Kinematics of the Universe

- *Robertson-Walker metric*
- *Metrical Tests*
- *Distance and Volume*
- *Time*
- *Velocity*
- *Acceleration*
- *Jerk*

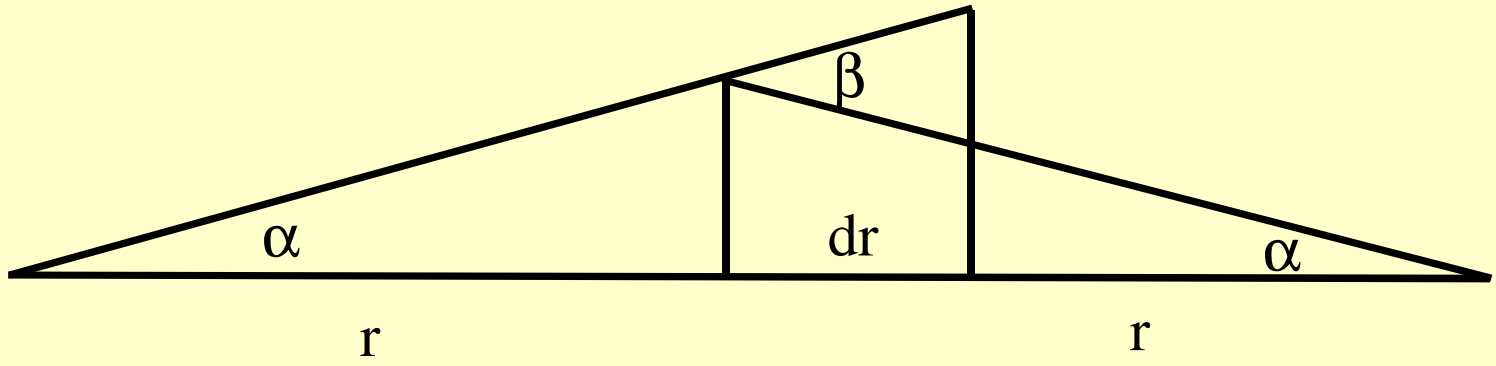
Fundamentals

- *Isotropy => homogeneity*
- *Cosmic time t*
- *Scale factor $a(t_0)=1$*
- *Symmetry=> Robertson-Walker metric*

$$ds^2 = dt^2 - a(t)^2 [dr^2 + d(r)^2 (d\theta^2 + \sin^2\theta \, d\phi^2)]$$

- *"radius" r*
- *"(Comoving angular diameter) distance" d(r)*

Maximally Symmetric 3-spaces



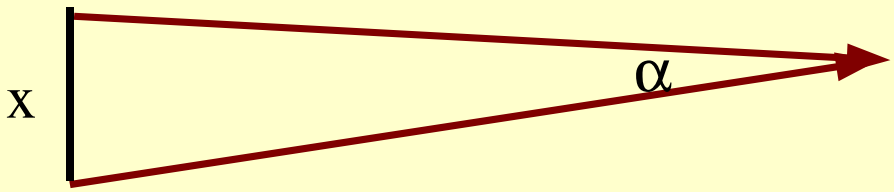
$$d(r+dr)\alpha - d(r-dr)\alpha = dr\,\beta$$

$$d(2r)\alpha = d(r)\beta$$

$$d(2r) = 2d(r)d'(r)$$

- => $d(r)=r$, $\mathcal{R}_0 \sin(h)r/\mathcal{R}_0$.

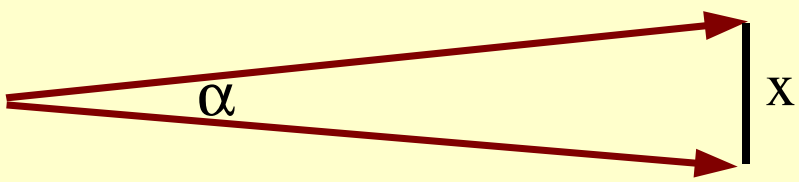
Distances



Angular diameter distance

$d_A = x(\text{then})/\alpha(\text{now})$

$= d\ a(t)$



Luminosity distance

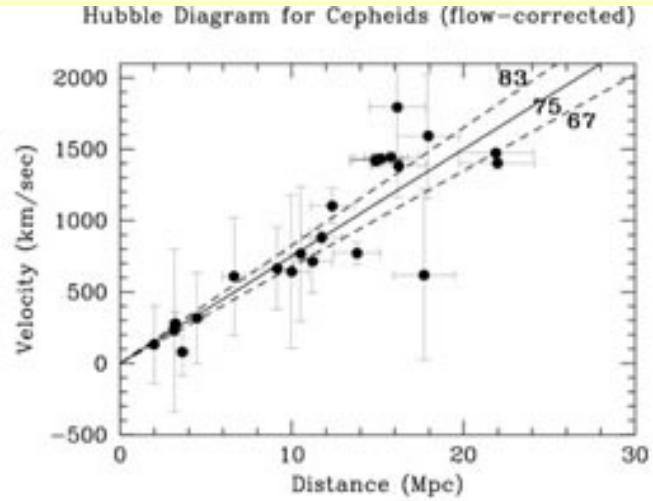
$d_L = [L(\text{then})/4\pi F(\text{now})]^{1/2}$.

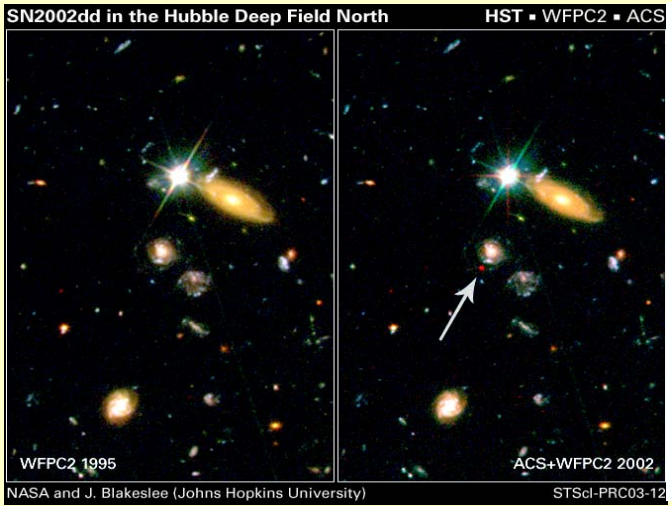
Allow for redshift of photon energy, rate

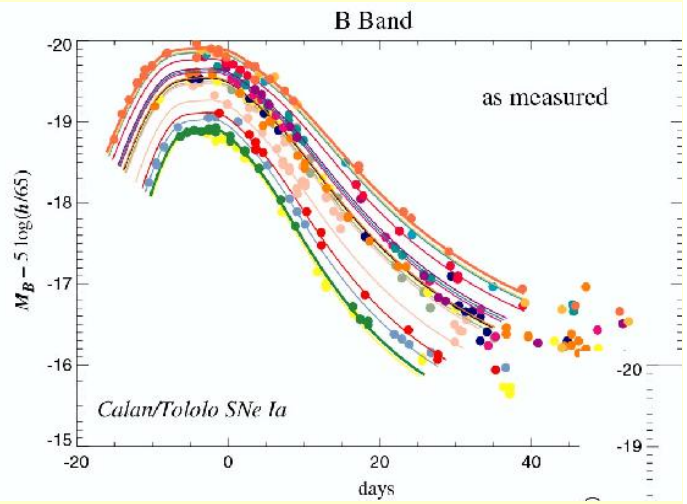
$d_L = d/a(t)$

Distance Measurement

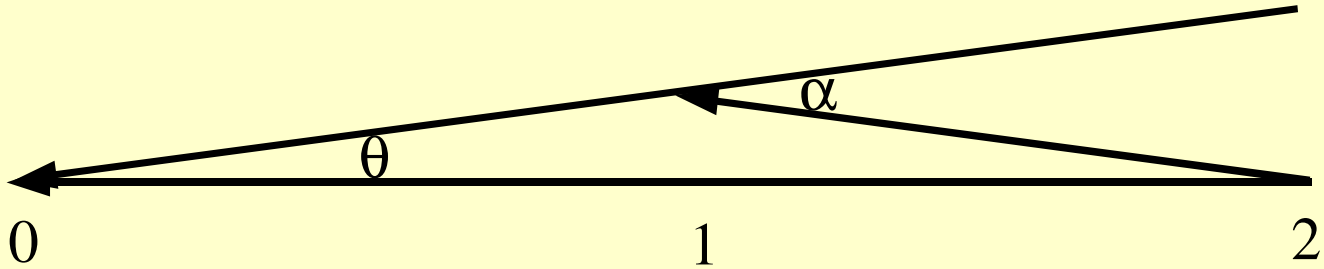
- Parallax(Hipparcos) to 100pc
- LMC main sequence fitting and SN1987a rings– 50kpc
- Virgo cluster etc Cepheids ($L \sim P^{1.3}$)15 Mpc
- Fisher-Tully for spirals ($L \sim V_c^4$)
- Type Ia SN, $L \sim 5\ 10^9\ L_o$; correct for rate of decline
- Type II supernova; modified black body formula with $R=Vt$
- Surface brightness fluctuations **Systematic Errors!**



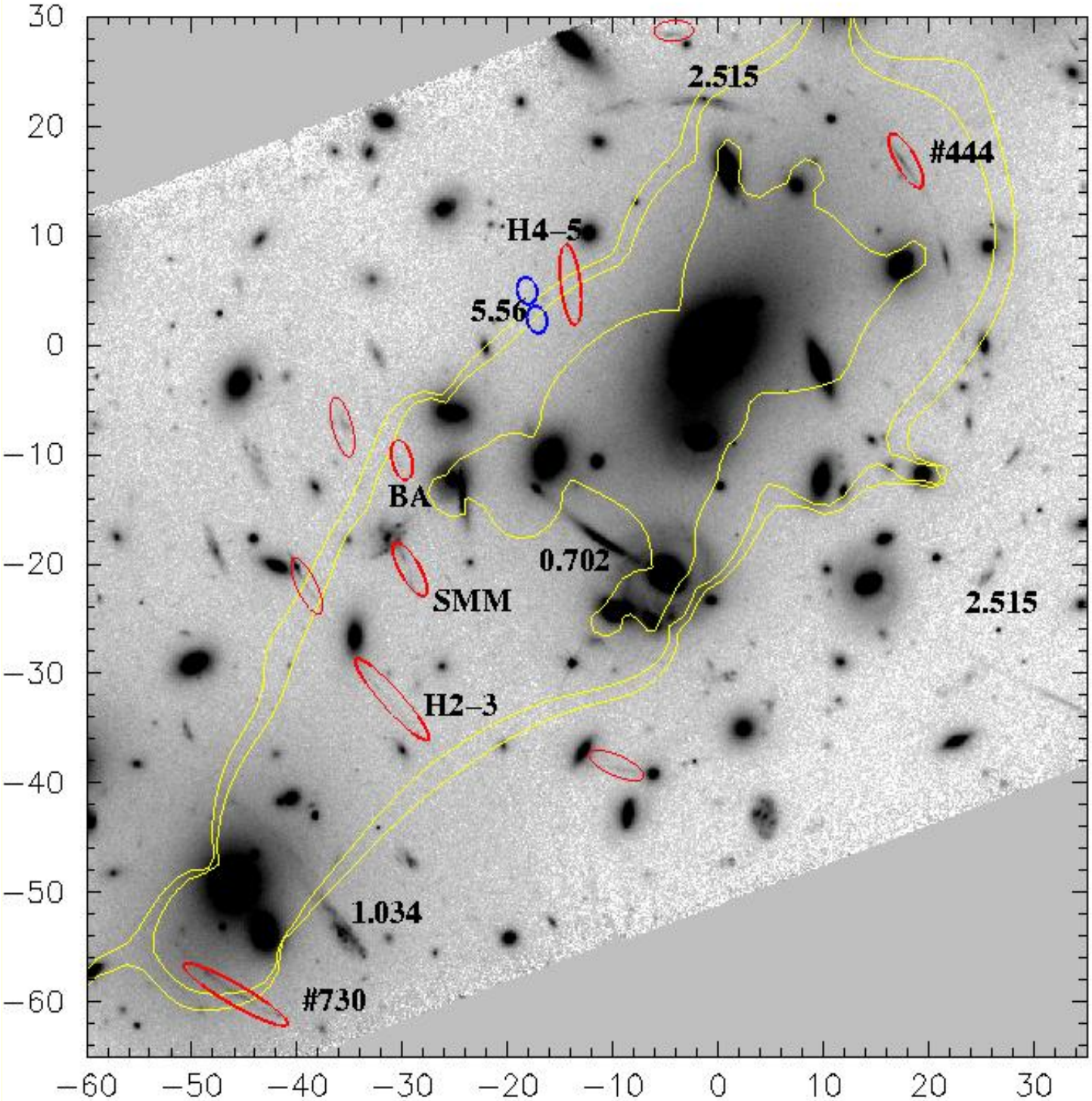




Surveying with Lenses



- $\theta = \alpha d_{12}/d_{02} = \alpha \mathcal{R}_{12}$.
- *With many multiple images can solve for* $\alpha(\theta)$
- *With many souce redshifts can obtain* $\mathcal{R}_{12}(z_2)$
- *With many lenses can obtain* $\mathcal{R}_{12}(z_1, z_2)$



- *A2218*

(Kneib et al)

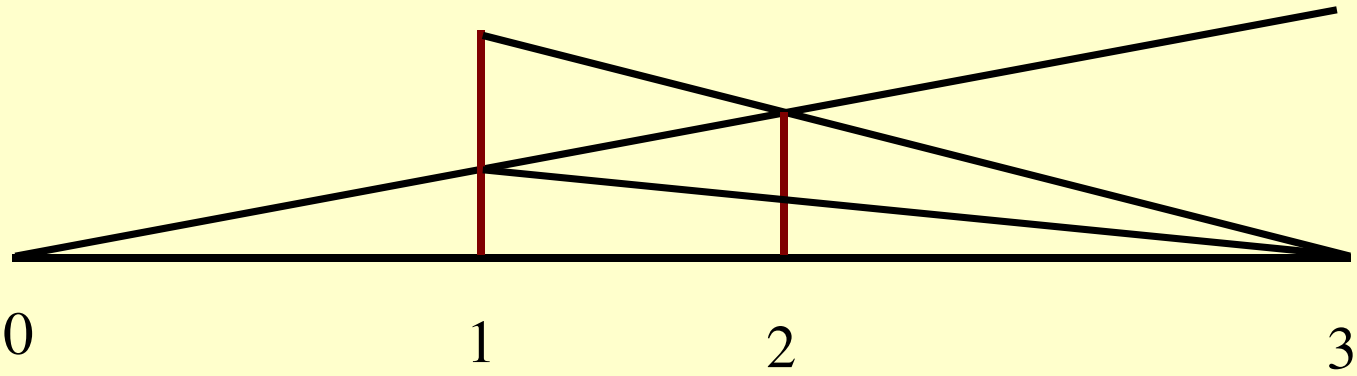
~100 sources

~10 redshifts

Accurate astrometry and redshifts

Limited by model accuracy

Metrical relationships



- *General relation for isotropic space*

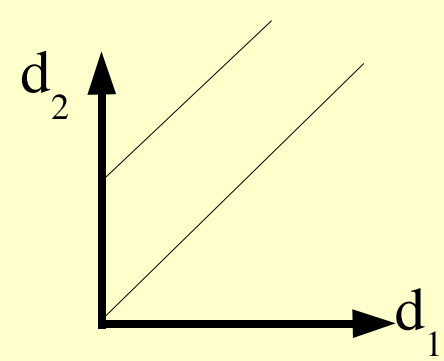
from lenses

$$R_{12} + \frac{d_{01}}{d_{02}} R_{23} = R_{13}$$

from supernovae

- *Relation for flat space*

$$R_{12} + \frac{d_{01}}{d_{02}} = 1$$



Comoving Volume

$$V = \Delta\,\Omega \int dr\; d^2$$

for flat space

$$V = \Delta\,\Omega\, d^3 / 3$$

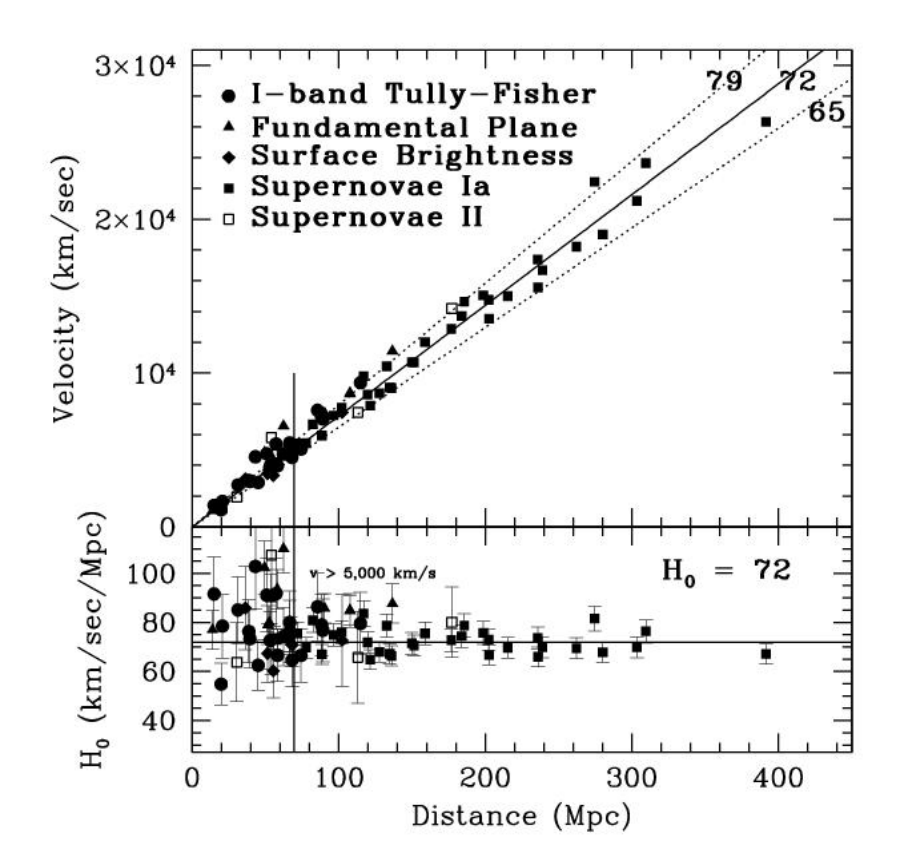
Relate to kinematics using dr=dt/a

Age of the Universe

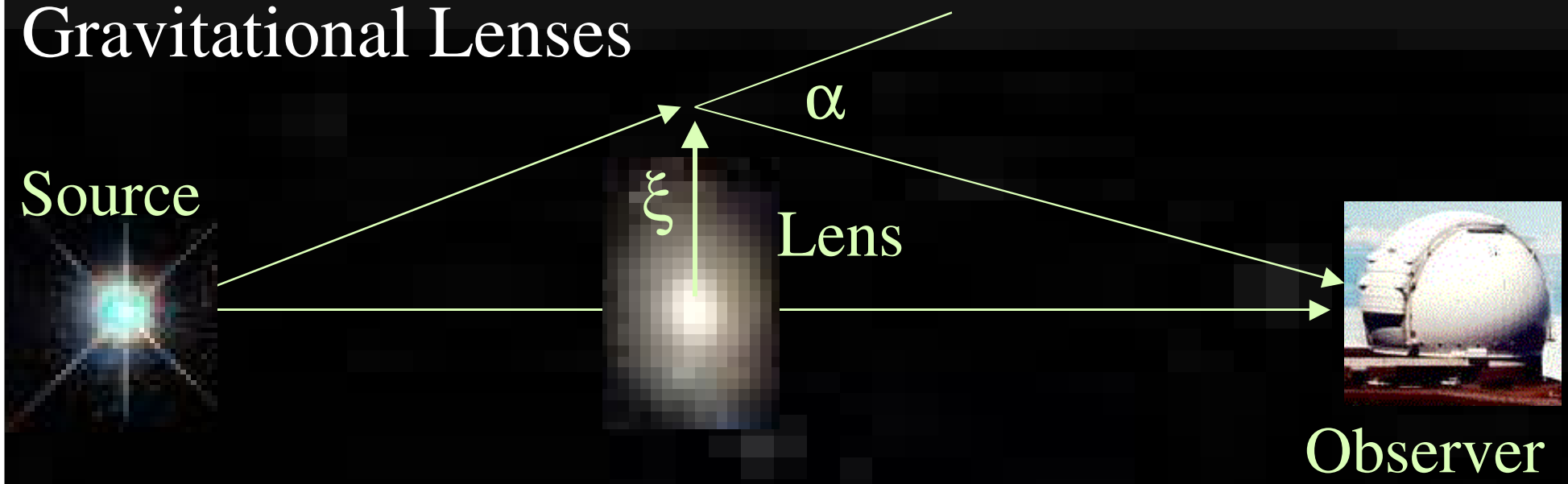
- *Radioactive dating using isotope pairs*
- *Stellar evolution in globular clusters*
- *White dwarf cooling*
- *Produces lower bounds but now consistent with indirect methods*

Velocities

- Define $\mathcal{H}(t)=a'/a = a'_{now}$
- $a=1-z+\ldots=1-\mathcal{H}_0(t_0-t)+\ldots \Rightarrow r=t_0-t=z/\mathcal{H}_0$. Hubble law



Gravitational Lenses



Time delay (relative to unperturbed ray)

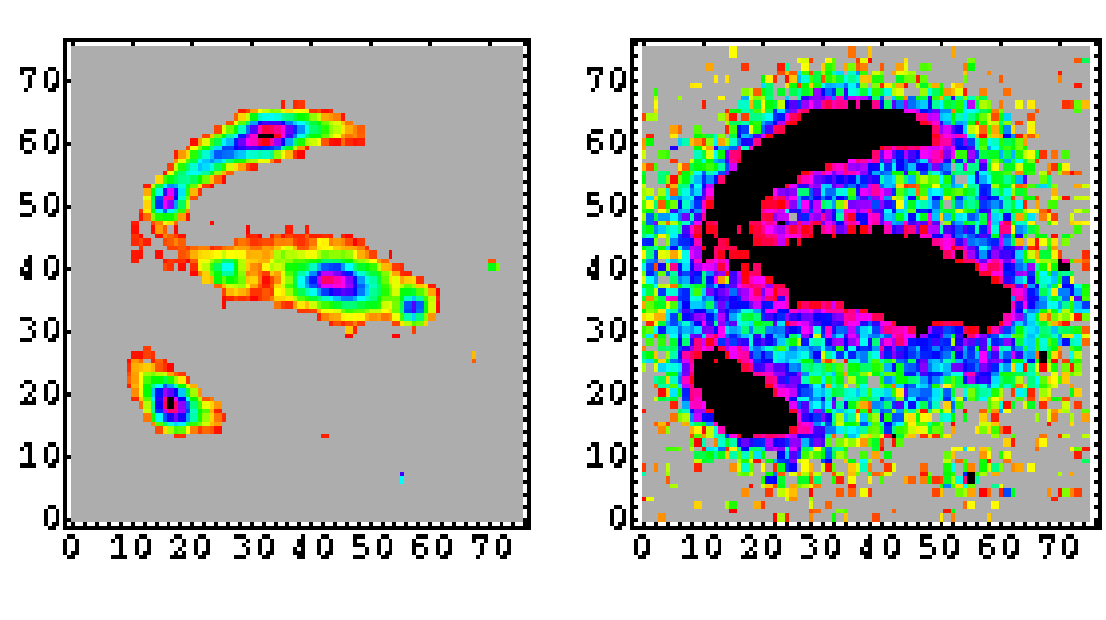
= geometric delay + gravitational delay

$= (1 + z) [\xi . \alpha / 2 c - \psi / c^3]$,

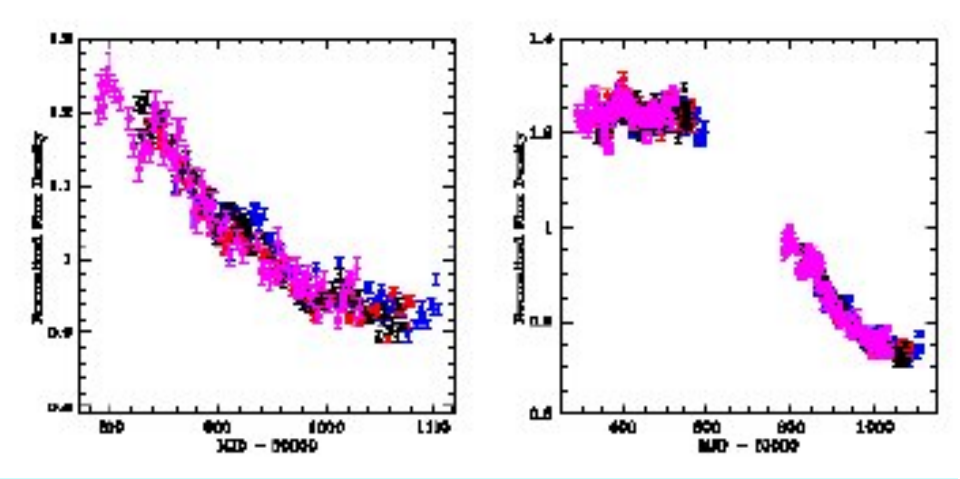
along each perturbed ray.

Time ~ Age of Universe x α^2 ~ 1 yr ~ H_0^{-1}

Frequency of occurence ~ Ω_{gal} ~ 0.003



B1608+656



Z_d = 0.63 E lens + merging companion

Z_s = 1.394 E+A radio source

$t_{A,C,D}$ = 26, 33, 73d; $\mu_{A,C,D}$ =2, 1, 0.35

Accuracy of H_0 limited by model

Acceleration

- $q=-aa''/a^2$
- *Flat space* $d = ad_L = r = \int \frac{dt}{a} = H_0^{-1}[z-(1-q_0/2)z^2+\ldots]$

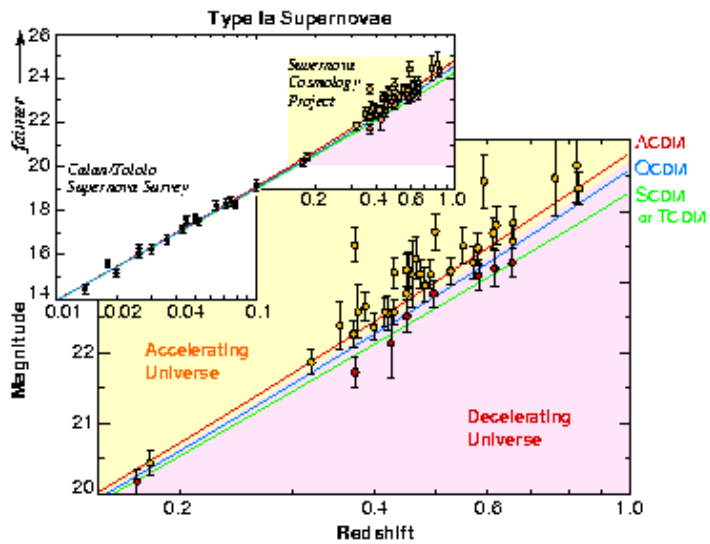


Figure 5: **Supernovae as standard candles**: the relation of observed brightness (in logarithmic units of "magnitude") vs. red shift for Type Ia

Kinematics Summary

- *Concordance of local kinematics with microwave background measurements*
- *Gravitational lenses provide excellent tool for surveying space but utility limited by model accuracy*
- *Natural kinematic models for a(t) involve* $j=a'''a^2/a'^3$
  - ΛCDM has j=1 and, in absence of credible dynamical model fit observations to j