On Linearization of Superon-Graviton Model (SGM)

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Referring to the supermultiplet of $N = 1$ supergravity(SUGRA) the linearization of $N = 1$ SGM action describing the nonlinear supersymmetric (NL SUSY) gravitational interaction of superon (Nambu–Goldstone (N-G) fermion) is attempted. The field contents of on-shell SUGRA supermultiplet are realized as the composites, though they have new SUSY transformations which closes on super-Poincaré (SP) algebra. Particular attentions are paid to the local Lorentz invariance.

1 Introduction

Extending the geometrical arguments of Einstein general relativity theory (EGRT) on Riemann space-time to new space-time where the coset space coordinates of $\frac{\text{superGL(4,R)}}{\text{GL(4,R)}}$ turning to the N-G fermion degrees of freedom (d.o.f.) are attached at every Riemann space-time point, we have proposed a new Einstein–Hilbert (E-H) type action [1]. The new E-H type action describes the NLSUSY [2] invariant gravitational interaction of N-G fermion *superon* in Riemann space-time.

In this paper we would like to discuss the linearization of the new E-H type action ($N = 1$ SGM action) to obtain the equivalent linear (L) SUSY [3] theory in the low energy, which is renormalizable. Considering a phenomenological potential of SGM, though qualitative and group theoretical, discussed in [4] based upon the composite picture of L SUSY representation and the recent interest in NLSUSY in superstring (membrane) world, the linearization of NLSUSY in curved space-time may be of some general interest.

The linearization of SGM is physically interesting in general, even if the consequent theory were a existing SUGRA-like, for the equivalence among them gives a new insight into the fundamental structure of nature. For the self-contained arguments we review the SGM action briefly. ($N = 1$) SGM action is given by [1];

$$L_{\text{SGM}} = -\frac{c^3}{16\pi G}|w|(\Omega + \Lambda), \tag{1}$$

$$|w| = \det w^a{}_\mu = \det(e^a{}_\mu + t^a{}_\mu), \qquad t^a{}_\mu = \frac{i}{2}\kappa^4(\bar\psi\gamma^a\partial_\mu\psi - \partial_\mu\bar\psi\gamma^a\psi), \tag{2}$$

where $e^a{}_\mu$ is the vierbein of EGRT, ψ is N-G fermion (superon), $\kappa^4 = \left(\frac{c^3\Lambda}{16\pi G}\right)^{-1}$ is the fundamental volume of four-dimensional space-time of Volkov–Akulov (V-A) model [2], and Λ is the *small* cosmological constant related to the strength of the superon-vacuum coupling constant. Ω is a new scalar curvature analogous to the Ricci scalar curvature R of EGRT, whose explicit expression is obtained by just replacing $e^a{}_\mu(x)$ by $w^a{}_\mu(x)$ in Ricci scalar R. These results can be understood intuitively by observing that $w^a{}_\mu(x) = e^a{}_\mu(x) + t^a{}_\mu(x)$ defined by $\omega^a = w^a{}_\mu dx^\mu$, where ω^a is the NL SUSY invariant differential one-form of V-A [2], is invertible,

and $s^{\mu\nu}(x) \equiv w_a{}^\mu(x)w^{a\nu}(x)$ are a unified vierbein and a unified metric tensor in SGM space-time [1,5]. The SGM action (1) is invariant at least under the following symmetry [6]; ordinary GL(4,R), the following new NLSUSY transformation:

$$\delta^{NL}\psi(x) = \frac{1}{\kappa^2}\zeta + i\kappa^2(\bar{\zeta}\gamma^\rho\psi(x))\partial_\rho\psi(x), \qquad \delta^{NL}e^a{}_\mu(x) = i\kappa^2(\bar{\zeta}\gamma^\rho\psi(x))\partial_{[\rho}e^a{}_{\mu]}(x), \qquad (3)$$

where ζ is a constant spinor and $\partial_{[\rho}e^a{}_{\mu]}(x) = \partial_\rho e^a{}_\mu - \partial_\mu e^a{}_\rho$, the following GL(4,R) transformations due to (3):

$$\delta_\zeta w^a{}_\mu = \xi^\nu \partial_\nu w^a{}_\mu + \partial_\mu \xi^\nu w^a{}_\nu, \qquad \delta_\zeta s_{\mu\nu} = \xi^\kappa \partial_\kappa s_{\mu\nu} + \partial_\mu \xi^\kappa s_{\kappa\nu} + \partial_\nu \xi^\kappa s_{\mu\kappa}, \qquad (4)$$

where $\xi^\rho = i\kappa^2\bar{\zeta}\gamma^\rho\psi(x)$ and $s_{\mu\nu} = w^a{}_\mu w_{a\nu}$, and the following local Lorentz transformation on $w^a{}_\mu$:

$$\delta_L w^a{}_\mu = \epsilon^a{}_b w^b{}_\mu \qquad (5)$$

with the local parameter $\epsilon_{ab} = (1/2)\epsilon_{[ab]}(x)$ or equivalently on ψ and $e^a{}_\mu$

$$\delta_L \psi(x) = -\frac{i}{2}\epsilon_{ab}\sigma^{ab}\psi, \qquad \delta_L e^a{}_\mu(x) = \epsilon^a{}_b e^b{}_\mu + \frac{\kappa^4}{4}\varepsilon^{abcd}\bar{\psi}\gamma_5\gamma_d\psi(\partial_\mu\epsilon_{bc}). \qquad (6)$$

The local Lorentz transformation forms a closed algebra, for example, on $e^a{}_\mu(x)$

$$[\delta_{L_1}, \delta_{L_2}]e^a{}_\mu = \beta^a{}_b e^b{}_\mu + \frac{\kappa^4}{4}\varepsilon^{abcd}\bar{\psi}\gamma_5\gamma_d\psi(\partial_\mu\beta_{bc}), \qquad (7)$$

where $\beta_{ab} = -\beta_{ba}$ is defined by $\beta_{ab} = \epsilon_{2ac}\epsilon_1{}^c{}_b - \epsilon_{2bc}\epsilon_1{}^c{}_a$. The commutators of two new NLSUSY transformations (3) on $\psi(x)$ and $e^a{}_\mu(x)$ are GL(4,R), i.e. new NLSUSY (3) is the square-root of GL(4,R):

$$[\delta_{\zeta_1}, \delta_{\zeta_2}]\psi = \Xi^\mu \partial_\mu \psi, \qquad [\delta_{\zeta_1}, \delta_{\zeta_2}]e^a{}_\mu = \Xi^\rho \partial_\rho e^a{}_\mu + e^a{}_\rho \partial_\mu \Xi^\rho, \qquad (8)$$

where $\Xi^\mu = 2i(\bar{\zeta}_2\gamma^\mu\zeta_1) - \xi_1^\rho\xi_2^\sigma e_a{}^\mu(\partial_{[\rho}e^a{}_{\sigma]})$. They show the closure of the algebra. SGM action (1) is invariant at least under [6]

$$[\text{global NLSUSY}] \otimes [\text{local GL}(4, \text{R})] \otimes [\text{local Lorentz}], \qquad (9)$$

which is isomorphic to SP whose single irreducible representation gives the group theoretical description of SGM [4].

2 Linearization of SGM

Here we just mention that the SGM action of equation (1) is a nontrivial generalization of E-H action. Interestingly, the following local spinor translation with a local parameter $\epsilon(x)$, $\delta\psi = \epsilon$, $\delta e^a{}_\mu = -i\kappa^4(\bar{\epsilon}\gamma^a\partial_\mu\psi + \bar{\psi}\gamma^a\partial_\mu\epsilon)$, gives $\delta w^a{}_\mu = 0 = \delta w_a{}^\mu$. However, this local spinor transformation cannot transform away the d.o.f. of ψ. Indeed, ψ seems to be transformed away if we choose $\delta\psi = \epsilon = -\psi$, but it is restored precisely in the unified vierbein $w^a{}_\mu$ by simultaneously transforming $e^a{}_\mu$, i.e., $w(e, \psi) = w(e + \delta e, \psi + \delta\psi) = w(e + t, 0)$ as indicated by $\delta w^a{}_\mu = 0$. And also the above local spinor transformation is a fake gauge transformation in a sense that, in contrast with the local Lorentz transformation on the coordinates in the vierbein formalism of EGRT, it cannot eliminate the d.o.f. of ψ since the unified vierbein $w^a{}_\mu = e^a{}_\mu + t^a{}_\mu$ is the only gauge field on SGM space-time and contains only integer spin. This confusing situation comes from the new geometrical formulation of SGM on unfamiliar SGM space-time, where

besides the Minkowski coordinates x^a, ψ is a Grassmann coordinate (i.e. the fundamental d.o.f.) defining the tangential space-time with $SO(3,1) \times SL(2,C)$ d.o.f. inspired by NLSUSY. The local spinor transformation ($\delta\psi = \epsilon(x)$) is just a coordinate transformation (redefinition) on SGM space-time. This situation can be understood easily by observing that the unified vierbein $w^a{}_\mu = e^a{}_\mu + t^a{}_\mu$ is defined by $\omega^a = dx^a + i\kappa^4\bar{\psi}\gamma^a d\psi = w^a{}_\mu dx^\mu$, where ω^a is the NLSUSY invariant differential one-form of V-A [2] and (x^a, ψ) are coordinates specifying the (SGM) flat space-time inspired by NLSUSY.

From these geometrical viewpoints (in SGM space-time) we can understand that ψ is a coordinate and would not be transformed away and the initial SGM space-time is preserved. Eliminating ψ by some arguments regarding the above local spinor translation as a gauge transformation leads to a different theory (ordinary E-H action) with a different vacuum (Minkowski flat space-time), which is another from SGM scenario assuming that the SGM space-time is an ultimate physical entity.

The linearization of such a highly nonlinear theory is interesting and inevitable for obtaining of a renormalizable field theory which is equivalent and describes the observed low energy physics.

The flat space linearization of $N = 1$ V-A model has been carried out, and it was proved that $N = 1$ V-A model is equivalent to $N = 1$ scalar supermultiplet [7] or $N = 1$ axial vector gauge supermultiplet of linear SUSY [8]. As a flat space exercise for the extended SGM linearization, we have carried out the linearization of $N = 2$ V-A model and have shown that it is equivalent to the spontaneously broken $N = 2$ linear SUSY *vector* $J^P = 1^-$ gauge supermultiplet model with SU(2) structure [9]. Interestingly, SU(2) algebraic gauge structure of the electroweak standard model (SM) may be explained for the first time provided that the electroweak gauge bosons are the composite fields of this (SGM) type in the low energy.

In these work the linearization is carried out by using the superfield formalism and/or by the heuristic and intuitive arguments on the relations between the component fields of L SUSY and NLSUSY. In either case it is crucial to discover the SUSY invariant relations, which connect the supermultiplets of L and NL theories and reproduce the SUSY transformations.

In above-mentioned cases of the global SUSY in flat space-time the SUSY invariant relations are obtained straightforwardly, for L and NL supermultiplets are well understood and the algebraic structures are the same SP.

The situation is rather different in SGM, for (i) the supermultiplet structure of the linearized theory of SGM is unknown except if it is expected to be a broken SUSY SUGRA-like theory containing graviton and a (massive) spin 3/2 field as dynamical d.o.f. and (ii) the algebraic structure (9) is changed into SP.

Therefore, by the heuristic arguments and referring to SUGRA we discuss for the moment the linearization of $N = 1$ SGM.

At first, we assume faithfully the SGM scenario that:

(i) the linearized theory should contain the spontaneously broken *global* (at least) SUSY;

(ii) graviton is an elementary field (not composite of superons corresponding to the vacuum of the Clifford algebra) in both L and NL theories;

(iii) the NLSUSY supermultiplet of SGM ($e^a{}_\mu(x)$, $\psi(x)$) should be connected to the composite supermultiplet ($\tilde{e}^a{}_\mu(e(x), \psi(x))$, $\tilde{\lambda}_\mu(e(x), \psi(x))$) for elementary graviton field and a composite (massive) spin 3/2 field of the SUGRA-like linearized theory.

From these assumptions and following the arguments used in the flat space cases we require that the SUGRA gauge transformation [10] with the global spinor parameter ζ should hold for the supermultiplet ($\tilde{e}^a{}_\mu(e, \psi)$, $\tilde{\lambda}_\mu(e, \psi)$) of the (SUGRA-like) linearized theory, i.e.,

$$\delta\tilde{e}^a{}_\mu(e, \psi) = i\kappa\bar{\zeta}\gamma^a\tilde{\lambda}_\mu(e, \psi), \tag{10}$$

$$\delta\tilde{\lambda}_\mu(e,\psi) = \frac{2}{\kappa}D_\mu\zeta = -\frac{i}{\kappa}\tilde{\omega}(e,\psi)_\mu{}^{ab}\sigma_{ab}\zeta, \tag{11}$$

where $\sigma^{ab} = \frac{i}{4}[\gamma^a,\gamma^b]$, $D_\mu = \partial_\mu - \frac{i}{2}\omega_\mu{}^{ab}(e,\psi)\sigma_{ab}$, ζ is a global spinor parameter and the variations in the left-hand side are induced by NLSUSY (3).

We put the following SUSY invariant relations, which connect $e^a{}_\mu$ to $\tilde{e}^a{}_\mu(e,\psi)$:

$$\tilde{e}^a{}_\mu(e,\psi) = e^a{}_\mu(x). \tag{12}$$

This relation (12) is the assumption (ii) and holds simply from the metric conditions. Consequently, the following covariant relation is obtained by substituting (12) into (10) and computing the variations under (3) [11]:

$$\tilde{\lambda}_\mu(e,\psi) = \kappa\gamma_a\gamma^\rho\psi(x)\partial_{[\rho}e^a{}_{\mu]}. \tag{13}$$

(As discussed later, these may be considered as the leading order of the expansions in κ of SUSY invariant relations. The expansions terminate with $(\psi)^4$. Now we see LSUSY transformation induced by (3) on the (composite) supermultiplet $(\tilde{e}^a{}_\mu(e,\psi), \tilde{\lambda}_\mu(e,\psi))$.)

The LSUSY transformation on $\tilde{e}^a{}_\mu$ becomes as follows. The left-hand side of (10) gives

$$\delta\tilde{e}^a{}_\mu(e,\psi) = \delta^{NL}e^a{}_\mu(x) = i\kappa^2(\bar{\zeta}\gamma^\rho\psi(x))\partial_{[\rho}e^a{}_{\mu]}(x). \tag{14}$$

While substituting (13) into the right-hand side of (10) we obtain

$$i\kappa^2(\bar{\zeta}\gamma^\rho\psi(x))\partial_{[\rho}e^a{}_{\mu]}(x) + \cdots \text{ (extra terms)}. \tag{15}$$

These results show that (12) and (13) are not SUSY invariant relations and reproduce (10) with unwanted extra terms, which should be identified with the auxiliary fields. The commutator of the two LSUSY transformations induces GL(4,R) with the field dependent parameters as follows:

$$[\delta_{\zeta_1},\delta_{\zeta_2}]\tilde{e}^a{}_\mu(e,\psi) = \Xi^\rho\partial_\rho\tilde{e}^a{}_\mu(e,\psi) + \tilde{e}^a{}_\rho(e,\psi)\partial_\mu\Xi^\rho, \tag{16}$$

where $\Xi^\mu = 2i(\bar{\zeta}_2\gamma^\mu\zeta_1) - \xi_1^\rho\xi_2^\sigma e_a{}^\mu(\partial_{[\rho}e^a{}_{\sigma]})$.

On $\tilde{\lambda}_\mu(e,\psi)$, the left-hand side of (11) becomes apparently rather complicated:

$$\begin{aligned}\delta\tilde{\lambda}_\mu(e,\psi) &= \kappa\delta(\gamma_a\gamma^\rho\psi(x)\partial_{[\rho}e^a{}_{\mu]})\\&= \kappa\gamma_a[\delta^{NL}\gamma^\rho\psi(x)\partial_{[\rho}e^a{}_{\mu]} + \gamma^\rho\delta^{NL}\psi(x)\partial_{[\rho}e^a{}_{\mu]} + \gamma^\rho\psi(x)\partial_{[\rho}\delta^{NL}e^a{}_{\mu]}].\end{aligned} \tag{17}$$

However, the commutator of the two LSUSY transformations induces the similar GL(4,R):

$$[\delta_{\zeta_1},\delta_{\zeta_2}]\tilde{\lambda}_\mu(e,\psi) = \Xi^\rho\partial_\rho\tilde{\lambda}_\mu(e,\psi) + \tilde{\lambda}_\rho(e,\psi)\partial_\mu\Xi^\rho. \tag{18}$$

These results indicate that it is necessary to generalize (10), (11) and (13) for obtaining SUSY invariant relations and for the closure of the algebra. Furthermore, due to the complicated expression of LSUSY (17) that makes the physical and mathematical structures obscure, we can hardly guess a linearized invariant action equivalent to SGM.

Now we generalize the linearization by considering the auxiliary fields such that LSUSY transformation on the linearized fields induces SP transformation.

By comparing (11) with (17) we understand that the local Lorentz transformation plays a crucial role. As for the local Lorentz transformation on the linearized asymptotic fields corresponding to the observed particles (in the low energy), it is natural to take (irrespective of (6)) the following forms

$$\delta_L\tilde{\lambda}_\mu(x) = -\frac{i}{2}\epsilon_{ab}\sigma^{ab}\tilde{\lambda}_\mu(x), \qquad \delta_L\tilde{e}^a{}_\mu(x) = \epsilon^a{}_b\tilde{e}^b{}_\mu, \tag{19}$$

where $\epsilon_{ab} = (1/2)\epsilon_{[ab]}(x)$ is a local parameter. The relation between (6), i.e. the ultimate Lorentz invariance encoded geometrically in SGM space-time, and (19), i.e. the Lorentz invariance defined on the (composite) asymptotic field in Riemann space-time, is unclear. In SGM the local Lorentz transformations (5) and (6), i.e. the local Lorentz invariant gravitational interaction of superon, are introduced by the geometrical arguments in SGM space-time [6] following EGRT. While in SUGRA theory the local Lorentz transformation invariance (19) is realized as usual by introducing the Lorentz spin connection $\omega_\mu{}^{ab}$. And the LSUSY transformation is defined successfully by the (Lorentz) covariant derivative containing the spin connection $\tilde{\omega}_\mu{}^{ab}(e, \psi)$ as seen in (11), which causes the super-Poincaré algebra on the commutator of SUSY and is convenient for constructing the invariant action. Therefore, in the linearized (SUGRA-like) theory the local Lorentz transformation invariance is expected to be realized as usual by defining (19) and introducing the Lorentz spin connection $\omega_\mu{}^{ab}$. We investigate how the spin connection $\tilde{\omega}_\mu{}^{ab}(e, \psi)$ appears in the linearized (SUGRA-like) theory through the linearization process. This is also crucial for constructing a nontrivial (interacting) linearized action, which has manifest invariances.

We discuss the Lorentz covariance of the transformation by comparing (17) with the right-hand side of (11). The direct computation of (11) by using SUSY invariant relations (12) and (13) under (3) produces complicated redundant terms as read off from (17). The local Lorentz invariance of the linearized theory may become ambiguous and lose the manifest invariance.

For a simple restoration of the manifest local Lorentz invariance we survey the possibility that such redundant terms may be adjusted by the d.o.f. of the auxiliary fields in the linearized supermultiplet. As for the auxiliary fields it is necessary for the closure of the off-shell superalgebra to include the equal number of the fermionic and the bosonic d.o.f. in the linearized supermultiplet. As new NLSUSY is a global symmetry, $\tilde{\lambda}_\mu$ has 16 fermionic d.o.f.. Therefore at least 4 bosonic d.o.f. must be added to the off-shell SUGRA supermultiplet with 12 d.o.f. [12] and a vector field may be a simple candidate. However, counting the bosonic d.o.f. present in the redundant terms corresponding to $\tilde{\omega}_\mu{}^{ab}(e, \psi)$, we may need a bigger supermultiplet e.g. $16 + 4 \cdot 16 = 80$ d.o.f., to carry out the linearization, in which case a rank-3 tensor $\phi_{\mu\nu\rho}$ and a rank-2 tensor-spinor $\lambda_{\mu\nu}$ may be candidates for the auxiliary fields.

Now we consider the simple modification of SUGRA transformations (algebra) by adjusting the (composite) structure of the (auxiliary) fields. We take, instead of (10) and (11),

$$\delta\tilde{e}^a{}_\mu(x) = i\kappa\bar{\zeta}\gamma^a\tilde{\lambda}_\mu(x) + \bar{\zeta}\tilde{\Lambda}^a{}_\mu, \tag{20}$$

$$\delta\tilde{\lambda}_\mu(x) = \frac{2}{\kappa}D_\mu\zeta + \tilde{\Phi}_\mu\zeta = -\frac{i}{\kappa}\tilde{\omega}_\mu{}^{ab}\sigma_{ab}\zeta + \tilde{\Phi}_\mu\zeta, \tag{21}$$

where $\tilde{\Lambda}^a{}_\mu$ and $\tilde{\Phi}_\mu$ represent symbolically the auxiliary fields $80 + 80$ and are functionals of $e^a{}_\mu$ and ψ. We need $\tilde{\Lambda}^a{}_\mu$ term in (20) to alter (14), (16), (17) and (18) toward that of super-Poincaré algebra of SUGRA. We attempt the restoration of the manifest local Lorentz invariance order by order by adjusting $\tilde{\Lambda}^a{}_\mu$ and $\tilde{\Phi}_\mu$. In fact, the Lorentz spin connection $\omega_\mu{}^{ab}(e)$ (i.e. the leading order terms of $\tilde{\omega}_\mu{}^{ab}(e, \psi)$) of (21) is reproduced by taking the following one

$$\tilde{\Lambda}^a{}_\mu = \frac{\kappa^2}{4}[ie_b{}^\rho\partial_{[\rho}e^b{}_{\mu]}\gamma^a\psi - \partial_{[\rho}e_{|b|\sigma]}e^b{}_\mu\gamma^a\sigma^{\rho\sigma}\psi], \tag{22}$$

which holds (16). Accordingly $\tilde{\lambda}_\mu(e, \psi)$ is determined up to the first order in ψ as follows:

$$\tilde{\lambda}_\mu(e, \psi) = \frac{1}{2i\kappa}(i\kappa^2\gamma_a\gamma^\rho\psi(x)\partial_{[\rho}e^a{}_{\mu]} - \gamma_a\tilde{\Lambda}^a{}_\mu) = -\frac{i\kappa}{2}\omega_\mu{}^{ab}(e)\sigma_{ab}\psi, \tag{23}$$

which indicates the minimal Lorentz covariant gravitational interaction of superon. Substituting (23) into (21) we obtain the following new LSUSY transformation of $\tilde{\lambda}_\mu$ (after Fiertz transformations)

$$\delta\tilde{\lambda}_\mu(e, \psi) = -\frac{i\kappa}{2}\{\delta^{NL}\omega_\mu{}^{ab}(e)\sigma_{ab}\psi + \omega_\mu{}^{ab}(e)\sigma_{ab}\delta^{NL}\psi\}$$

$$= -\frac{i}{2\kappa}\omega_\mu{}^{ab}(e)\sigma_{ab}\zeta + \frac{i\kappa}{2}\left\{\tilde{\epsilon}^{ab}(e,\psi)\sigma_{ab}\cdot\omega_\mu{}^{cd}(e)\sigma_{cd}\psi + \cdots\right\}. \tag{24}$$

Remarkably, the local Lorentz transformations of $\tilde{\lambda}_\mu(e,\psi)$ (i.e. the second term) with the field dependent antisymmetric parameters $\tilde{\epsilon}^{ab}(e,\psi)$ are induced in addition to the intended ordinary global SUSY transformation. (23) are the SUSY invariant relations for $\tilde{\lambda}_\mu(e,\psi)$ in the lowest order with ψ, for the SUSY transformation of (23) gives the right hand side of (21) with the consequent auxiliary fields. Interestingly the commutator of the two LSUSY transformations on (23) induces GL(4,R):

$$[\delta_{\zeta_1},\delta_{\zeta_2}]\tilde{\lambda}_\mu(e,\psi) = \Xi^\rho\partial_\rho\tilde{\lambda}_\mu(e,\psi) + \partial_\mu\Xi^\rho\tilde{\lambda}_\rho(e,\psi), \tag{25}$$

where Ξ^ρ is the same field dependent parameter as given in (16). (16) and (25) show the closure of the algebra on SP algebra provided that the SUSY invariant relations (12) and (23) are adopted. These phenomena coincide with SGM scenario [1,4] from the algebraic point of view, i.e. they are the superon-graviton composite (eigenstates) corresponding to the linear representations of SP algebra. As for the redundant terms in (24) with $(\psi)^2$ the SUSY transformations we can recast them by considering the modified spin connection $\tilde{\omega}_\mu{}^{ab}(e,\psi)$ particularly with the contorsion terms and (the auxiliary field) $\tilde{\Phi}_\mu(e,\psi)$. In fact, we have proved that the contributions to (24) from the SUGRA-inspired contorsion terms:

$$\begin{aligned}
K_{\mu ab}\sigma^{ab}\psi &= \frac{i\kappa^2}{4}(\bar{\tilde{\lambda}}_a\gamma_b\tilde{\lambda}_\mu - \bar{\tilde{\lambda}}_b\gamma_a\tilde{\lambda}_\mu + \bar{\tilde{\lambda}}_a\gamma_\mu\tilde{\lambda}_b)\sigma^{ab}\psi \\
&= \frac{i\kappa^4}{16}\{(e_a{}^\mu\bar{\psi}\sigma^{cd}\omega_{\nu cd}(e)\gamma_b\omega_{\mu fg}(e)\sigma^{fg}\psi - [a\leftrightarrow b] + \cdots\}\sigma^{ab}\psi,
\end{aligned} \tag{26}$$

satisfies

$$[\delta_{\zeta_1},\delta_{\zeta_2}](K_{\mu ab}\sigma^{ab}\psi) = \Xi^\rho\partial_\rho(K_{\mu ab}\sigma^{ab}\psi) + \partial_\mu\Xi^\rho(K_{\rho ab}\sigma^{ab}\psi). \tag{27}$$

We can obtain the complete linearized (off-shell) supermultiplets of the super-Poincaré algebra by repeating the similar procedures (including the auxiliary fields $80+80$) order by order which terminates with $(\psi)^4$. The complicated procedures has been carried out successfully up to $O(\psi^2)$ [13].

Finally, we mention another way of the systematic linearization by using the superfield formalism applied to the coupled system of V-A action with SUGRA [14]. We can define on such a coupled system a local spinor gauge symmetry, which induces a super-Higgs mechanism [15] converting V-A field to the longitudinal component of massive spin $3/2$ field. The consequent Lagrangian obtained may be an analogue of what we anticipated in above discussions but with the elementary spin $3/2$ field.

The linearization of the new E-H type action (1) with the extra dimensions, which gives another unification framework describing the observed particles as elementary fields, is open. And the linearization of SGM action for spin $3/2$ N-G fermion field [16] (with extra dimensions) may be in the same scope.

3   Discussion

Now we summarize the results as follows. Referring to SUGRA transformations we have attempted explicitly the linearization of $N=1$ SGM up to $O(\psi^2)$ in the (SUGRA-like) LSUSY transformations. The closure of the new LSUSY transformations (20) and (24) on the linearized supermultiplet, which are different from SUGRA transformations, can be proved order by order with ψ by introducing the auxiliary fields. It is interesting that the simple relation

$\lambda_\mu = e^a{}_\mu \gamma_a \psi + \cdots$, which is suggested by the flat space-time linearization, seems disfavour with the SGM linearization. As conjectured before, what LSUSY SP may be to SGM in quantum field theory, what $O(4)$ symmetry is to the relativistic hydrogen model in quantum mechanics, which is tested by the linearization. The linearization of SGM is physically interesting in general, even if it were a existing SUGRA-like theory, for the consequent broken LSUSY theory is shown to be equivalent and gives a new insight into the fundamental structure of nature (like the relation between Landau–Ginzberg theory and BCS theory for superconductivity). The complete linearization to all orders, which can be anticipated by the systematics emerging in the present study, needs specifications of the auxiliary fields and remains to be studied. They are now in progress.

Acknowledgements

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