

COSMOLOGY WITH EXTRA DIMENSIONS

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Randall - Sundrum

Utilize gravitational red shift to give hierarchy of energy scales (eg. $m_{\text{Planck}}/m_{\text{weak}} \sim 10^{16}$)

$$\frac{\omega_1}{\omega_2} = \frac{g_{00}(r_1)}{g_{00}(r_2)}$$

Atom 1

$$\frac{\omega_1}{\omega_2} = \frac{g_{00}(r_1)}{g_{00}(r_2)}$$

Atom 2



$$\frac{\omega_1}{\omega_2} = \sqrt{\frac{g_{00}(r_1)}{g_{00}(r_2)}} \rightarrow \infty \text{ if } g_{00}(r_2) \rightarrow 0$$

(ie if r_2 near a horizon)

Boldly replace a QM system by a 4d QFT (localized on a brane) at a fixed position in 5d curved space



QFT_1, y_1

→ 5th dim



QFT_2, y_2

$$\rightarrow \text{ratio of mass scales } \frac{m_1}{m_2} = \sqrt{\frac{g_{00}(y_1)}{g_{00}(y_2)}} \rightarrow \infty \text{ if } y_2 \text{ near horizon}$$

Involves 'warped' metric

$$ds_5^2 = e^{-2\ln y/L} ds_4^2 + dy^2$$

5th coord $y \in (0, \pi R)$

usual 4d line element (Mink or FRW...)

- two length scales

L = curvature length (of 5d Anti-de-Sitter space)

πR = size of 5th dimension, and at $y=0$ and $y=\pi R$ have 4d 'boundaries'

- the effective action that describes this system ...

$$S_{\text{bulk}} = \int d^4x dy \sqrt{-G} \left\{ R[G] \frac{M_s^3}{16\pi} - \Lambda_5 + \dots \right\}$$

det of 5d metric 5d Ricci 5d "Planck mass" 5d cosm. const

$$S_{\text{brane}} = \int d^4x dy \sqrt{-G} \left\{ \delta(y) \mathcal{L}_{\text{eff}T_1} + \delta(y-\pi R) \mathcal{L}_{\text{eff}T_2} \right\}$$

eg, hidden sector eg, the SM

Why 4d gravity at $r > L$ and not 5d gravity ??

- Need to know spectrum of 4d excitations of 5d graviton³

In 5d $(\partial_\mu \partial^\mu - \partial_z^2 + V(z)) h(x^\mu, z) = 0$

decompose into 4d plane waves

$$h(x, z) = e^{-ip \cdot x} \hat{h}(z) \quad \text{of mass in 4d}$$

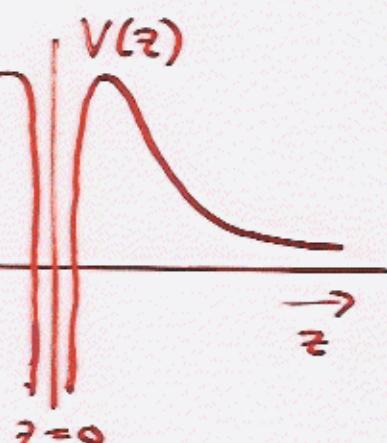
$$m^2 = p^2$$

\hat{h} satisfies

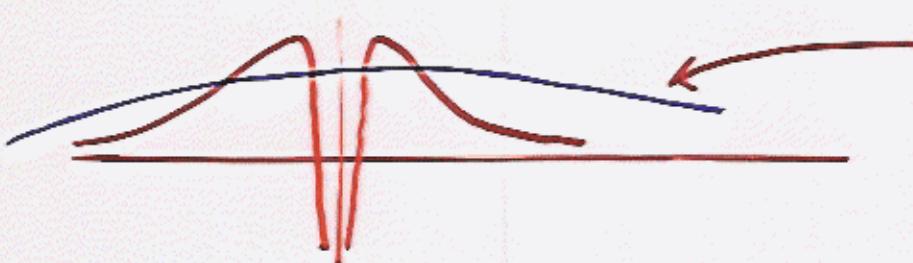
$$\left\{ -\frac{d^2}{dz^2} + V(z) \right\} \hat{h} = -m^2 \hat{h}$$

just like a schrodinger eqn

with $V(z) = \frac{15}{8(1z|+L)^2} - \frac{3\delta(z)}{2L}$

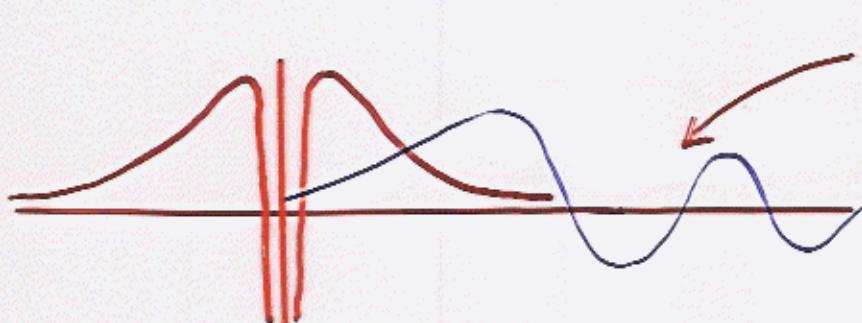


two types of solutions



zero mode $\hat{h}_c(z)$
 $m^2 = 0$ (and spin 2)

massless 4d graviton



massive (spin 2) kink modes
Form a continuum with
 $m^2 > 0$ if $\frac{\pi R}{L} \rightarrow \infty$

• Exchange of $p^2=0$ mode \rightarrow 4d GR

subst. $\hat{h}_0(z)$ into 5d action and match with 4d E-H giving

$$M_{\text{planck}}^2 = M_5^3 L (1 - e^{-\pi R/L})$$



usual 4d Planck

$$\text{mass } G_N = \frac{1}{M_{\text{Pl}}^2}$$

• Note still get sensible 4d gravity in $R \rightarrow \infty$ limit!

"decompactification" limit (RS2) with no IR brane

In this $R \rightarrow \infty$ case exchange of continuum KK modes $p^2 > 0$ (with no mass gap!) leads to power-law modification of 4d grav

$$V_{\text{grav}}(r) = \frac{m_1 m_2}{M_{\text{Pl}}^2 r} \left(1 + \frac{L^2}{r^2} \right) \quad r \gg L$$

harmless if $L \leq 0.1$ mm

and $V_{\text{grav}}(r) = \frac{m_1 m_2}{M_5^3 r^2}$ for $r \ll L$

5d grav. law

For finite R get discrete spectrum of $p^2 > 0$ KK spin 2 modes with mass gap and spacing

$$\Delta m_{\text{KK}} \sim e^{-\pi R/L} M_5$$

The leading modification to 4d grav is the usual Yukawa term

$$V(r) = \frac{m_1 m_2}{M_{Pl}^2 r} \left(1 + \alpha e^{-r/\lambda} \right)$$

$$\text{with } \lambda \sim 1/\Delta m_{\text{KK}} \sim \frac{e^{\pi R/L}}{M_5}$$

What sizes of parameters?

due to red shift mass scales on two branes at $y=0$ and $y=\pi R$ are related by

$$\frac{m_{QFT_1}}{m_{QFT_2}} = \sqrt{\frac{1}{e^{-2\pi R/L}}} = e^{\pi R/L}$$

The idea of Randall and Sundrum was to use this to explain M_{Planck} to m_{weak} hierarchy, thus

$$e^{\pi R/L} \sim 10^{16} \Rightarrow R/L \sim 10$$

(a mild hierarchy leads to a large hierarchy, cf. 'dimensional transmutation' in YM theories $\Lambda_{\text{QCD}} \sim M_{Pl} e^{-8\pi^2/g^2}$)

If this is to explain m_{weak} , the SM (or at least some of it) must be localized on brane #2 — the 'weak brane' or 'IR brane'

→ 'minimal' RS1 puts all L_{SM} on IR brane

What are constraints on RS1?

From our SM perspective

$$L_{\text{eff, grav}} = -T_{\text{SM}}^{\mu\nu} \left\{ \frac{h_{\mu\nu}^{(0)}}{M_{\text{Pl}}} + \sum_n \frac{h_{\mu\nu}^{(n)}}{\Lambda_\pi} \right\}$$

usual graviton coupling coupling to discrete tower of spin 2 KK modes
 $\Lambda_\pi = M_S e^{-\pi R/L}$
 $m_n \approx x_n \frac{1}{M_{\text{Pl}} L} \cdot \Lambda_\pi$
 zeroes of Bessel $J_1(x_n) = 0$
 $x_n \approx 3.8, 7.0, \dots$

So far default values of params

- Spin 2 KK modes have \sim TeV masses and are strongly coupled ($\Lambda_\pi \sim$ TeV too) to IR brane

note : leading order (semiclassical) analysis etc

when 5d curvature small compared to 5d, M_S ,

$$\Rightarrow \frac{1}{M_{\text{Pl}} L} \lesssim 0.1$$

- because of Bessel function suppression at Planck brane ('UV brane'), the KK graviton modes are very weakly coupled to UV brane (ie, Z_{QFT})

What about cosmology/astrophysics?

Most interesting case

RS2



'Planck' UV
brane



brane $\rightarrow \infty$

Does not provide sol'n to hierarchy problem

BUT RS2 is excellent example of

theory of modified gravity in IR (and UV)

- Consequences for cosmology?

Usually have Friedmann eqn

$$\left(\frac{\dot{R}}{R}\right)^2 = \frac{3\rho}{8\pi M_{pl}^2}$$

R = our 4d scale factor

Is this still true?

To find answer useful to think about evolution of R in new way

From perspective of astrophysical/cosmological constraints^{7a}
RSI is pretty safe, as all astro/cosmo physics that is
well understood (excepting super GZK cosmic rays) is
kinematically limited

earliest cosmo epoch directly constrained

$$\text{BBN} \quad T_{\text{BBN}} \sim \text{MeV} \ll \Delta m_{\text{rec}} \text{ in RSI}$$

hottest astro objects

$$\text{red giants, SN's, ...} \quad T_{\text{SN}} \lesssim 50 \text{ keV} \\ \ll \Delta m_{\text{rec}}$$

so no direct limits on RSI from cosmo/astro comparable
to collider constraints

Metric on brane is the induced metric from $G_{IJ}^{(5)}(x, y)$ evaluated at $y = y_{\text{BRANE}}$

— so \dot{R} can be due to motion of brane in a static background!

'mirage cosmology'

Brane motion is determined by Israel junction condition

$$[K_{\mu\nu}] = -\frac{1}{8\pi M_s^3} \left(T_{\mu\nu} - \frac{1}{3} T^\rho_\rho g_{\mu\nu}^{(4)} \right)$$

difference of
extrinsic curvatures
across brane

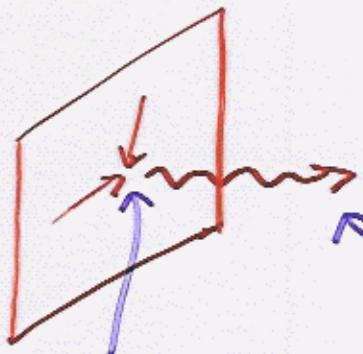
4d energy momentum
tensor of matter on
brane

For AdS bulk position of brane is constant $\rightarrow \dot{R} = 0$

so need a different background for cosmology ...

Can understand correct one on physical grounds 9

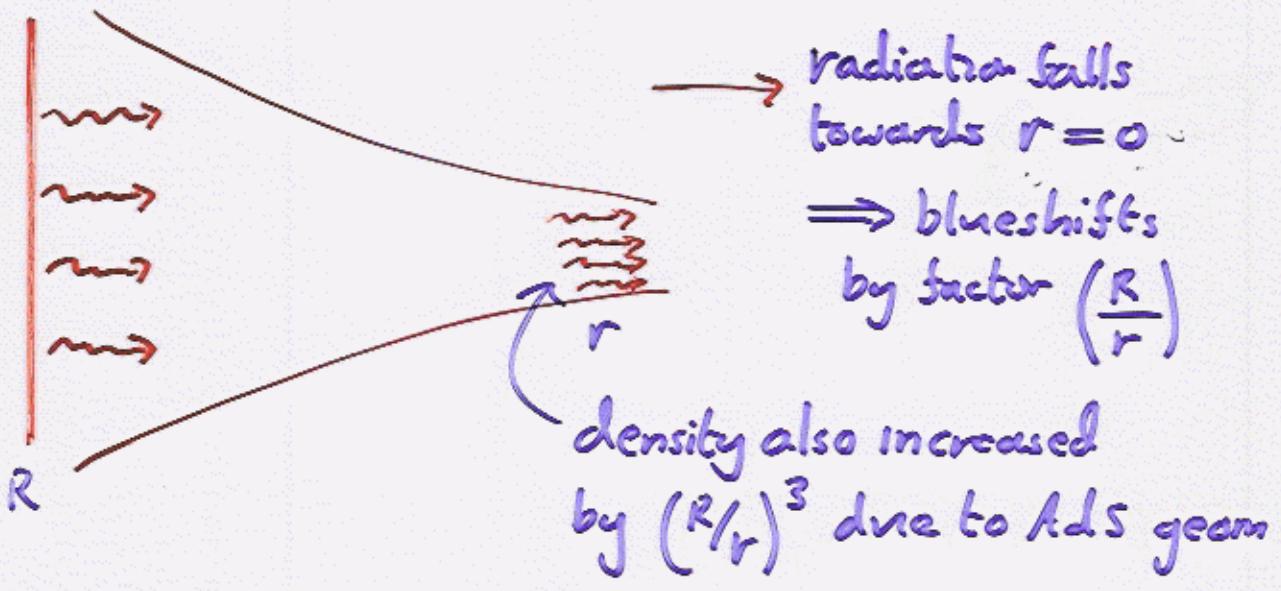
- to get BBN and CMBR, etc, ... our matter must be thermal with temp $T \gtrsim$ few MeV



SM matter
couple (weakly)
to bulk KK gravitons

inevitably get production
of bulk radiation

What happens to this radiation?



$$\rho(r) = \rho(R) \left(\frac{R}{r}\right)^4$$

\Rightarrow no matter how small $\rho(R)$ is, $\rho(r)$ diverges as $r \rightarrow 0$

Eventually a horizon forms around a BH.

Bulk metric is AdS - Schwarzschild

$$ds^2 = f(r) dt^2 - r^2 (d\vec{x})^2 - f(r)^{-1} dr^2$$

$$f(r) = \frac{r^2}{L^2} \left(1 - \frac{r_H^4}{r^4}\right)$$

BH horizon radius

with this background Israel junction condition

$$\Rightarrow \boxed{\frac{3M_{Pl}^2}{8\pi} \left(\frac{\dot{R}}{R}\right)^2 = \rho \left(1 + \frac{\rho L^2}{12M_{Pl}^2}\right) + 3\frac{M_{Pl}^2}{L^2} \left(\frac{r_H}{R}\right)^4}$$

usual FRW RHS (ρ = density of brane matter)

1) NEW ρ^2 term

2) NEW $1/R^4$ term

Friedmann eqn is modified

$r=R(t)$
brane motion

$r=0$ BH singularity
 $r=r_H$ horizon

So cosmology at sufficiently early times is differentⁿ

- both new terms involve mass scale M_c^4

$$M_c^4 = M_{\text{Pl}}^2 / L^2$$

$$(M_{\text{Pl}}^2 = L M_s^3)$$

how low can M_c be?

from sub-mm grav tests know $L \lesssim 0.1 \text{ mm}$

$$\Rightarrow M_c \gtrsim 1 \text{ TeV}$$

- for $T_{\text{SM}} \ll M_c$ have $\rho \sim g_{\text{XSM}} T^4 \ll M_c^4$

and ρ^2 modification to FRW is small

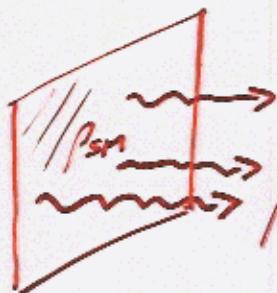
\Rightarrow recover Friedmann eqn, almost...

$$\frac{3M_{\text{Pl}}^2}{8\pi} \left(\frac{\dot{R}}{R} \right)^2 = \rho + 3M_c^4 \left(\frac{r_H}{R} \right)^4$$

$\underbrace{\quad}_{\text{additional 'dark radiation' term}}$
(since $\sim R^{-4}$ behavior)

- interpretation and size of dark rad'n?

the dark rad'n is nothing but the energy radiated to the continuum bulk graviton modes from the hot SM brane!



$$\rho_d \sim R^{-4} \text{ due to AdS geom}$$

Since we know couplings of KK modes to $T_{\mu\nu}^{SM}$
can calculate how much ρ_d

$$\Delta \dot{\rho}_d = \frac{1}{2} \langle \sigma \cdot V_{rel} \cdot E \rangle n_{SM}^2$$

$$\sigma_i(s) = C_i \frac{\sqrt{s}}{M_s^3}$$

$$C_S = \frac{1}{12}, \quad C_F = \frac{1}{16}, \quad C_V = \frac{1}{4}$$

$$= \frac{2 T^8}{S \pi^4 M_s^3} (C_i K_i^2) \Gamma(\frac{3}{2}) \Gamma(\frac{9}{2}) \tilde{\gamma}(\frac{7}{2}) \tilde{\gamma}(\frac{9}{2})$$

↑
of spin dof for sm particle
type i

Can integrate this eqn to find

$$\Omega_d \equiv \frac{\rho_d}{\rho + \rho_d} = \int_{\tau_i}^{\infty} d\tau \left(\frac{\dot{\rho}_d}{\rho} \right)$$

$$= \frac{15C}{g_* \pi^2} \left(\frac{3\rho_i}{M_c^4} \right)$$

time at which evolution starts (reheating of our brane)

$$C \approx 0.112$$

Maximum value from linear-in- ρ period ($\rho_i < 12 M_c^4$)

$$\Omega_d = \frac{90C}{g_* \pi^3}$$

note suppressed by $g_{*,SM}$

and at BBN epoch using const. entropy for bulk modes

$$\Omega_{d,BBN} = \left(\frac{g_{*,BBN}}{g_*} \right)^{1/3} \Omega_d$$

$$g_{*,BBN} = 10.75$$

$\lesssim 0.0044$ # of light dof in SM at BBN

So $\Omega_{dark} < 0.5\%$ and well within limits at present

- this result good for RS2!

also allows for cosmological ρ^2 period

during ρ^2 period (when ρ^2 dominates) Friedmann eqn becomes

$$6M_s^3 H = \rho$$

and integration of Ω_d different

$$\Omega_{d,\rho^2} = \int_{\tilde{\tau}_0}^{\tilde{\tau}_1} d\tilde{\tau} \left(\frac{d\rho}{\rho} \right)$$

end of ρ^2 epoch when
 $\rho = 12M_c^4$

time at start of
 ρ^2 epoch, $\rho_0 \lesssim M_s^4$

$$\approx \underbrace{\frac{90C}{g_* \pi^2}}_{\text{previous result}} \cdot \frac{1}{3} \ln \left(\frac{M_{pl} L}{12^{3/2}} \right) \cdot \alpha$$

enhancement!

kinematic factor
due to brane motion
 $\alpha \gtrsim 5\pi/32$

$$\Rightarrow \boxed{\Omega_{d, \text{BBN}} \approx 0.05}$$

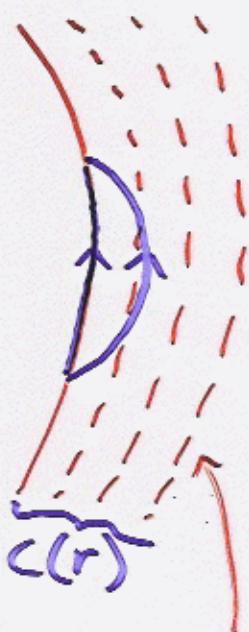
interesting # - on edge of testability w/
BBN / WMAP data!

Lorentz-symmetry violation

We've learnt that AdS-Schwarzschild is correct background for cosmo evolution

$$ds^2 = \frac{r^2}{L^2} \left(1 - \frac{r_H^4}{r^4}\right) dt^2 - \frac{L^2}{r^2} \left(1 - \frac{r_H^4}{r^4}\right)^{-1} dr^2 - \frac{r^2}{L^2} (d\vec{x})^2$$

a consequence is Lorentz violation



$$\text{Local speed of light} \\ = \left(1 - \frac{r_H^4}{r^4}\right)^{1/2}$$

Since varies with r , quanta that propagate in r have Lorentz-violating Lef

each const r surface
is Lorentz-inv w/ speed
of light $c(r)$

Some papers have claimed that this could be used to explain super GZK events...

Let's see why not true!

Holography and AdS/CFT correspondence

To understand cosmology in RS2 very useful to use a very different description

there exists a purely 4d formulation (no 5th dimension!)

usual 4d gravity coupled to
a 4d conformal field theory (CFT_4)

$$S_{\text{eff}} = \int d^4x \sqrt{-g} R(g) + \int d^4x L_{\text{CFT}} \sqrt{-g} + \dots$$

- a trivial example of a CFT_4 is QED below the electron mass
- more usefully, QCD above Λ_{QCD} is almost a CFT (almost scale invariant) if quark masses ($m_{\text{top}}, \text{etc...}$) are set to zero

Dictionary for AdS/CFT

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5d description

continuum of KK
bulk graviton modes



4d CFT description

continuum of CFT
states (e.g., glueballs in limit
 $\Lambda_{\text{QCD}} \rightarrow 0$)

zero mode graviton \longleftrightarrow usual 4d graviton

position in 5th dimension \longleftrightarrow energy scale (size) of
excitation in CFT

$y \rightarrow 0$ \longleftrightarrow UV of CFT

$y \rightarrow \infty$ \longleftrightarrow IR of CFT

Bulk functional
integral \longleftrightarrow CFT+4d grav functional S_{4d}
with source $\hat{\phi}(x)$

$$\left[[d\varphi(x,y)] e^{-S_{\text{EH}}[\varphi(x,y)]} \right]_{\varphi(x,0)=\hat{\phi}(x)} \quad Z[\hat{\phi}(x)]$$

L, M_s parameters



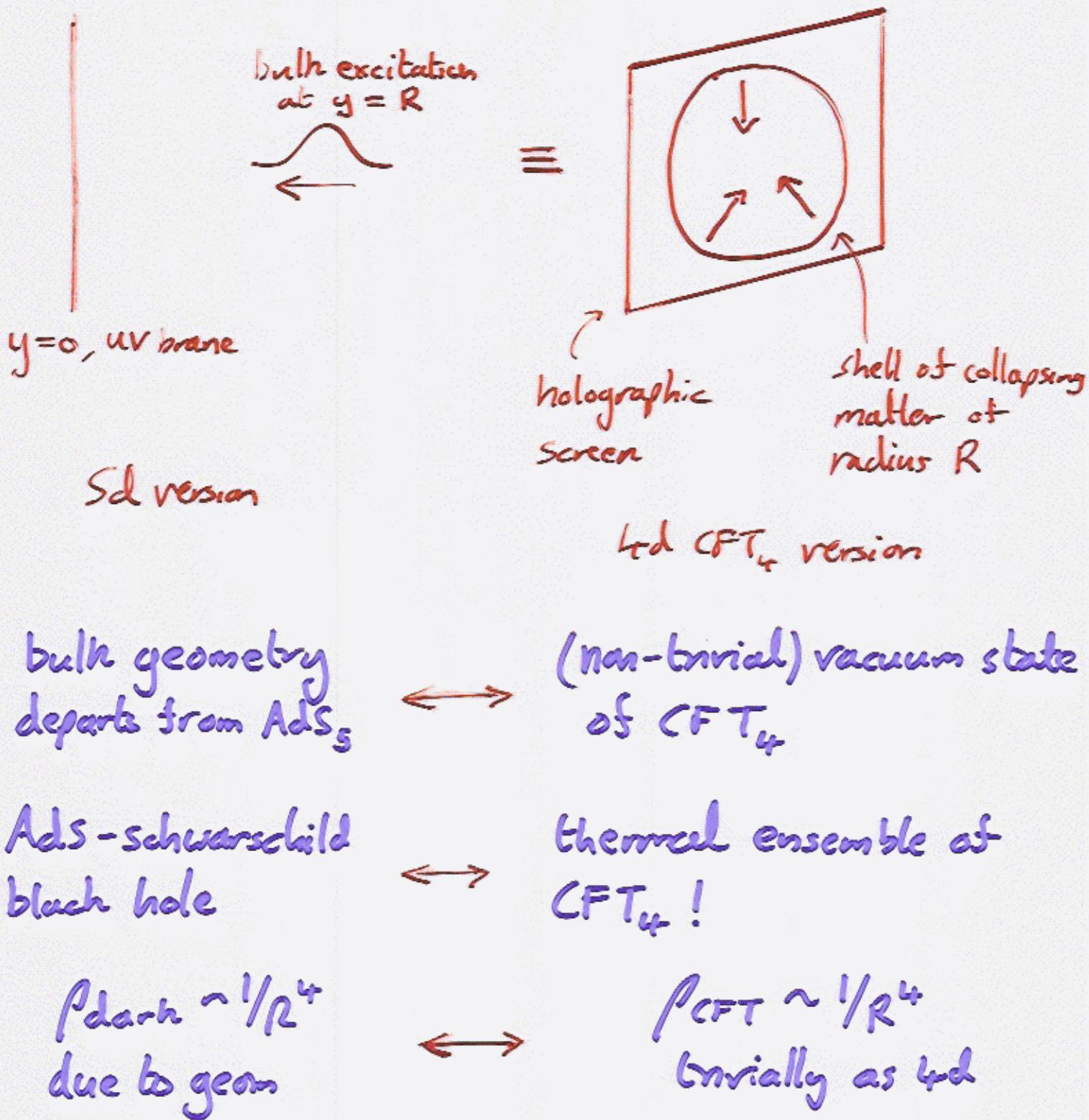
$$M_{\text{pl}} = \sqrt{L M_s^3}$$

$$(C) = 2\pi^2 (M_s L)^3$$

'central charge' of CFT

\sim # of dof of CFT
 $\lesssim 10^{62} !!$

A picture



We can now understand Lorentz symm violation
of Sd picture fully... 19

AdS-Schwarzschild
geom of Sd \longleftrightarrow thermal state of
 CFT_4 (the 'dark
radiation')

CFT_4 thermal state defines a preferred
rest frame (\nLeftarrow CMBR!)

Interactions of states with this thermal
state 'break' Lorentz symm in trivial (IR)
way

of interactions of SM particles or
greater with CMBR

Lorentz symm violation is harmless as

- 1) $P_{\text{dark}} \ll P_{\text{CMBR}}$
- 2) CFT_4 only couples via gravity to SM
- much weaker than ∂ -SM coupling

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We can also understand ρ^2 term in new Friedmann eqn ...

4d grav + CFT action is more precisely

$$S_{4d}[g_4, \varphi] = \int d^4x \sqrt{g_4} \left\{ -\frac{1}{2} M_{Pl}^2 R[g_4] \right.$$

$$\left. - b_4 \left(R^2/24 - R^{\mu\nu} R_{\mu\nu}/8 \right) + \dots \right\}$$

This higher-derivative term
is calculable and is due to
internal anomaly ($\sim c$)
of CFT

b_4 term leads to ρ^2 correction

leading behavior $H^2 \sim \rho/M_4^2$

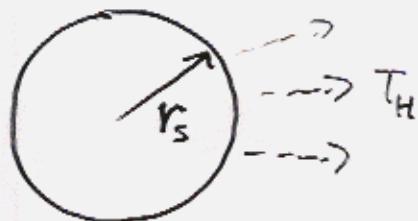
$$\Rightarrow b_4 \left(R^2/24 - \dots \right) \sim c H^4 \sim \frac{L^2 \rho^2}{M_4^2} \sim \frac{\rho^2}{M_c^4} \checkmark$$

ρ^2 term is due to (very large if) CFT dof.
giving anomalously large 4-derivative terms in
Lagr.

Modification of properties of astrophysical BHs

The very large # of CFT dof has an important effect on BH evolution

In 4d grav + CFT_4 picture clear that usual Schwarzschild BH exists



and Hawking evaporates with temp T_H

But now have additional

$$c \sim 10^{62} \left(\frac{0.1 \text{ mm}}{L} \right)^2 \text{ modes!}$$

\Rightarrow BH lifetimes much reduced (naively)!

$$\tau \sim \frac{10^{64}}{c} \left(\frac{M}{M_\odot} \right)^3 \text{ yr}$$

$$\sim \left(\frac{M}{M_\odot} \right)^3 \left(\frac{0.1 \text{ mm}}{L} \right)^2 10^2 \text{ yr}$$

AdS/CFT₄ map is valid for sufficiently large BHs

$$\frac{M}{M_{\text{Pl}}^2} \geq L \quad (r_s \geq L)$$

$$\Rightarrow M \geq 10^{-8} \left(\frac{L}{0.1 \text{ mm}} \right) M_\odot$$

well within range of astro BHs.

Thus, ignoring accretion, BH survival (and even more so ang. mom'm) puts stringent limits on L.

However accretion important and best estimate of limits including this are

$$L \leq 1.3 \times 10^{-2} \text{ mm} \quad (\text{conservative})$$

$$\leq 2.4 \times 10^{-4} \text{ mm} \quad (\text{if old BH exist})$$

$$\leq 10^{-4} \text{ mm} \quad (\text{non-zero ang. mom'm of BH})$$

Conclusions

only covered one v. small area of extra-dimensional cosmo/astro physics

- → there is a VAST amount more to say!
- → already clear that much fascinating and important phenomena can be studied
- → astro/cosmo provide most stringent limits on standard (ADD) large extra dimension scenarios (and strongly disfavor them)
- → An exciting time in theoretical/experimental physics!

HAVE FUN!

