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THE MUON DECAY AT THE TWO LOOP LEVEL OF THE ELECTROWEAK INTERACTIONS

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ABSTRACT

The progress in the evaluation of the muon decay lifetime is reviewed. The electroweak bosonic corrections are given together with the respective shift of the W boson mass. After inclusion of the recently corrected fermionic contributions, this requires an updated fitting formula for the prediction of M_W.[1]

1 Introduction

The muon lifetime is one of the key observables of today's particle physics. Not only is it measured very precisely, since the current experimental error is 18 ppm, but can be described to competing accuracy within the Standard Model, giving rise to a strong correlation between the masses of the heavy gauge bosons. As a low energy process, the decay is expected to be governed by an effective interaction involving only the electron, muon and their respective neutrinos. The dynamics of the system should be corrected mostly by QED, whereas the electroweak interactions determine solely the size of the coupling constant.

The calculation at the two loop level has been completed recently with the evaluation of the electroweak bosonic corrections [1]. Subsequently, the fermionic part, which was first obtained in [2] has been recalculated [3]. A difference between

[1]The results presented in this contribution were obtained in collaboration with M. Awramik.

the two results has been found and the original result [2] corrected. There is now agreement on all parts of the two loop contributions.

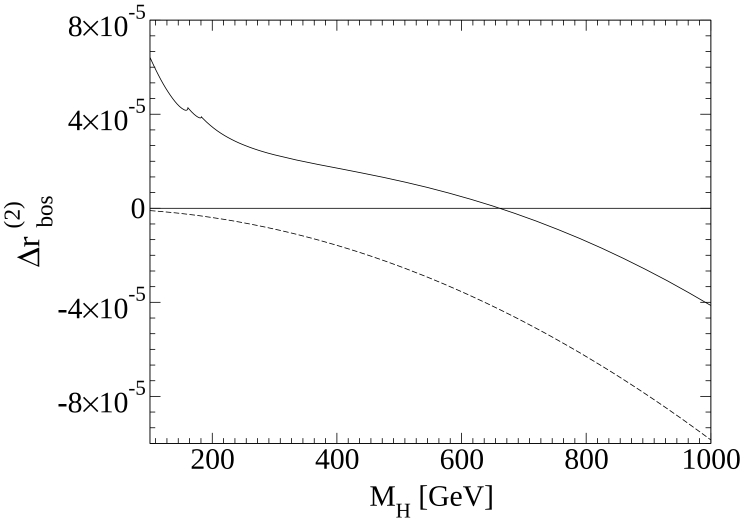

Figure 1: *The complete bosonic corrections (solid line) together with the leading term (dashed line) in the large Higgs boson mass expansion.*

2 Two loop contributions

The main complication in the calculation of the bosonic contributions is connected with the separation of the QED corrections to the Fermi model. This problem has been solved with the use of the effective theory language, where the full lagrangian

$$\mathcal{L}_{\rm SM} = \mathcal{L}_{\rm SM}(\alpha, \alpha_s, m_\nu, m_l, m_q, M_W, M_Z, M_H, m_t, V_{\rm MSN}, V_{\rm CKM}), \tag{1}$$

has been replaced with the Fermi lagrangian

$$\begin{aligned} \mathcal{L}_{\rm eff} &= \mathcal{L}_{\rm kin}(\nu) + \mathcal{L}_{\rm QED}(\alpha^0, m_l^0, m_q^0, l^0, q^0, A_\mu^0) + \mathcal{L}_{\rm QCD}(\alpha_s^0, m_q^0, q^0, A_\mu^{a,0}) \\ &+ \frac{G_F}{\sqrt{2}}\ \overline{e^0}\gamma^\alpha(1-\gamma_5)\mu^0 \times \overline{\nu_\mu}\gamma_\alpha(1-\gamma_5)\nu_e, \end{aligned} \tag{2}$$

and the amputated renormalized Green functions in both theories have been required to be equal up to higher order terms in an expansion in the W boson mass

$$\mathcal{G}_{\rm SM} = \mathcal{G}_{\rm eff} + \mathcal{O}(1/M_W^4). \tag{3}$$

The above *matching equation* has the advantage that one can take any suitable masses and momenta of the light particles and subsequently expand in them. For the calculation, all of these parameters were put to zero and no expansion was necessary. This lead to problems with the Dirac gamma matrix structure in the four-fermion operators, which were solved by a suitable projection operator. However, the procedure has also been tested with fermion masses as infrared regulators and agreement was found.

The result for the pure bosonic contributions is given in Fig. 1. The impact of this class on the prediction of the W boson mass as well as that of the fermionic part is summarized in Fig. 2.

	$\Delta r^{(\alpha)} + \Delta r^{(\alpha^2)}_{\text{ferm}}$	$+ \Delta r^{(\alpha\alpha_s)} + \Delta r^{(\alpha\alpha_s^2)}$		$+\Delta r^{(2)}_{\text{bos}}$	
M_H [GeV]	M_W [FHWW] [GeV]	M_W [GeV]	ΔM_W [MeV]	M_W [GeV]	ΔM_W [MeV]
100	80.3771	80.3758	-1.3	80.3747	-2.4
200	80.3338	80.3326	-1.2	80.3321	-1.7
600	80.2521	80.2509	-1.2	80.2508	-1.3
1000	80.2135	80.2122	-1.3	80.2129	-0.6

Figure 2: *The impact of the bosonic and the corrected fermionic contributions on the prediction of the W boson mass. The second column gives the result from [2].*

3 Acknowledgments

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