Proceedings of Institute of Mathematics of NAS of Ukraine 2004, Vol. 50, Part 2, 593–600

Hamiltonian of Superstring and Super p-Branes with Enhanced Supersymmetry

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The Hamiltonian and covariant generators of κ-symmetry and extra bosonic gauge symmetries of superstring and super p-branes preserving 3/4 of the $D=4$ $N=1$ supersymmetry are presented.

1 Introduction

As shown in [1] exotic BPS states preserving $\frac{\mathsf{M}-1}{\mathsf{M}}$ fraction of $N=1$ supersymmetry can be realized by static configurations of free tensionless super p-branes ($p=1,2,\ldots$) with the action linear in derivatives[1]. These static configurations were described by general solutions of the equations of motion of super p-branes evolving in superspace extended by tensor central charge (TCC) coordinates. Because of the $OSp(1|2\mathsf{M})$ global symmetry of the model, its static p-brane solution was formulated in terms of supertwistor previously used to formulate superparticle models [3–5] and forming a subspace of the $Sp(2\mathsf{M})$ invariant symplectic space [6, 7]. As a result, the static form of the discussed supertwistor representation of the BPS brane solution is not static in terms of the original superspace-time and TCC coordinates. It is static only modulo transformations of enhanced κ-symmetry and its accompanying local symmetries, since the supertwistor components are invariant under these gauge symmetries, as shown in [8]. In the bosonic sector the unphysical p-brane motions related to the gauge symmetries were geometrically realized as the Abelian shifts [8] of the space-time and TCC coordinates by the Lorentz bivectors (generally multivectors) generalizing vector light-like Penrose shifts of the standard space-time coordinates [9]. Being inessential on the classical level of consideration, these shifts may turn out to be essential in the quantum dynamics of strings and branes. This necessitates quantum treatment of the model [1] in the original variables that belong to the superspace extended by TCC coordinates and auxiliary spinor fields. Interest to this problem is stimulated by a conjectured relation of the tensionless strings with higher spin theories and free conformal SYM theories [10, 11, 7], as well as by the presence of higher spin states in the quantized $OSp(1|2\mathsf{M})$ invariant model of superparticle [12].

In this talk we start study of the formulated problem on the example of $D=4$ $N=1$ super p-brane model [1] and consider its Hamiltonian description, based on the covariant approach to the first- and second-class constraints division [13, 14], correlating with the description [15] in the fixed light-cone gauge. The covariant first-class constraints which are generators of the local symmetries of the model, as well as, its Hamiltonian are presented. Further development of the results towards the exploration of quantum dynamics and symmetries of the string/brane model is discussed.

[1]New Wess–Zumino like super p-brane models nonlinear in derivatives and preserving $\frac{\mathsf{M}-1}{\mathsf{M}}$ fraction of supersymmetry were recently proposed in [2].

2 Lagrangians for strings and branes with enhanced supersymmetry and symplectic twistor

A new simple model [1] describes tensionless strings and p-branes spreading in the symplectic superspace $\mathcal{M}^{\rm susy}_{\sf M}$. For $\mathsf{M} = 2^{[\frac{D}{2}]}$ ($D = 2, 3, 4$ mod 8) this superspace naturally associates with D-dimensional Minkowski space-time extended by the Majorana spinor θ_a ($a = 1, 2, \ldots, 2^{[\frac{D}{2}]}$) and the tensor central charge coordinates z_{ab} additively unified with the standard space-time coordinates $x_{ab} = x^m(\gamma_m C^{-1})_{ab}$ in the symmetric spin-tensor Y_{ab}. The supersymmetric and reparametrization invariant action of the model [1]

$$S_p = \frac{1}{2}\int d\tau d^p\sigma\ \rho^\mu U^a W_{\mu ab} U^b \tag{1}$$

includes the world-volume pullback

$$W_{\mu ab} = \partial_\mu Y_{ab} - 2i(\partial_\mu\theta_a\theta_b + \partial_\mu\theta_b\theta_a),$$

of the supersymmetric Cartan differential one-form $W_{ab} = W_{\mu ab}d\xi^\mu$, where $\partial_\mu \equiv \frac{\partial}{\partial\xi^\mu}$ and $\xi^\mu = (\tau, \sigma^M) \equiv (\tau, \vec\sigma)$, $(M = 1, 2, \ldots, p)$ are world-volume coordinates. The local auxiliary Majorana spinor $U^a(\tau, \sigma^M)$ parametrizes the generalized momentum $P^{ab} = \rho^\tau U^a U^b$ of the tensionless p-brane and $\rho^\mu(\tau, \sigma^M)$ is the world-volume vector density providing the reparametrization invariance of S_p. This action has $(\mathsf{M} - 1)$ κ-symmetries and consequently preserves $\frac{\mathsf{M}-1}{\mathsf{M}}$ fraction of the original global supersymmetry.

By the generalized Penrose transformation of variables

$$Y_{ab}U^b = i\tilde Y_a + \tilde\eta\theta_a, \qquad \tilde\eta = -2i(U^a\theta_a), \tag{2}$$

where $\tilde\eta$ is real Goldstone fermion associated with the spontaneous breakdown of $\frac{1}{\mathsf{M}}$ supersymmetry, the differential one-form $U^a W_{ab} U^b$ is presented as

$$U^a W_{ab} U^b = i\{U^a d\tilde Y_a - dU^a \tilde Y_a + d\tilde\eta\tilde\eta\} \equiv dY^\Lambda G_{\Lambda\Sigma} Y^\Sigma. \tag{3}$$

The new object $Y^\Lambda = (iU^a, \tilde Y_a, \tilde\eta)$ in (2), (3) is $OSp(1|2\mathsf{M})$ supertwistor and $G_{\Lambda\Sigma} = (-)^{\Lambda\Sigma+1}G_{\Sigma\Lambda}$ is $OSp(1|2\mathsf{M})$ invariant supersymplectic metric

$$G_{\Lambda\Sigma} = \begin{pmatrix} \omega^{(2\mathsf{M})} & 0 \\ 0 & i \end{pmatrix} = \begin{pmatrix} 0 & -\delta_a{}^b & 0 \\ \delta^a{}_b & 0 & 0 \\ 0 & 0 & i \end{pmatrix},$$

which is the supersymmetric generalization of $Sp(2\mathsf{M})$ symplectic metric $\omega^{(2\mathsf{M})}$. In view of (2) and (3), the action S_p (1) is presented in the supertwistor form

$$S_p = \frac{1}{2}\int d\tau d^p\sigma\ \rho^\mu \partial_\mu Y^\Lambda G_{\Lambda\Sigma} Y^\Sigma \tag{4}$$

that is apparently invariant under global $OSp(1|2\mathsf{M})$ symmetry. For the particular case of $D = 11$ the action (4) is invariant under $OSp(1|64)$ generalized superconformal symmetry [16].

The original action (1) is invariant under $(\mathsf{M} - 1)$ κ-symmetries since the transformation parameter $\kappa_a(\tau, \vec\sigma)$ is restricted by only one real condition

$$U^a\kappa_a = 0, \tag{5}$$

as it follows from the transformation rules of the primary variables

$$\delta_\kappa\theta_a = \kappa_a, \qquad \delta_\kappa Y_{ab} = -2i(\theta_a\kappa_b + \theta_b\kappa_a), \qquad \delta_\kappa U^a = 0. \tag{6}$$

It is easy to show [8] that all components of the supertwistor $Y^\Lambda = (iU^a, \tilde{Y}_a, \tilde{\eta})$ are invariant under κ-symmetry transformations (5), (6)

$$\delta_\kappa \tilde{Y}_a = 0, \qquad \delta_\kappa \tilde{\eta} = 0, \qquad \delta_\kappa U^a = 0,$$

so that new representation of S_p (4) includes only κ-invariant variables. Note that in 4-dimensional space-time Y^Λ contains only 9 real variables that is twice less than the number of the original variables Y_{ab}, θ_a, U^a.

3 Example of $OSp(1|8)$ string/brane. Primary constraints

$OSp(1|8)$ is the global supersymmetry of the massless fields of all spins in $D = 4$ space-time extended by TCC coordinates [6, 7]. Therefore, we study $D = 4$ example of the string/brane model (1) formulated in generalized $(4+6)$-dimensional space $\mathcal{M}_{4+6}$ extended by the Grassmannian Majorana bispinor θ_a. In this case the $D = 4$ $N = 1$ superalgebra

$$\{Q_a, Q_b\} = \left(\gamma^m C^{-1}\right)_{ab} P_m + i\left(\gamma^{mn} C^{-1}\right)_{ab} Z_{mn}$$

includes the TCC 2−form generator Z_{mn}, and the matrix coordinates Y_{ab} are

$$Y_{ab} = x_{ab} + z_{ab},$$

where

$$x_{ab} = x_m\left(\gamma^m C^{-1}\right)_{ab}, \qquad z_{ab} = z_{mn}\left(\gamma^{mn} C^{-1}\right)_{ab}$$

with the charge conjugation matrix C chosen to be imaginary in the Majorana representation. Here we use the same agreements about the spinor algebra as in [1]. In the Weyl basis real symmetric 4×4 matrix Y_{ab} is presented as

$$Y_a{}^b = Y_{ad} C^{db} = \begin{pmatrix} z_\alpha{}^\beta & x_{\alpha\dot{\beta}} \\ \tilde{x}^{\dot{\alpha}\beta} & \bar{z}^{\dot{\alpha}}{}_{\dot{\beta}} \end{pmatrix}, \qquad C^{ab} = \begin{pmatrix} \epsilon^{\alpha\beta} & 0 \\ 0 & \epsilon_{\dot{\alpha}\dot{\beta}} \end{pmatrix}.$$

In the $D = 4$ case the auxiliary Majorana spinor $U^a(\tau, \sigma^M)$ complemented by another auxiliary Majorana spinor $V^a(\tau, \sigma^M)$ forms a spinor basis together with the spinors $(\gamma_5 U)_a$ and $(\gamma_5 V)_a$

$$U_a = \begin{pmatrix} u_\alpha \\ \bar{u}^{\dot{\alpha}} \end{pmatrix}, \qquad V_a = \begin{pmatrix} v_\alpha \\ \bar{v}^{\dot{\alpha}} \end{pmatrix}, \qquad (U\gamma_5 V) = -2i, \qquad (UV) = 0,$$

where the γ_5-matrix is

$$\gamma_5 = \begin{pmatrix} -i\delta_\alpha^\beta & 0 \\ 0 & i\delta^{\dot{\alpha}}_{\dot{\beta}} \end{pmatrix}.$$

Respectively the linear independent Weyl spinors u^α and v^α attached to the string/brane world volume may be identified with the local Newman–Penrose dyad [9]

$$u^\alpha v_\alpha \equiv u^\alpha \varepsilon_{\alpha\beta} v^\beta = 1, \qquad u^\alpha u_\alpha = v^\alpha v_\alpha = 0.$$

In the Weyl basis the action (1) acquires the form

$$S_p = \frac{1}{2}\int d\tau d^p\sigma\ \rho^\mu \left(2u^\alpha \omega_{\mu\alpha\dot{\alpha}} \bar{u}^{\dot{\alpha}} + u^\alpha \omega_{\mu\alpha\beta} u^\beta + \bar{u}^{\dot{\alpha}} \bar{\omega}_{\mu\dot{\alpha}\dot{\beta}} \bar{u}^{\dot{\beta}}\right), \tag{7}$$

where the supersymmetric one-forms $\omega_{\mu\alpha\dot\alpha}$ and $\omega_{\mu\alpha\beta}$ are

$$\omega_{\mu\alpha\dot\alpha} = \partial_\mu x_{\alpha\dot\alpha} + 2i(\partial_\mu\theta_\alpha\bar\theta_{\dot\alpha} + \partial_\mu\bar\theta_{\dot\alpha}\theta_\alpha),$$
$$\omega_{\mu\alpha\beta} = -\partial_\mu z_{\alpha\beta} - 2i(\partial_\mu\theta_\alpha\theta_\beta + \partial_\mu\theta_\beta\theta_\alpha),$$
$$\bar\omega_{\mu\dot\alpha\dot\beta} = -\partial_\mu\bar z_{\dot\alpha\dot\beta} - 2i(\partial_\mu\bar\theta_{\dot\alpha}\bar\theta_{\dot\beta} + \partial_\mu\bar\theta_{\dot\beta}\bar\theta_{\dot\alpha}).$$

The momenta densities $\mathcal{P}^{\mathfrak{M}}(\tau,\sigma^M)$

$$\mathcal{P}^{\mathfrak{M}} = \frac{\partial L}{\partial\dot{\mathcal{Q}}_{\mathfrak{M}}} = \left(P^{\dot\alpha\alpha}, \pi^{\alpha\beta}, \bar\pi^{\dot\alpha\dot\beta}, \pi^\alpha, \bar\pi^{\dot\alpha}, P_u^\alpha, \bar P_u^{\dot\alpha}, P_v^\alpha, \bar P_v^{\dot\alpha}, P_\mu^{(\rho)}\right)$$

are canonically conjugate to the coordinates

$$\mathcal{Q}_{\mathfrak{M}} = (x_{\alpha\dot\alpha}, z_{\alpha\beta}, \bar z_{\dot\alpha\dot\beta}, u_\alpha, \bar u_{\dot\alpha}, v_\alpha, \bar v_{\dot\alpha}, \rho^\mu)$$

with respect to the Poisson brackets

$$\{\mathcal{P}^{\mathfrak{M}}(\vec\sigma), \mathcal{Q}_{\mathfrak{N}}(\vec\sigma')\}_{\text{P.B.}} = \delta^{\mathfrak{M}}_{\mathfrak{N}}\delta^p(\vec\sigma - \vec\sigma')$$

with the periodic δ-function $\delta^p(\vec\sigma-\vec\sigma')$, where $\vec\sigma = (\sigma^1,\ldots,\sigma^p)$, for the case of closed string/brane studied here.

As far as S_p (7) is linear in the proper time derivatives, it is characterized by the presence of the primary constraints. These constraints may be divided into four sectors.

The bosonic Φ-sector includes the constraints $\Phi \equiv \left(\Phi^{\dot\alpha\alpha}, \Phi^{\alpha\beta}, \bar\Phi^{\dot\alpha\dot\beta}\right)$ with

$$\begin{aligned} \Phi^{\dot\alpha\alpha} &= P^{\dot\alpha\alpha} - \rho^\tau u^\alpha \bar u^{\dot\alpha} \approx 0, \\ \Phi^{\alpha\beta} &= \pi^{\alpha\beta} + \frac{1}{2}\rho^\tau u^\alpha u^\beta \approx 0, \\ \bar\Phi^{\dot\alpha\dot\beta} &= \bar\pi^{\dot\alpha\dot\beta} + \frac{1}{2}\rho^\tau \bar u^{\dot\alpha}\bar u^{\dot\beta} \approx 0. \end{aligned} \tag{8}$$

The constraints from the Grassmannian Ψ-sector, where $\Psi = (\Psi^\alpha, \bar\Psi^{\dot\alpha})$, are given by

$$\begin{aligned} \Psi^\alpha &= \pi^\alpha - 2i\bar\theta_{\dot\alpha}P^{\dot\alpha\alpha} - 4i\pi^{\alpha\beta}\theta_\beta \approx 0, \\ \bar\Psi^{\dot\alpha} &= -(\Psi^\alpha)^* = \bar\pi^{\dot\alpha} - 2iP^{\dot\alpha\alpha}\theta_\alpha - 4i\bar\pi^{\dot\alpha\dot\beta}\bar\theta_{\dot\beta} \approx 0. \end{aligned} \tag{9}$$

The dyad or (u,v)-sector is formed by the constraints

$$\begin{aligned} &P_u^\alpha \approx 0, \qquad \bar P_u^{\dot\alpha} \approx 0, \qquad P_v^\alpha \approx 0, \qquad \bar P_v^{\dot\alpha} \approx 0, \\ &\Xi \equiv u^\alpha v_\alpha - 1 \approx 0, \qquad \bar\Xi \equiv \bar u^{\dot\alpha}\bar v_{\dot\alpha} - 1 \approx 0. \end{aligned} \tag{10}$$

Finally, the $\rho-$sector includes the constraints

$$P_\mu^{(\rho)} \approx 0, \qquad \mu = (\tau, M), \qquad M = (1,\ldots,p). \tag{11}$$

To find all local symmetries of the brane action, it is necessary to split the constraints (8)–(11) into the first- and the second-class sets. Then the first-class constraints will generate the local symmetries on the Poisson brackets in accordance with the Dirac prescription.

4 The first-class constraints

To begin with consider the first-class constraints from the Ψ-sector (9) that generate κ-symmetry transformations (6). To find their explicit form it is necessary to solve equation (5) restricting the transformation parameter $\kappa_a(\tau, \vec{\sigma})$. Taking into account its decomposition over the dyad basis

$$\kappa_\alpha = \kappa u_\alpha + \tilde{\kappa} v_\alpha, \qquad \bar{\kappa}_{\dot{\alpha}} = \bar{\kappa}\bar{u}_{\dot{\alpha}} + \bar{\tilde{\kappa}}\bar{v}_{\dot{\alpha}}$$

it follows from equation (5) that $\tilde{\kappa} = \bar{\tilde{\kappa}}$ thus $\tilde{\kappa} = \tilde{\kappa}_R$ is real. Then the 3-parametric κ-symmetry transformation laws acquire the form

$$\begin{aligned}
&\delta_\kappa \theta_\alpha = \kappa u_\alpha, \qquad \delta_\kappa \bar{\theta}_{\dot{\alpha}} = \bar{\kappa}\bar{u}_{\dot{\alpha}}, \\
&\delta_\kappa x_{\alpha\dot{\alpha}} = -2i(\kappa u_\alpha \bar{\theta}_{\dot{\alpha}} + \bar{\kappa}\bar{u}_{\dot{\alpha}}\theta_\alpha), \qquad \delta_\kappa z_{\alpha\beta} = -2i\kappa(u_\alpha\theta_\beta + u_\beta\theta_\alpha); \\
&\delta_{\tilde{\kappa}_R}\theta_\alpha = \tilde{\kappa}_R v_\alpha, \qquad \delta_{\tilde{\kappa}_R}\bar{\theta}_{\dot{\alpha}} = \tilde{\kappa}_R \bar{v}_{\dot{\alpha}}, \\
&\delta_{\tilde{\kappa}_R} x_{\alpha\dot{\alpha}} = -2i\tilde{\kappa}_R(v_\alpha\bar{\theta}_{\dot{\alpha}} + \bar{v}_{\dot{\alpha}}\theta_\alpha), \qquad \delta_{\tilde{\kappa}_R} z_{\alpha\beta} = -2i\tilde{\kappa}_R(v_\alpha\theta_\beta + v_\beta\theta_\alpha).
\end{aligned} \tag{12}$$

Transformations (12) are generated on the Poisson brackets by three constraints

$$\Psi^{(u)} \equiv \Psi^\alpha u_\alpha \approx 0, \qquad \bar{\Psi}^{(u)} \equiv \bar{\Psi}^{\dot{\alpha}}\bar{u}_{\dot{\alpha}} \approx 0, \qquad \Psi_R^{(v)} \equiv \Psi^\alpha v_\alpha + \bar{\Psi}^{\dot{\alpha}}\bar{v}_{\dot{\alpha}} \approx 0$$

that belong to the first-class. The Poisson brackets of these κ-symmetry generators

$$\begin{aligned}
&\{\Psi^{(u)}(\vec{\sigma}), \bar{\Psi}^{(u)}(\vec{\sigma}')\}_{\text{P.B.}} = -4i\Phi^{(u)}\delta^p(\vec{\sigma} - \vec{\sigma}') \approx 0, \\
&\{\Psi^{(u)}(\vec{\sigma}), \Psi^{(u)}(\vec{\sigma}')\}_{\text{P.B.}} = -8iT^{(u)}\delta^p(\vec{\sigma} - \vec{\sigma}') \approx 0, \\
&\{\bar{\Psi}^{(u)}(\vec{\sigma}), \bar{\Psi}^{(u)}(\vec{\sigma}')\}_{\text{P.B.}} = -8i\bar{T}^{(u)}\delta^p(\vec{\sigma} - \vec{\sigma}') \approx 0, \\
&\{\Psi_R^{(v)}(\vec{\sigma}), \Psi_R^{(v)}(\vec{\sigma}')\}_{\text{P.B.}} = -8i\tilde{T}_R^{(v)}\delta^p(\vec{\sigma} - \vec{\sigma}') \approx 0, \\
&\{\Psi_R^{(v)}(\vec{\sigma}), \Psi^{(u)}(\vec{\sigma}')\}_{\text{P.B.}} = -2i(\tilde{T}^{(+)} - i\tilde{T}^{(-)})\delta^p(\vec{\sigma} - \vec{\sigma}') \approx 0, \\
&\{\Psi_R^{(v)}(\vec{\sigma}), \bar{\Psi}^{(u)}(\vec{\sigma}')\}_{\text{P.B.}} = -2i(\tilde{T}^{(+)} + i\tilde{T}^{(-)})\delta^p(\vec{\sigma} - \vec{\sigma}') \approx 0
\end{aligned}$$

are closed by 6 bosonic first-class constraints from the Φ-sector that were chosen as follows

$$T^{(u)} \equiv u_\alpha \Phi^{\alpha\beta} u_\beta \approx 0, \qquad \bar{T}^{(u)} = (T^{(u)})^*, \tag{13}$$

$$\Phi^{(u)} \equiv u_\alpha \Phi^{\alpha\dot{\beta}}\bar{u}_{\dot{\beta}}, \tag{14}$$

$$\tilde{T}_R^{(v)} \equiv v_\alpha \Phi^{\alpha\beta} v_\beta + \bar{v}_{\dot{\alpha}}\bar{\Phi}^{\dot{\alpha}\dot{\beta}}\bar{v}_{\dot{\beta}} + \bar{v}_{\dot{\alpha}}\Phi^{\dot{\alpha}\alpha}v_\alpha \approx 0, \tag{15}$$

$$\tilde{T}^{(+)} \equiv \Phi^{\dot{\alpha}\alpha}(u_\alpha\bar{v}_{\dot{\alpha}} + v_\alpha\bar{u}_{\dot{\alpha}}) + 2\left(\Phi^{\alpha\beta}u_{\{\alpha}v_{\beta\}} + \bar{\Phi}^{\dot{\alpha}\dot{\beta}}\bar{u}_{\{\dot{\alpha}}\bar{v}_{\dot{\beta}\}}\right) \approx 0, \tag{16}$$

$$\tilde{T}^{(-)} \equiv i\left[\Phi^{\dot{\alpha}\alpha}(u_\alpha\bar{v}_{\dot{\alpha}} - v_\alpha\bar{u}_{\dot{\alpha}}) + 2\left(\Phi^{\alpha\beta}u_{\{\alpha}v_{\beta\}} - \bar{\Phi}^{\dot{\alpha}\dot{\beta}}\bar{u}_{\{\dot{\alpha}}\bar{v}_{\dot{\beta}\}}\right)\right] \approx 0. \tag{17}$$

The meaning of the constraints (13)–(17) is that they generate local shifts of space-time and TCC coordinates along the directions defined by the moving tetrade attached to super p-brane world volume [8]. Indeed, constraints (13) correspond to local shifts of TCC coordinates along the conjugate momentum

$$\delta_{T^{(u)}} z_{\alpha\beta} = \epsilon_{T^{(u)}} u_\alpha u_\beta \approx -2\frac{\epsilon_{T^{(u)}}}{\rho^\tau}\pi_{\alpha\beta}, \qquad \delta_{\bar{T}^{(u)}}\bar{z}_{\dot{\alpha}\dot{\beta}} = \bar{\epsilon}_{\bar{T}^{(u)}}\bar{u}_{\dot{\alpha}}\bar{u}_{\dot{\beta}} \approx -2\frac{\bar{\epsilon}_{\bar{T}^{(u)}}}{\rho^\tau}\bar{\pi}_{\dot{\alpha}\dot{\beta}},$$

whereas the constraint (14) generates analogous shift for ordinary space-time coordinates

$$\delta_{\Phi^{(u)}} x_{\alpha\dot{\alpha}} = \epsilon_{\Phi^{(u)}} u_\alpha \bar{u}_{\dot{\alpha}} \approx \frac{\epsilon_{\Phi^{(u)}}}{\rho^\tau} P_{\alpha\dot{\alpha}}.$$

The local symmetry generated by the constraint (15) is the shift of space-time and TCC coordinates along another light-like direction parametrized by the dyad component v_α

$$\delta_{\tilde{T}_R^{(v)}} x_{\alpha\dot{\alpha}} = \epsilon_{\tilde{T}_R^{(v)}} v_\alpha \bar{v}_{\dot{\alpha}}, \qquad \delta_{\tilde{T}_R^{(v)}} z_{\alpha\beta} = \epsilon_{\tilde{T}_R^{(v)}} v_\alpha v_\beta, \qquad \delta_{\tilde{T}_R^{(v)}} \bar{z}_{\dot{\alpha}\dot{\beta}} = \epsilon_{\tilde{T}_R^{(v)}} \bar{v}_{\dot{\alpha}} \bar{v}_{\dot{\beta}}.$$

The action (7) is invariant under this transformation due to mutual cancellation of contributions to the variation coming from space-time and TCC coordinates. Finally constraints (16), (17) correspond to local shifts of space-time coordinates along the directions transverse to the world volume

$$\delta_{\tilde{T}^{(+)}} x_{\alpha\dot{\alpha}} = \epsilon_{\tilde{T}^{(+)}} m^{(+)}_{\alpha\dot{\alpha}}, \qquad \delta_{\tilde{T}^{(-)}} x_{\alpha\dot{\alpha}} = \epsilon_{\tilde{T}^{(-)}} m^{(-)}_{\alpha\dot{\alpha}}$$

compensated by appropriate shifts of TCC coordinates

$$\begin{aligned} &\delta_{\tilde{T}^{(+)}} z_{\alpha\beta} = 2\epsilon_{\tilde{T}^{(+)}} u_{\{\alpha} v_{\beta\}}, \qquad \delta_{\tilde{T}^{(+)}} \bar{z}_{\dot{\alpha}\dot{\beta}} = 2\epsilon_{\tilde{T}^{(+)}} \bar{u}_{\{\dot{\alpha}} \bar{v}_{\dot{\beta}\}}; \\ &\delta_{\tilde{T}^{(-)}} z_{\alpha\beta} = 2i\epsilon_{\tilde{T}^{(-)}} u_{\{\alpha} v_{\beta\}}, \qquad \delta_{\tilde{T}^{(-)}} \bar{z}_{\dot{\alpha}\dot{\beta}} = -2i\epsilon_{\tilde{T}^{(-)}} \bar{u}_{\{\dot{\alpha}} \bar{v}_{\dot{\beta}\}}, \end{aligned}$$

where $u_{\{\alpha} v_{\beta\}} = \frac{1}{2}(u_\alpha v_\beta + u_\beta v_\alpha)$.

In the dyad sector there present are two first-class constraints

$$P_v^{(u)} \equiv P_v^\alpha u_\alpha \approx 0, \qquad \bar{P}_v^{(u)} \equiv \bar{P}_v^{\dot{\alpha}} \bar{u}_{\dot{\alpha}} \approx 0$$

generating shifts of v_α ($\bar{v}_{\dot{\alpha}}$) dyad components along u_α ($\bar{u}_{\dot{\alpha}}$). These shifts constitute apparent local symmetry of the action (7) as S_p does not contain v_α ($\bar{v}_{\dot{\alpha}}$).

The first-class constraints of the ρ-sector are

$$P_M^{(\rho)} \approx 0, \qquad M = (1, \ldots, p).$$

They commute with all other constraints since canonically conjugate variables ρ^M do not enter the primary constraints (8)–(11). $P_M^{(\rho)}$ constraints correspond to redefinition of p space components ρ^M of the $(p+1)$-dimensional world-volume vector density $\rho^\mu(\tau, \vec{\sigma})$

$$\delta_\epsilon \rho^M = \epsilon^M(\tau, \vec{\sigma}). \tag{18}$$

The transformations (18) are new local symmetries of the action S_p due to arbitrariness in the definition of ρ^M.

The first-class constraint that includes primary ones from the different sectors is provided by the Weyl symmetry generator

$$\Delta_W \equiv (P_u^\alpha u_\alpha + \bar{P}_u^{\dot{\alpha}} \bar{u}_{\dot{\alpha}}) - (P_v^\alpha v_\alpha + \bar{P}_v^{\dot{\alpha}} \bar{v}_{\dot{\alpha}}) - 2\rho^\mu P_\mu^{(\rho)} \approx 0.$$

The local transformation generated by Δ_W is the dilation that acts only on auxiliary variables u_α, v_α and ρ^μ

$$\begin{aligned} &\rho'^\mu = e^{-2\Lambda} \rho^\mu, \qquad u'_\alpha = e^\Lambda u_\alpha, \qquad v'_\alpha = e^{-\Lambda} v_\alpha, \\ &x'_{\alpha\dot{\alpha}} = x_{\alpha\dot{\alpha}},\ z'_{\alpha\beta} = z_{\alpha\beta}, \qquad \theta'_\alpha = \theta_\alpha. \end{aligned} \tag{19}$$

The transformation (19) is identified with the Weyl symmetry of the p-brane action. From the string point of view, the Weyl invariants $\rho^\mu u_\alpha \bar{u}_{\dot{\alpha}}$ and $\rho^\mu u_\alpha u_\beta$ constructed from auxiliary world-volume fields are similar to conventional Weyl invariant of tensile string

$$\sqrt{-g} g^{\mu\nu} \Leftrightarrow \rho^\mu u_\alpha \bar{u}_{\dot{\alpha}},$$

but here (19) is the symmetry the tensionless super p-brane action.

Finally the set of the first-class constraints comprises p space-like reparametrization generators

$$L_M = P^{\dot\alpha\alpha}\omega_{M\alpha\dot\alpha} - \pi^{\alpha\beta}\omega_{M\alpha\beta} - \bar\pi^{\dot\alpha\dot\beta}\bar\omega_{M\dot\alpha\dot\beta} + \partial_M\theta_\alpha\Psi^\alpha + \partial_M\bar\theta_{\dot\alpha}\bar\Psi^{\dot\alpha} + (P_u^\alpha\partial_M u_\alpha + P_v^\alpha\partial_M v_\alpha) + (\bar P_u^{\dot\alpha}\partial_M\bar u_{\dot\alpha} + \bar P_v^{\dot\alpha}\partial_M\bar v_{\dot\alpha}) - \partial_M P_\nu^{(\rho)}\rho^\nu \approx 0$$

that are the secondary constraints. The remaining time-like τ-reparametrization constraint is not independent and is constructed from the constraints $T^{(u)}$ (13), $\Phi^{(u)}$ (14) for $z_{\alpha\beta}$, $x_{\alpha\dot\alpha}$ and some algebraic combinations of the other first-class constraints for the remaining generalized coordinates.

5 The Hamiltonian

Having completed the construction of the first-class constraints we present the total gauge independent Hamiltonian density of the super p-brane in the form of their linear combination

$$H_T(\tau,\vec\sigma) = \kappa_u\Psi^{(u)} + \bar\kappa_u\bar\Psi^{(u)} + \kappa_R\Psi_R^{(v)} + a_u\Phi^{(u)} + b_u T^{(u)} + \bar b_u\bar T^{(u)} + c^{(+)}\tilde T^{(+)} + c^{(-)}\tilde T^{(-)} + c_R^{(v)}\tilde T_R^{(v)} + eP_v^{(u)} + \bar e\bar P_v^{(u)} + \omega\Delta_W + f^M P_M^{(\rho)} + \tilde\rho^M L_M \approx 0,$$

where the functions κ, a, b, c, e, f, ω and $\tilde\rho$ form the set of $(9+2p)_B + 3_F$ real Lagrange multipliers. The above Hamiltonian density yields the covariant equations of motion.

6 Conclusion

The Hamiltonian of the simplest $D = 4$ $N = 1$ super p-brane model of which general solution describes the BPS state with exotic 3/4 fraction of supersymmetry was constructed. To separate the first-class constraints in the manifestly Lorentz covariant way there has been introduced additional Weyl spinor field $v_\alpha(\tau,\vec\sigma)$ forming the local Newman–Penrose dyad [9] together with $u_\alpha(\tau,\vec\sigma)$. It was shown that the model possesses 3 fermionic first-class constraints that are generators of enhanced κ-symmetry transformations and $2p+9$ bosonic first-class constraints corresponding to space-like reparametrizations (p constraints), redefinitions of p space components of the auxiliary world-volume vector density $\rho^\mu = (\rho^\tau, \rho^M)$, shifts of the dyad components v_α along u_α (2 constraints), local Weyl symmetry (1 constraint) and 6 Abelian shifts of the space-time and TCC coordinates along the directions specified by the bilinear combinations of the dyad components. The first-class constraints fix the number of physical degrees of freedom of the p-brane model to be $n_{\rm phys} = 2(4-p)_B + 1_F$ $(0 \le p \le 4)$ and encode them by the $OSp(1|8)$ supertwistor, which is invariant under $8_B + 3_F$ gauge symmetries, as was proved in [8]. So, in the supertwistor representation $8_B + 3_F$ redundant superspace-time degrees of freedom are covariantly excluded just by the variables transformation without any gauge fixing.

The results obtained here constitute a reliable basis for the covariant separation of the second-class constraints, construction of the Dirac brackets and performing the canonical quantization of super p-brane model [1]. These results are presented in the recent papers [19].

Acknowledgements

A.Z. thanks Fysikum at the Stockholm University for kind hospitality and Ingemar Bengtsson for useful discussions. The work was partially supported by the grant of the Royal Swedish Academy of Sciences and SFFR of Ukraine under project 02.07/276.

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