ_8 are comparable to those of the standard model. The systematic error from the unknown β_{TM} inflates the error of σ_8 but no other parameter for the current sample sizes. However a sample size of 100 objects is ridiculously small, so systematic errors will dominate statistical errors for future larger samples. See Section 3 for a discussion of the prospects of obtaining such samples and possible measures to reduce the systematics.

2.3 Baryon Fraction and Its Evolution

The fundamental assumption of this method is that clusters contain a representative sample of all the material in the universe. This is a reasonable assumption, since clusters are the most massive bound objects known. The cluster gas fraction is measured from X-ray observations of relaxed clusters. The gas mass comes directly from the X-ray observations, since that is the material producing the X-rays. The total mass is inferred from assuming hydrostatic equilibrium, that is how much mass is required to contain the hot X-ray producing gas (equation 4). If the X-ray gas were the only baryons in the cluster and the distance to the cluster were known, then $\mathrm{f_{gas}} = \Omega_{\mathrm{b0}}/\Omega_{\mathrm{m0}}$, which leads to a constraint on Ω_{m0} if Ω_{b0} can be estimated independently. Ω_{b0} can in fact be so estimated from the observed abundances of light elements such as Deuterium or Helium that are produced very early after the Big Bang (called primordial nucleosynthesis). After making a small correction for baryons in the cluster galaxies ($\mathrm{f_{baryon}} = \mathrm{f_{gas}}(1 + 0.19\sqrt{\mathrm{h}})$), this relation becomes $\mathrm{f_{gas}}(1 + 0.19\sqrt{\mathrm{h}}) = \Omega_{\mathrm{b0}}/\Omega_{\mathrm{m0}}$. More generally, the distances are not known so converting the observables to gas fraction depends on redshift and cosmology and the final relation is

$$\mathrm{f_{gas}}(1 + 0.19\sqrt{\mathrm{h}}) = \frac{\Omega_{\mathrm{b0}}}{\Omega_{\mathrm{m0}}} \frac{(\mathrm{h}/0.5)^{1.5} \mathrm{D_A^{1.5}}(1, 0, -1, \mathrm{z})}{\mathrm{D_A^{1.5}}(\Omega_{\mathrm{m0}}, \Omega_{\Lambda 0}, \mathrm{w}, \mathrm{z})} \tag{9}$$

where $\mathrm{D_A}$ is the angular diameter distance. This relation yields constraints on both Ω_{m0} and $\Omega_{\Lambda 0}$ with the additional assumption that the baryon fraction is independent of redshift. References 29,64–70 provide an introduction and current status of this method.

Observations of cluster baryon fractions have the potential to give tight cosmological constraints if the two assumptions that clusters contain a representative sample of the material in the universe and the cluster baryon fractions does not evolve are correct. Both of these assumptions seem reasonable but astronomy is replete with reasonable assumptions that turn out to be incorrect. What do we know? The baryon fraction is a function of the temperature of the cluster, but that function appears to becomes weaker or levels off above a temperature of 4 keV.[70–72] The average baryon fraction of a sample of 10 hot clusters with average redshift 0.058 is 0.175 ± 0.011 (for h=0.7, $\Omega_{m0} = 0.3$, $\Omega_{\Lambda 0} = 0.7$).[70] Contrast these results with the average baryon fraction of 6 hot clustes with average redshift 0.302 of 0.143 ± 0.010 (asssuming the same cosmology).[68] In deriving these quantities we have made two corrections that nearly cancel. First, a baryon depletion in clusters relative to the universe of 0.84 ± 0.03. Second, the hot gas is not uniform but is clumped, which enhances the cluster's luminosity over the uniform value and leads to an overestimate of the amount of hot gas by a factor of 1.09 ± 0.04. Both corrections come from numerical hydrodynamic simulation of clusters.[73,74] The low and high redshift baryon fractions agree with each other and with the standard model value of $0.166^{+0.012}_{-0.013}$. Similar weak evolution was observed as a function of temperature for the same cosmology, but there was significant evolution if the cosmology was assumed to be $\Omega_{m0} = 1$, $\Omega_{\Lambda 0} = 0$.[28] We conclude that what data exists provides some support for the two assumptions of this method, but systematic errors on the order of 10% are possible from depletion and/or clumping of the X-ray gas.

2.4 Spatial Power Spectrum of Clusters of Galaxies

Consider the fluctuations in the mass density of the universe about its average: δ(x) = ρ(x)/$\rho_{\rm avg}$ -1. The power spectrum, P(k), is the squared modulus of the Fourier transform of δ(x). If δ(x) has a Gaussian distribution, then all quantities describing the fluctuation field may be obtained from P(k). The normalization of the power spectrum is $\sigma_8 \approx \sqrt{\mathrm{P(0.172hMpc^{-1})/3879h^{-3}Mpc^3}}$. Since most of the mass of the universe is dark, P(k) must be measured using some visible tracer, usually galaxies or clusters. In practice the distances (from redshifts) to a complete sample of objects must be measured from a (nearly) contiguous region of the sky. This topic has a long and venerable history. There are many more references than I can possibly give here. Some relating specifically to clusters of galaxies are (Ref. 75–80).

In terms of our interest, universes with different cosmologies have different power spectra. Astronomers say "The cosmology of the universe is written on the sky." Of course the reality is not so elegant or simple. The power spectra measured using galaxies or clusters as the visible tracers are different. This fact has both good news and bad news. The good news is the signal from clusters is about ten times larger than from galaxies, implying about one hundred times fewer clusters are needed compared to galaxies for the same signal to noise. The bad news is at least one class of object does not trace the matter power spectrum!

Even worse, the power spectrum is a third example of "What you see is not what you have". The observed power spectrum is a *biased* and *distorted* version of the mass power spectrum that gives the cosmological information.

P(k) is biased because high peaks or high masses cluster more than low masses, $\mathrm{P_{biased}(k) = b^2(z, M)P(k)}$ where b is the value of the bias and M is the mass of the tracer. Galaxies form via nonlinear gravitational, dissipative and radiative processes that probably produce complicated biases. Clusters, on the other hand, are expected to have a comparatively simple bias mainly driven by gravity. There is at least the hope that b can be calculated for them. We adopt the following form for the bias.

$$b(z, M) = 1 + \frac{1}{\delta_c}[a\nu^2 - 1] + \frac{2p}{\delta_c}\frac{1}{1 + (a\nu^2)^p} \tag{10}$$

Different authors calculated different values of the parameters a and p; a=1, p=0[81] or a=0.707, p=0.3.[57]

P(k) is distorted because the cluster's distance is not known, only the redshift or line of sight velocity. This velocity includes the Hubble velocity, related to distance, the peculiar velocity induced by other masses ($\sigma_{\rm p}$) and redshift measurement errors ($\sigma_{\rm z}$).

$$P_{z-space} = P(k)\int_0^1 \frac{(1 + \beta\mu^2)^2}{1 + k^2\mu^2\sigma^2/[2H^2]} \tag{11}$$

where μ is the cosine of the angle between k and the line of sight, $\sigma = \sqrt{\sigma_{\rm p}^2/2 + \sigma_{\rm z}^2}$ and $\beta = \Omega_{m0}^{0.6}/b$.

P(k) measured with tracers that are evolving nonlinearly ($\delta > 1$) is different from that predicted by linear evolution because at least some of P(k) comes from the density profiles of individual nonlinear fluctuations. See section 16.4 of Peacock's book[82] for the details.

The power spectrum has been measured from 426 REFLEX X-ray selected clusters in 4.24 sr and z $\leq$ 0.365, and with L(0.1,2.4) $\geq 2.5{\times}10^{42}{\rm h}^{-2}$ ${\rm erg\ s}^{-1}$ and F(0.1,2.4) $\geq$ 3.0$\times 10^{-12}{\rm erg\ cm}^{-2}\ {\rm s}^{-1}$ (Ref. 80). The constraints on σ_8 and Ω_{m0} have smaller statistical errors than those on the standard model values, but only agree with the standard model at about the $\sim 1.8\sigma$ level. These results are insensitive to variations in h, ${\rm n_s}$ and Ω_b so long as they vary within the standard model error range, that is the best fit point stays within the original 1σ contour. But the best fit point can move outside the 3σ contour depending on the precise M-L relation used. This latter effect is a variant of the $\beta_{\rm TM}$ problem of the temperature function.

3 Future Prospects

Science marches on. The WMAP continues to acquire data and the Planck mission is coming. The Sloan Digital Sky Survey also continues to acquire data and will eventually have $\sim 10^6$ galaxies with redshifts from which many thousands of clusters will come. Observations of supernovae continue as well and a satellite dedicated to supernovae studies, SNAP, is under discussion.

Some advances in X-ray studies of clusters can be expected. The ROSAT archive has not been fully exploited. Extending the REFLEX sample to the northern hemisphere and going slightly deeper will yield about 1500 clusters over the whole sky (outside the plane of the Milky Way) with redshift less than 0.4. Deeper non-contiguous samples from the archive add ~ 500 more with redshift less than 1.3 for about 2000 objects from ROSAT in total. Similar non-contiguous searches from XMM-Newton and Chandra will add ~ 500 more with redshift less than 1.5.

In addition, X-rays are not the only way to find clusters. There are huge existing optical data bases and ever more objective and quantifiable cluster searches are being made of those data bases. A few dozen clusters have been discovered so far from the weak lensing of background objects. Planck and the South Pole Telescope will find many thousands of clusters from the influence of their hot electrons on the microwave background photons, the Sunyaev-Zeldovich effect.

Except for X-ray techniques all of the above work may be done from the ground. Most of it can be done better from space, but X-ray studies require it be done there. Unfortunately it is very expensive to build and fly spacecraft. But it is almost certain that a dedicated satellite will be required to make large advances in the field of X-ray cluster cosmology. At least six such missions have been proposed, five to NASA and one to ESA. The details of each proposal vary, but there are generally two subsurveys. A wide survey of $\sim 10,000\ \mathrm{deg}^2$ to a flux limit of $\sim 3 \times 10^{-14} \mathrm{erg\ s}^{-1}\mathrm{cm}^{-2}$ in the 0.5-2.0 keV band that will find $\sim 10,000$ clusters and a deep survey of $\sim 100\ \mathrm{deg}^2$ to a flux limit of $\sim 3 \times 10^{-15} \mathrm{erg\ s}^{-1}\mathrm{cm}^{-2}$ that will find $\sim 2,000$ clusters.

As mentioned at the end of Section 2.2, systematics will likely dominate over statistical uncertainities for these large samples. A new concept of self calibration has recently been advanced to deal with this issue. An arbitrary mass-observable relation with arbitrary evolution (both power laws so far, but this assumption is not crucial) may be calibrated from the survey itself without a large increase in the errors on the cosmological parameters if the sample is large and deep.[83–86] This happy situation occurs because the effect of the mass-observable evolution is not degenerate with that from cosmology, there is a large dynamic range of masses that are sampled and the observable depends on the mass (e.g. temperature or power spectrum bias) yielding an indirect measurement of that mass. Furthermore, accurate masses are expected for $\sim 1\%$ of the sample from other data that will jump start the self calibration.

All of this implies that a generic X-ray cluster survey will provide allowed regions in the cosmological parameter hyperspace that are of comparable size to those from expected advances in other cosmology techniques. Like existing work, the constraints from different techniques will be independent and complementary.

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