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CPT Symmetry and Properties of the Exact and Approximate Effective Hamiltonians for the Neutral Kaon Complex[1]

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We show that the diagonal matrix elements of the exact effective Hamiltonian governing the time evolution in the subspace of states of an unstable particle and its antiparticle need not be equal at for $t > t_0$ (t_0 is the instant of creation of the pair) when the total system under consideration is CPT invariant but CP noninvariant. The unusual consequence of this result is that, contrary to the properties of stable particles, the masses of the unstable particle "1" and its antiparticle "2" need not be equal for $t \gg t_0$ in the case of preserved CPT and violated CP symmetries.

1 Introduction

The problem of testing CPT-invariance experimentally has attracted the attention of physicist, practically since the discovery of antiparticles. CPT symmetry is a fundamental theorem of axiomatic quantum field theory which follows from locality, Lorentz invariance, and unitarity [2]. Many tests of CPT-invariance consist in searching for decay process of neutral kaons. All known CP- and hypothetically possible CPT-violation effects in neutral kaon complex are described by solving the Schrödinger-like evolution equation [3–7] (we use $\hbar = c = 1$ units)

$$i\frac{\partial}{\partial t}|\psi;t\rangle_{\parallel} = H_{\parallel}|\psi;t\rangle_{\parallel} \tag{1}$$

for $|\psi;t\rangle_{\parallel}$ belonging to the subspace $\mathcal{H}_{\parallel} \subset \mathcal{H}$ (where $\mathcal{H}$ is the state space of the physical system under investigation), e.g., spanned by orthonormal neutral kaons states $|K_0\rangle$, $|\overline{K}_0\rangle$, and so on, (then states corresponding with the decay products belong to $\mathcal{H} \ominus \mathcal{H}_{\parallel} \stackrel{\text{def}}{=} \mathcal{H}_{\perp}$), and non-Hermitian effective Hamiltonian $H_{\parallel}$ obtained usually by means of Lee–Oehme–Yang (LOY) approach (within the Weisskopf–Wigner approximation (WW)) [3–5, 7]:

$$H_{\parallel} \equiv M - \frac{i}{2}\Gamma, \tag{2}$$

where $M = M^{+}$, $\Gamma = \Gamma^{+}$, are (2×2) matrices.

Solutions of equation (1) can be written in matrix form and such a matrix defines the evolution operator (which is usually nonunitary) $U_{\parallel}(t)$ acting in $\mathcal{H}_{\parallel}$:

$$|\psi;t\rangle_{\parallel} = U_{\parallel}(t)|\psi;t_0 = 0\rangle_{\parallel} \stackrel{\text{def}}{=} U_{\parallel}(t)|\psi\rangle_{\parallel}, \tag{3}$$

where,

$$|\psi\rangle_{\parallel} \equiv q_1|\mathbf{1}\rangle + q_2|\mathbf{2}\rangle, \tag{4}$$

[1] This paper is a shortened version of [1].

and $|\mathbf{1}\rangle$ stands for the vectors of the $|K_0\rangle$, $|B_0\rangle$ type and $|\mathbf{2}\rangle$ denotes antiparticles of the particle "1": $|\overline{K}_0\rangle$, $|\overline{B}_0\rangle$, $\langle \boldsymbol{j} \,|\, \boldsymbol{k}\rangle = \delta_{jk}$, $j,k = 1,2$.

In many papers it is assumed that the real parts, $\Re(\cdot)$, of the diagonal matrix elements of $H_{\|}$, $\Re(h_{jj}) \equiv M_{jj}$, $(j = 1,2)$, where

$$h_{jk} = \langle \boldsymbol{j} \,|\, H_{\|} |\boldsymbol{k}\rangle, \qquad (j,k = 1,2), \tag{5}$$

correspond to the masses of particle "1" and its antiparticle "2" respectively [3–7], (and such an interpretation of $\Re(h_{11})$ and $\Re(h_{22})$ will be used in this paper), whereas the imaginary parts, $\Im(\cdot)$, $\Im(h_{jj}) \equiv -\frac{1}{2}\Gamma_{jj}$, $(j = 1,2)$ interpreted as the decay widths of these particles [3–7]. Such an interpretation seems to be consistent with the recent and the early experimental data for neutral kaon and similar complexes [8].

Relations between matrix elements of $H_{\|}$ implied by CPT-transformation properties of the Hamiltonian H of the total system, containing neutral kaon complex as a subsystem, are crucial for designing CPT-invariance and CP-violation tests and for proper interpretation of their results. The aim of this talk is to examine the properties of the approximate and exact $H_{\|}$ generated by the CPT-symmetry of the total system under consideration and independent of the approximation used.

2 H_{LOY} and CPT-symmetry

Now, let us consider briefly some properties of the LOY model. Let H be total (self-adjoint) Hamiltonian, acting in $\mathcal{H}$ – then the total unitary evolution operator $U(t)$ fulfills the Schrödinger equation

$$i\frac{\partial}{\partial t}U(t)|\phi\rangle = HU(t)|\phi\rangle, \qquad U(0) = I, \tag{6}$$

where I is the unit operator in $\mathcal{H}$, $|\phi\rangle \equiv |\phi; t_0 = 0\rangle \in \mathcal{H}$ is the initial state of the system:

$$|\phi\rangle \equiv |\psi\rangle_{\|}. \tag{7}$$

In our case $U(t)|\phi\rangle \equiv |\phi; t\rangle$.

Let P denote the projection operator onto the subspace $\mathcal{H}_{\|}$: $P\mathcal{H} = \mathcal{H}_{\|}$, $P = P^2 = P^+$, then the subspace of decay products $\mathcal{H}_{\perp}$ equals $\mathcal{H}_{\perp} = (I - P)\mathcal{H} \stackrel{\text{def}}{=} Q\mathcal{H}$, and $Q \equiv I - P$. For the case of neutral kaons or neutral B-mesons, etc., the projector P can be chosen as follows:

$$P \equiv |\mathbf{1}\rangle\langle\mathbf{1}| + |\mathbf{2}\rangle\langle\mathbf{2}|. \tag{8}$$

We assume that time-independent basis vectors $|K_0\rangle$ and $|\overline{K}_0\rangle$ are defined analogously to corresponding vectors used in LOY theory of time evolution in neutral kaon complex [3]. In the LOY approach it is assumed that vectors $|\mathbf{1}\rangle$, $|\mathbf{2}\rangle$ considered above are eigenstates of $H^{(0)}$ for 2-fold degenerate eigenvalue m_0:

$$H^{(0)}|\boldsymbol{j}\rangle = m_0|\boldsymbol{j}\rangle, \qquad j = 1,2, \tag{9}$$

where $H^{(0)}$ is a so called free Hamiltonian, $H^{(0)} \equiv H_{\text{strong}} = H - H_W$, and H_W denotes weak and other interactions which are responsible for transitions between eigenvectors of $H^{(0)}$, i.e., for the decay process.

The condition guaranteeing the occurrence of transitions between subspaces $\mathcal{H}_{\|}$ and $\mathcal{H}_{\perp}$, i.e., a decay process of states in $\mathcal{H}_{\|}$, can be written as follows $[P, H_W] \neq 0$, that is

$$[P, H] \neq 0. \tag{10}$$

Usually, in LOY and related approaches, it is assumed that $\Theta H^{(0)}\Theta^{-1} = H^{(0)^+} \equiv H^{(0)}$, where Θ is the antiunitary operator, $\Theta \stackrel{\text{def}}{=} \mathcal{CPT}$. The subspace of neutral kaons $\mathcal{H}_{\|}$ is assumed to be invariant under Θ:

$$\Theta P \Theta^{-1} = P^{+} \equiv P. \tag{11}$$

In the kaon rest frame, the time evolution is governed by the Schrödinger equation (6), where the initial state of the system has the form (7), (4). Within assumptions (9), (10) the WW approach, which is the source of the LOY method, leads to the following formula for $\mathrm{H}_{\mathrm{LOY}}$ (e.g., see [3–5, 7]):

$$H_{\mathrm{LOY}} = m_0 P - \Sigma(m_0) \equiv PHP - \Sigma(m_0) = M_{\mathrm{LOY}} - \frac{i}{2}\Gamma_{\mathrm{LOY}}, \tag{12}$$

where it has been assumed that $\langle \mathbf{1} \,|\, H_W \,|\, \mathbf{2}\rangle = \langle \mathbf{1} \,|\, H_W \,|\, \mathbf{2}\rangle^* = 0$ (see [3–7]). Here

$$\Sigma(\epsilon) = PHQ \frac{1}{QHQ - \epsilon - i0} QHP. \tag{13}$$

The matrix elements h_{jk}^{LOY} of H_{LOY} are

$$h_{jk}^{\mathrm{LOY}} = H_{jk} - \Sigma_{jk}(m_0) = M_{jk}^{\mathrm{LOY}} - \frac{i}{2}\Gamma_{jk}^{\mathrm{LOY}} \qquad (j,k = 1,2), \tag{14}$$

where, in this case,

$$H_{jk} = \langle \boldsymbol{j} \,|\, H \,|\, \boldsymbol{k}\rangle \equiv \langle \boldsymbol{j} \,|\, (H^{(0)} + H_W) \,|\, \boldsymbol{k}\rangle \equiv m_0 \delta_{jk} + \langle \boldsymbol{j} \,|\, H_W \,|\, \boldsymbol{k}\rangle, \tag{15}$$

and $\Sigma_{jk}(\epsilon) = \langle \boldsymbol{j} \,|\, \Sigma(\epsilon) \,|\, \boldsymbol{k}\rangle$.

Now, if $\Theta H_W \Theta^{-1} = H_W^+ \equiv H_W$, that is if

$$[\Theta, H] = 0, \tag{16}$$

then using, e.g., the following phase convention [4–7]

$$\Theta|\mathbf{1}\rangle \stackrel{\text{def}}{=} -|\mathbf{2}\rangle, \qquad \Theta|\mathbf{2}\rangle \stackrel{\text{def}}{=} -|\mathbf{1}\rangle, \tag{17}$$

and taking into account that $\langle \psi \,|\, \varphi \rangle = \langle \Theta\varphi \,|\, \Theta\psi\rangle$, one easily finds from (12)–(15) that

$$h_{11}^{\mathrm{LOY}^{\Theta}} - h_{22}^{\mathrm{LOY}^{\Theta}} = 0, \tag{18}$$

and thus

$$M_{11}^{\mathrm{LOY}} = M_{22}^{\mathrm{LOY}}, \tag{19}$$

(where $h_{jk}^{\mathrm{LOY}^{\Theta}}$ denotes the matrix elements of $H_{\mathrm{LOY}}^{\Theta}$ – of the LOY effective Hamiltonian when the relation (16) holds), in the CPT-invariant system. This is the standard result of the LOY approach and this is the picture which one meets in the literature [3–6, 8].

3 Beyond the LOY approximation

The more accurate approximate formulae for $H_{\|}(t)$ have been derived in [9] using the Krolikowski–Rzewuski equation for the projection of a state vector [10], which results from the Schrödinger equation (6) for the total system under consideration, and, in the case of initial conditions of the type (7), takes the following form

$$\left(i\frac{\partial}{\partial t} - PHP\right) U_{||}(t) = -i\int_0^\infty K(t-\tau)U_{||}(\tau)d\tau, \tag{20}$$

where $U_{||}(0) = P$, $K(t) = \Theta(t)PHQ\exp(-itQHQ)QHP$, and $\Theta(t) = \{1 \text{ for } t \geq 0,\;\; 0 \text{ for } t < 0\}$.

The integro-differential equation (20) can be replaced by the following differential one (see [9–13])

$$\left(i\frac{\partial}{\partial t} - PHP - V_{||}(t)\right) U_{||}(t) = 0, \tag{21}$$

where $PHP + V_{||}(t) \stackrel{\text{def}}{=} H_{||}(t)$. Taking into account (20) and (21) or (1) one finds

$$V_{||}(t)U_{||}(t) = -i\int_0^\infty K(t-\tau)U_{||}(\tau)d\tau. \tag{22}$$

Using this relation, one finds to the lowest nontrivial order that [9]

$$V_{||}(t) \cong V_{||}^{(1)}(t) \stackrel{\text{def}}{=} -i\int_0^\infty K(t-\tau)\exp\left[i(t-\tau)PHP\right]d\tau. \tag{23}$$

From (22) one can conclude that [9, 13]

$$H_{||}(0) \equiv PHP, \qquad V_{||}(0) = 0, \qquad V_{||}(t \to 0) \simeq -itPHQHP. \tag{24}$$

In the case of conserved CPT and of H such that $H_{12} = H_{21} = 0$, from (23) one finds that $H_{||} = m_0 P - \Sigma(m_0) \equiv H_{\text{LOY}}$.

On the other hand, in the case

$$H_{12} = H_{21}^* \neq 0, \tag{25}$$

the form of $H_{||}$ is much more complicated. For example in the case of conserved CPT, formula (23) leads to matrix elements of $H_{||}$ [7, 14], which differ from (14). Indeed, in the case of preserved CPT-symmetry (16), one finds $H_{11} = H_{22}$, which implies that $H_{11} \equiv H_{22} \stackrel{\text{def}}{=} H_0$, and [9] $\Sigma_{11}(\varepsilon = \varepsilon^*) \equiv \Sigma_{22}(\varepsilon = \varepsilon^*) \stackrel{\text{def}}{=} \Sigma_0(\varepsilon = \varepsilon^*)$. Therefore matrix elements $v_{jk}^{\Theta} \equiv \langle \boldsymbol{j} \,|\, V_{||}^{\Theta} \,|\, \boldsymbol{k}\rangle$, $(j,k = 1,2)$ of operator $V_{||}^{\Theta}$ (here $V_{||}^{\Theta}$ denotes $V_{||}$ when (16) occurs and $V_{||} \stackrel{\text{def}}{=} \lim_{t\to\infty} V_{||}^{(1)}(t)$) take the following form

$$\begin{aligned}
v_{j1}^{\Theta} = -\frac{1}{2}\Big\{&\Sigma_{j1}(H_0 + |H_{12}|) + \Sigma_{j1}(H_0 - |H_{12}|) \\
&+ \frac{H_{21}}{|H_{12}|}\Sigma_{j2}(H_0 + |H_{12}|) - \frac{H_{21}}{|H_{12}|}\Sigma_{j2}(H_0 - |H_{12}|)\Big\}, \\
v_{j2}^{\Theta} = -\frac{1}{2}\Big\{&\Sigma_{j2}(H_0 + |H_{12}|) + \Sigma_{j2}(H_0 - |H_{12}|) \\
&+ \frac{H_{12}}{|H_{12}|}\Sigma_{j1}(H_0 + |H_{12}|) - \frac{H_{12}}{|H_{12}|}\Sigma_{j1}(H_0 - |H_{12}|)\Big\}.
\end{aligned} \tag{26}$$

Finally, assuming $|H_{12}| \ll |H_0|$, and using relation (5) and the expression (14), we obtain for the CPT–invariant system [15, 16]

$$h_{j1}^{\Theta} \simeq h_{j1}^{\text{LOY}} - H_{21}\frac{\partial\Sigma_{j2}(x)}{\partial x}\bigg|_{x=H_0}, \qquad h_{j2}^{\Theta} \simeq h_{j2}^{\text{LOY}} - H_{12}\frac{\partial\Sigma_{j1}(x)}{\partial x}\bigg|_{x=H_0}, \tag{27}$$

where $h_{jk} = H_{jk} + v_{jk}$ and $j = 1, 2$. From these formulae we conclude that, e.g., the difference between diagonal matrix elements of $H_{||}^{\Theta}$, which plays an important role in designing CPT-invariance tests for the neutral kaons system, equals

$$\Delta h \stackrel{\text{def}}{=} h_{11} - h_{22} \simeq H_{12} \frac{\partial \Sigma_{21}(x)}{\partial x}\bigg|_{x=H_0} - H_{21} \frac{\partial \Sigma_{12}(x)}{\partial x}\bigg|_{x=H_0} \neq 0, \tag{28}$$

which differs from the LOY results (18), (19).

4 CPT and the exact effective Hamiltonian

The aim of this Section is to show, that contrary to the LOY conclusion (18), diagonal matrix elements of the exact effective Hamiltonian $H_{||}$ cannot be equal when the total system under consideration is CPT invariant but CP noninvariant. This will be done by means of the method used in [17].

Universal properties of the (unstable) particle–antiparticle subsystem of the system described by the Hamiltonian H, for which the relation (16) holds, can be extracted from the matrix elements of the exact $U_{||}(t)$ appearing in (3). Such $U_{||}(t)$ has the following form

$$U_{||}(t) = PU(t)P, \tag{29}$$

where P is defined by the relation (8), and $U(t)$ is the total unitary evolution operator $U(t)$, which solves the Schrödinger equation (6). Operator $U_{||}(t)$ acts in the subspace of unstable states $\mathcal{H}_{||} \equiv P\mathcal{H}$. Of course, $U_{||}(t)$ has nontrivial form only if (10) holds, and only then transitions of states from $\mathcal{H}_{||}$ into $\mathcal{H}_{\perp}$ and vice versa, i.e., decay and regeneration processes, are allowed.

Using the matrix representation one finds

$$U_{||}(t) \equiv \begin{pmatrix} \boldsymbol{A}(t) & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{pmatrix}, \qquad \boldsymbol{A}(t) = \begin{pmatrix} A_{11}(t) & A_{12}(t) \\ A_{21}(t) & A_{22}(t) \end{pmatrix}, \tag{30}$$

where $\mathbf{0}$ denotes the suitable zero submatrices and a submatrix $\boldsymbol{A}(t)$ is the (2×2) matrix acting in $\mathcal{H}_{||}$ and $A_{jk}(t) = \langle \boldsymbol{j} \,|\, U_{||}(t) \,|\, \boldsymbol{k} \rangle \equiv \langle \boldsymbol{j} \,|\, U(t) \,|\, \boldsymbol{k} \rangle$, $(j, k = 1, 2)$.

Now assuming (16) and using the phase convention (17), [3–5], one easily finds that [6,18,19]

$$A_{11}(t) = A_{22}(t). \tag{31}$$

Note that assumptions (16) and (17) give no relations between $A_{12}(t)$ and $A_{21}(t)$.

The important relation between amplitudes $A_{12}(t)$ and $A_{21}(t)$ follows from the famous Khalfin's Theorem [6,18,19]. This Theorem states that in the case of unstable states, if amplitudes $A_{12}(t)$ and $A_{21}(t)$ have the same time dependence

$$r(t) \stackrel{\text{def}}{=} \frac{A_{12}(t)}{A_{21}(t)} = \text{const} \equiv r, \tag{32}$$

then it must be $|r| = 1$.

For unstable particles the relation (31) means that decay laws $p_j(t) \stackrel{\text{def}}{=} |A_{jj}(t)|^2$, (where $j = 1, 2$), of the particle $|\mathbf{1}\rangle$ and its antiparticle $|\mathbf{2}\rangle$ are equal, $p_1(t) \equiv p_2(t)$. The consequence of this latter property is that the decay rates of the particle $|\mathbf{1}\rangle$ and its antiparticle $|\mathbf{2}\rangle$ must be equal too. From (31) it does not follow that the masses of the particle "1" and the antiparticle "2" should be equal.

More conclusions about the properties of the matrix elements of $H_{||}$ one can infer analyzing the following identity [20, 11–13]

$$H_{||} \equiv H_{||}(t) = i\frac{\partial U_{||}(t)}{\partial t}[U_{||}(t)]^{-1} \equiv i\frac{\partial \boldsymbol{A}(t)}{\partial t}[\boldsymbol{A}(t)]^{-1}. \tag{33}$$

Relation (33) must be fulfilled by the exact as well as by every approximate effective Hamiltonian governing the time evolution in every two-dimensional subspace $\mathcal{H}_{||}$ of states $\mathcal{H}$ [20, 11–13].

It is easy to find from (33) the general formulae for the diagonal matrix elements, $h_{jj}(t)$, of $H_{||}(t)$, in which we are interested. We have

$$h_{11}(t) = \frac{i}{\det \boldsymbol{A}(t)}\left(\frac{\partial A_{11}(t)}{\partial t}A_{22}(t) - \frac{\partial A_{12}(t)}{\partial t}A_{21}(t)\right), \tag{34}$$

$$h_{22}(t) = \frac{i}{\det \boldsymbol{A}(t)}\left(-\frac{\partial A_{21}(t)}{\partial t}A_{12}(t) + \frac{\partial A_{22}(t)}{\partial t}A_{11}(t)\right). \tag{35}$$

Now, assuming (16) and using the consequence (31) of this assumption, one finds

$$h_{11}(t) - h_{22}(t) = -i\frac{A_{12}(t)A_{21}(t)}{\det \boldsymbol{A}(t)}\frac{\partial}{\partial t}\ln\left(\frac{A_{12}(t)}{A_{21}(t)}\right). \tag{36}$$

This result means that in the considered case for $t > 0$ the following Theorem holds:

$$h_{11}(t) - h_{22}(t) = 0 \;\Leftrightarrow\; \frac{A_{12}(t)}{A_{21}(t)} = \text{const}, \qquad (t > 0). \tag{37}$$

Thus for $t > 0$ the problem under studies is reduced to the Khalfin's Theorem (see the relation (32)).

From (34) and (35) it is easy to see that at $t = 0$

$$h_{jj}(0) = \langle \boldsymbol{j} \,|\, H \,|\, \boldsymbol{j} \rangle, \qquad (j = 1, 2), \tag{38}$$

which means that in a CPT invariant system (16) in the case of pairs of unstable particles, for which transformations of type (17) hold, the unstable particles "1" and "2" are created at $t = t_0 \equiv 0$ as particles with equal masses: $M_{11}(0) = M_{22}(0) \equiv \langle \mathbf{1} \,|\, H \,|\, \mathbf{1} \rangle$.

Now let us go on to analyze conclusions following from the Khalfin's Theorem. CP noninvariance requires that $|r| \neq 1$ [6, 19] (see also [3–5, 8]). This means that in such a case it must be $r = r(t) \neq$ const. So, if in the system considered the property (16) holds but $[\mathcal{CP}, H] \neq 0$, and the unstable states "1" and "2" are connected by a relation of type (17), then at $t > 0$ it must be $(h_{11}(t) - h_{22}(t)) \neq 0$ in this system.

Let us focus our attention on $\Re\,(h_{11}(t) - h_{22}(t))$. Following the method used in [12] and using assumption (11) and the identity (33), after some algebra, one finds [21]

$$[\Theta, H_{||}(t)] = \mathcal{A}(t) + \mathcal{B}(t), \tag{39}$$

where:

$$\mathcal{A}(t) = P[\Theta, H]U(t)P(U_{||}(t))^{-1}, \qquad \mathcal{B}(t) = \left\{PH - H_{||}(t)P\right\}[\Theta, U(t)]P(U_{||}(t))^{-1}. \tag{40}$$

From (39) we find

$$\Theta H_{||}(t)\Theta^{-1} - H_{||}(t) \equiv (\mathcal{A}(t) + \mathcal{B}(t))\Theta^{-1}. \tag{41}$$

Using this latter relation one finds that

$$h_{11}(t)^* - h_{22}(t) = \langle \mathbf{2} \,|\, (\mathcal{A}(t) + \mathcal{B}(t))\Theta^{-1} \,|\, \mathbf{2} \rangle. \tag{42}$$

Adding expression (42) to its complex conjugate one gets

$$\Re(h_{11}(t) - h_{22}(t)) = \Re\langle \mathbf{2} \,|\, (\mathcal{A}(t) + \mathcal{B}(t))\Theta^{-1} \,|\, \mathbf{2}\rangle. \tag{43}$$

We are considering the case of unstable states, i.e., states $|\mathbf{1}\rangle$, $|\mathbf{2}\rangle$, which lead to such projection operator P (8) that condition (10) holds. In this case we have $\Theta U(t) = U^{+}(t)\Theta$, which gives $\Theta U_{\|}(t) = U_{\|}^{+}(t)\Theta$, $\Theta U_{\|}^{-1}(t) = (U_{\|}^{+}(t))^{-1}\Theta$, and $[\Theta, U(t)] = -2i(\Im U(t))\Theta$. This relation leads to the following result in the case of the conserved CPT-symmetry

$$\mathcal{B}(t) = -2iP\big\{H - H_{\|}(t)P\big\}(\mathrm{Im}\, U(t))P(U_{\|}^{+}(t))^{-1}\Theta. \tag{44}$$

Formula (44) allows us to conclude that $\langle \mathbf{2} \,|\, \mathcal{B}(0)\Theta^{-1} \,|\, \mathbf{2}\rangle = 0$ and $\Re(\langle \mathbf{2} \,|\, \mathcal{B}(t > 0)\Theta^{-1} \,|\, \mathbf{2}\rangle) \neq 0$, if condition (16) holds. This means that in this case it must be $\Re(h_{11}(t)) \neq \Re(h_{22}(t))$ for $t > 0$. So, there is no possibility for $\Re(h_{11})$ to equal $\Re(h_{22})$ for $t > 0$ in the considered case of P fulfilling the condition (10) (i.e., for unstable states) when CPT-symmetry is conserved: It must be $\Re(h_{11}) \neq \Re(h_{22})$.

Assuming the LOY interpretation of $\Re(h_{jj}(t))$, $(j = 1, 2)$, one can conclude from the Khalfin's Theorem and from the properties (37), (43), (44) that if $A_{12}(t)$, $A_{21}(t) \neq 0$ for $t > 0$ and if the total system considered is CPT-invariant, but CP-noninvariant, then $M_{11}(t) \neq M_{22}(t)$ for $t > 0$, that is, that contrary to the case of stable particles (the bound states), the masses of the simultaneously created unstable particle "1" and its antiparticle "2", which are connected by the relation (17), need not be equal for $t > t_0 = 0$. Of course, such a conclusion contradicts the standard LOY result (18), (19). However, one should remember that the LOY description of neutral K mesons and similar complexes is only an approximate one, and that the LOY approximation is not perfect. On the other hand the relation (37) and the Khalfin's Theorem follow from the basic principles of the quantum theory and are rigorous. Consequently, their implications should also be considered as rigorous.

5 Final remarks

Note that properties of the more accurate approximation described in Section 3 are consistent with the general properties and conclusions obtained in Section 4 for the exact effective Hamiltonian – compare (24) and (38) and relations (37) with (28). From the result (28) it follows that in the case of the approximate $H_{||}$, $\Delta h = 0$ can be achieved only if $H_{12} = H_{21} = 0$. This means that if the first order $|\Delta S| = 2$ interactions are forbidden in the $K_0, \overline{K}_0$ complex then predictions following from the use of the mentioned more accurate approximation and from the LOY theory should lead to the the same masses for K_0 and for $\overline{K}_0$. This does not contradict the results of Section 3 derived for the exact $H_{||}$: the mass difference is very, very small and should arise at higher orders of the more accurate approximation. On the other hand from (28) it follows that $\Delta h \neq 0$ if and only if $H_{12} \neq 0$. This means that if measurable deviations from the LOY predictions concerning the masses of, e.g. K_0, $\overline{K}_0$ mesons are ever detected, then the most plausible interpretation of this result will be the existence of first order $|\Delta S| = 2$ interactions in the system considered.

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