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The Sigma-Model Representation for the Duality-Symmetric $D = 11$ Supergravity

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In this paper which is based on the results obtained in collaboration with Igor Bandos and Dmitri Sorokin I present the extention of the "doubled fields" formalism by Cremmer, Julia, Lü and Pope to the supersymmetric case and lifting this construction onto the level of the proper duality-symmetric action for $D = 11$ supergravity. Further extension of the doubled field formulation to include dynamical sources is also discussed.

1 Introduction

This year we can truly commemorate the 25th anniversary from the day when eleven-dimensional supergravity [1] was born. The importance of this event is worth mentioning since supergravity in $D = 11$ space-time dimensions is a kind of special theory possessing a number of non-trivial features. First of all, this theory is the maximally-dimensional supersymmetric theory of gravity interacting with antisymmetric tensor fields for the standard signature of metric with one time-like direction. Second, the theory has relatively simple structure of the action in comparison to other low-dimensional maximal supergravities. The third point consists in the observation that the massless spectrum of a theory derived upon a compactification from $D = 11$ down to $D = 4$ does not contain particles with spin higher than two that is an indication of having a consistent four-dimensional interacting theory. Last but not least the eleven-dimensional supergravity is the low-energy limit of M-theory unifying non-perturbatively five different theories of superstrings. Since not much has been known about M-theory, $D = 11$ supergravity is very useful tool to investigate the M-theory structure.

One of the ways to do so is to consider different routes of dimensional reduction to establish the connection between different low-energy effective actions for superstrings which are nothing but supergravities and to find a relevant scheme that would be in accordance with four-dimensional phenomenology. Following this way another notion, the duality, becomes important and allows us to establish the symmetry structure in reduced theories which is originally hidden. Moreover, the duality gives a fresh look on the symmetry structure in higher-dimensional supergravities that allows one to find new way for describing in a uniform duality invariant manner gauge and internal symmetries of these theories.

It should be emphasized that the notion of duality in the context of searching new symmetries in dimensionally reduced supergravities [2] appeared one year after discovering the eleven-dimensional supergravity. There discovered was a hidden symmetry structure of maximal low-dimensional supergravities that extends the naively expected global symmetry groups arising in the process of dimensional reduction on multidimensional tori. Some important observations were made that were crucial to develop the technique to derive later on the low-dimensional theories of supergravities by reduction on spheres and manifolds with non-trivial holonomy groups.

The aim of this paper is twofold. First, we would like to review the old results reminding ideas and methods that led to the discovery of hidden global symmetry structure in maximal supergravities and to trace how these ideas and methods naturally get transformed for the description of local symmetries in such theories [4]. And second, we present new results [8] which generalize the recent construction of local symmetries in uniform duality invariant manner. Though we will note only the key points that are necessary for the discussion in what follows, all the details can be read off original papers [2–6, 8].

2 Hidden global symmetries in maximal supergravities

Let me get started with recalling that D-dimensional maximal supergravity is the theory which follows from D=11 supergravity by dimensional reduction on torus $T^{(11-D)}$. Typically, such a theory is expected to be invariant under the product of a non-compact global group and a compact local group $SL(11-D,\mathbb{R})_{\text{Global}} \otimes SO(11-D)_{\text{Local}}$. The former group acts on tensors and scalars, while the latter one acts on fermions and scalars. However, close inspection reveals the hidden symmetry group structure extending the structure above. This is due to "conspiracy" of scalars and tensors coming in the reduction process from gravitational and tensor gauge fields sectors and of scalars and tensors appearing after dualizing a higher p-rank tensor field with $p \geq D/2$ into a $(D-2-p)$-rank tensor field. Recasting the fields the following symmetry structure can be recovered [6]

D	G_{Global}	H_{Local}
$D = 11$	$E_0 \sim \mathbf{1}$	$\mathbf{1}$
$D = 10$	$E_1 \sim SO(1,1)/Z_2$	$\mathbf{1}$
$D = 9$	$E_2 = GL(2,\mathbb{R})$	$SO(2)$
$D = 8$	$E_{3(+3)} = SL(3,\mathbb{R}) \otimes SL(2,\mathbb{R})$	$U(2) = SO(3) \otimes SO(2)$
$D = 7$	$E_{4(+4)} = SL(5,\mathbb{R})$	$Usp(4) = SO(5)$
$D = 6$	$E_{5(+5)} = SO(5,5)$	$Usp(4) \otimes Usp(4)$
$D = 5$	$E_{6(+6)}$	$Usp(8)$
$D = 4$	$E_{7(+7)}$	$SU(8)$
$D = 3$	$E_{8(+8)}$	$SO(16)$

Let me now introduce the basic notion of a twisted self-duality condition which will be important in what follows. To this end note that when the dimension D is odd the E_{11-D} symmetry is realized on the gauge potentials and is an invariance of the Lagrangian. However, as soon as the space-time dimension is even, the E_{11-D} symmetry for field strengths of degree $D/2$ is realized on the field strengths rather than on the potentials and is an invariance of the equations of motion and Bianchi identities only. But one can still realize such a symmetry at the level of equations of motion on the potentials if one introduces their additional dual partners. After that the equations of motion take the form of the twisted self-duality conditions [2]

$$\mathcal{V}\mathcal{F}^{(D/2)} = \Omega\mathcal{V} * \mathcal{F}^{(D/2)}, \tag{1}$$

where Ω is the symplectic metric of the Sp group that contains the E_{11-D} as a subgroup, $\mathcal{V}$ is the matrix of scalars in the SL subgroup of the E_{11-D} and $*$ stands for D-dimensional Hodge operator.

The meaning of the twisted self-duality conditions is clear. As soon as we introduced additional dual potentials, the number of degrees of freedom for corresponding tensor fields became twice higher than we need. The twisted self-duality condition reduces this number going back to the standard situation. However, curiously, the corresponding equations of motion for our theory have been encoded in these conditions.

What we have learned from these observations? We should emphasize the role of duality in the establishing the global hidden symmetries in maximal supergravities. Later on dualities were recognized as a basic tool to establish hidden non-perturbative symmetries of M-theory. The next point is that we have another description for the dynamics of self-dual fields having the field strengths of the degree of $D/2$. Two questions arise: Could we apply similar approach to describe the dynamics of fields dual to each other with respect to the Hodge operation and which are not necessary self-dual? And could we generalize the construction to describe local symmetries of the theory?

The answer came from the observation made by Cremmer, Julia, Lü and Pope [4]. Motivated by studying the global symmetry groups in maximal supergravities in the context of the M-theory conjecture, the authors observed that doubling the gauge fields, as well as dilatonic scalars and axionic scalars by their duals one can present the bosonic equations of motion for this sector of supergravities, starting from $D = 11$ supergravity, in the form which generalizes the twisted self-duality conditions

$$*\mathcal{G} = \mathcal{S}\mathcal{G}. \tag{2}$$

Here $\mathcal{G}$ is a combined single field written in terms of the exponential of linear combinations of generators for each field and its double, with all the potentials as coefficients. $\mathcal{S}$ is a pseudo-involution that exchanges the generators for fields and those for their partners under the doubling. Complete construction is a remnant of G/H coset structure of scalar fields well known in supergravities and can be regarded as its generalization. Since the dynamics of scalars is described by the G/H sigma-model action one can expect the sigma-model structure for "doubled fields action" constructed out of $\mathcal{G}$, if the action does exist. To find such an action, let me be more precise confining myself to the dimension eleven space-time and presenting the details of the construction by Cremmer, Julia, Lü and Pope for $D = 11$ supergravity. Although this case is simple in comparison with even ten-dimensional type IIA supergravity obtained from $D = 11$ supergravity by dimensional reduction, it is generic and possesses all features for the doubled field approach.

3 Doubled field formulation of $D = 11$ supergravity

From the Lagrangian for the bosonic sector of $D = 11$ supergravity [1]

$$\mathcal{L} = -\sqrt{g}\, R - \frac{1}{48}\sqrt{g}\, F_{mnpq}F^{mnpq} - \frac{1}{3}A^{(3)} \wedge F^{(4)} \wedge F^{(4)}, \qquad F^{(4)} = dA^{(3)} \tag{3}$$

we get the second order equation of motion for the $A^{(3)}$ gauge field

$$d(*F^{(4)} - A^{(3)} \wedge F^{(4)}) = 0 \tag{4}$$

that can be presented as the Bianchi identity for the dual field $A^{(6)}$

$$dF^{(7)} = 0, \qquad F^{(7)} = dA^{(6)} + A^{(3)} \wedge F^{(4)}. \tag{5}$$

Now forget for a while the dynamical origin of $F^{(7)} = dA^{(6)} + A^{(3)} \wedge F^{(4)}$ and introduce it as an independent partner of $F^{(4)}$. These field strengths are invariant under the local gauge transformations

$$\delta A^{(3)} = \Lambda^{(3)}, \qquad \delta A^{(6)} = \Lambda^{(6)} - \Lambda^{(3)} \wedge A^{(3)} \tag{6}$$

with closed forms $\Lambda^{(3)}$, $\Lambda^{(6)}$ associated with the so-called large gauge transformations [7]. Because of the presence of "bare" $A^{(3)}$ in $F^{(7)}$ and therefore in $\delta A^{(6)}$ that is traced back to the presence of the Chern–Simons term in the Lagrangian (3) the large gauge transformations are non-Abelian

$$[\delta_{\Lambda_1^{(3)}}, \delta_{\Lambda_2^{(3)}}] = \delta_{\Lambda^{(6)}}, \qquad [\delta_{\Lambda^{(3)}}, \delta_{\Lambda^{(6)}}] = [\delta_{\Lambda_1^{(6)}}, \delta_{\Lambda_2^{(6)}}] = 0. \tag{7}$$

These relations can be associated with a superalgebra generated by a "Grassmann-odd" generator t_3 and a commuting generator t_6

$$\{t_3, t_3\} = -2t_6, \qquad [t_3, t_6] = [t_6, t_6] = 0 \tag{8}$$

after that one can realize an element of the supergroup

$$\mathcal{A} = e^{t_3 A^{(3)}} e^{t_6 A^{(6)}} \tag{9}$$

and introduce the Cartan form

$$\mathcal{G} = d\mathcal{A}\mathcal{A}^{-1} = F^{(4)} t_3 + F^{(7)} t_6, \tag{10}$$

which by definition satisfies the Maurer–Cartan equation called sometimes the zero-curvature condition

$$d\mathcal{G} + \mathcal{G} \wedge \mathcal{G} = 0. \tag{11}$$

To impose the duality relation between a priori independent field strengths and arriving therefore at the standard number of degrees of freedom one introduces the pseudo-involution operator $\mathcal{S}$ which interchanges the generators t_3 and t_6

$$\mathcal{S} t_3 = t_6, \qquad \mathcal{S} t_6 = t_3, \qquad \mathcal{S}^2 = 1. \tag{12}$$

Using $\mathcal{S}$ and the Hodge operator one can immediately check that the following condition

$$*\mathcal{G} = \mathcal{S}\mathcal{G} \tag{13}$$

reproduces correctly the duality relations between the field strenghts and therefore reduces tensors' degrees of freedom to the correct number. Moreover, when this condition holds the Maurer-Cartan equation amounts to second order equations of motion for $F^{(4)}$ and $F^{(7)}$.

Since the relation (13) enjoys all necessary properties of the condition (1) it also shares the name of the twisted self-duality condition.

So, we have explicitly described the doubled field approach to the bosonic sector of $D = 11$ supergravity. However, we should still add the fermions and construct a proper action for the doubled fields. We are now turning to these points.

4 Generalization of doubled field approach to fermions and lifting it onto the level of action

Adding the fermions is a simple task since what one should do is to extend $\mathcal{G}$ with the superalgebra valued element $\mathcal{C} = -C^{(4)} t_3 + C^{(7)} t_6$ [8]

$$\mathcal{G} \longrightarrow \mathcal{G} + \mathcal{C}, \tag{14}$$

where $C^{(4)}$ and $C^{(7)}$ are defined as

$$C^{(4)} = -\frac{1}{4}\bar{\psi} \wedge \Gamma^{(2)} \wedge \psi, \qquad C^{(7)} = \frac{i}{4}\bar{\psi} \wedge \Gamma^{(5)} \wedge \psi,$$
$$\Gamma^{(n)} = \frac{1}{n!} dx^{m_n} \wedge \cdots \wedge dx^{m_1} \Gamma^{(n)}_{m_1 \cdots m_n}, \tag{15}$$

and ψ^α is the one-form associated with the Majorana spin 3/2 gravitino ψ^α_m.

After this replacement the twisted self-duality condition becomes

$$*(\mathcal{G}+\mathcal{C}) = \mathcal{S}(\mathcal{G}+\mathcal{C}) \longrightarrow (\mathcal{S}-*)(\mathcal{G}+\mathcal{C}) = 0. \tag{16}$$

Therefore, if one finds a way to construct the action from which the twisted self-duality condition (16) will follow as an equation of motion the task will be completed. Fortunately, this way exists and is based on the PST technique [9] developed to construct the Lagrangians in theories with self-dual or duality-symmetric fields.

Applying the PST technique the twisted self-duality condition is reproduced from the following action [8]

$$\begin{aligned} S = S_{EH} + S_\psi - \mathrm{Tr}\int_{\mathcal{M}^{11}} \frac{1}{2}\left(\mathcal{G}+\frac{1}{2}\mathcal{C}\right) \wedge (\mathcal{S}-*)\mathcal{C} \\ - \mathrm{Tr}\int_{\mathcal{M}^{11}} \left[\frac{1}{4} * \mathcal{G}\wedge\mathcal{G} - \frac{1}{12}\mathcal{G}\wedge\mathcal{S}\mathcal{G} - \frac{1}{4} * i_v(\mathcal{S}-*)(\mathcal{G}+\mathcal{C}) \wedge i_v(\mathcal{S}-*)(\mathcal{G}+\mathcal{C})\right], \end{aligned} \tag{17}$$

where S_{EH} and S_ψ stand for the Einstein–Hilbert action and for the fermionic kinetic term [1],

$$v = \frac{da(x)}{\sqrt{-(\partial a)^2}} \tag{18}$$

is the one-form constructed out the PST scalar auxiliary field (cf. [9]) which ensures the covariance of the action and

$$\mathrm{Tr}\,(t_3 t_3) = -\mathrm{Tr}\,(t_6 t_6) = -1, \qquad \mathrm{Tr}\,(t_3 t_6) = 0. \tag{19}$$

As usual we have denoted by i_v the inner product of the vector field v_m with a form.

It is worth mentioning that the action just presented is the generalization of the sigma-model action having typically the form

$$S \propto \int_{\mathcal{M}^D} \mathrm{Tr}\,(*\mathcal{G}\wedge\mathcal{G}) \tag{20}$$

and therefore we have derived the result which we expected from the beginning.

It is an instructive exercise to rewrite the sigma-model action to the standard for $D = 11$ supergravity form. After some manipulations with taking into account the definitions of $\mathcal{G}$ and $\mathcal{S}$ one can arrive at

$$S = S_{CJS} + \int_{\mathcal{M}^{11}} \frac{1}{2} i_v\mathcal{F}^{(4)} \wedge * i_v \mathcal{F}^{(4)} \tag{21}$$

with S_{CJS} being the standard action by Cremmer, Julia and Scherk [1] and $\mathcal{F}^{(4)} = F^{(4)} - *F^{(7)}$. This is the action for the duality-symmetic $D = 11$ supergravity [10] from which one can dynamically derive the duality condition $\mathcal{F}^{(4)} = 0$. Note also that this action depends on both $A^{(3)}$ and $A^{(6)}$ potentials therefore the name duality-symmetric, and on the shell of the duality condition $\mathcal{F}^{(4)} = 0$ it coincides with the Cremmer–Julia–Scherk action.

5 The sigma-model representation of $D = 11$ supergravity coupling to the M-sources

It is well known that a M2 and a M5 branes are dynamical sources for $A^{(3)}$ and $A^{(6)}$ $D = 11$ supergravity gauge fields [10–12] and therefore one may wonder what is the way of extension of the doubled fields formalism to describe dynamical coupling of $D = 11$ supergravity to the M-branes' sources? To get the answer let me consider the case of coupling of $D = 11$ supergravity to single closed M2 and M5 branes which is naively described by the following set of Bianchi identities and equations of motion (cf. [11, 10, 13, 14])

$$
\begin{aligned}
&d\hat{F}^{(4)} = *J_m^{(6)},\\
&d*\hat{F}^{(4)} = \hat{F}^{(4)} \wedge \hat{F}^{(4)} - 2h^{(3)} \wedge *J_m^{(6)} + *J_e^{(3)},\\
&h^{(3)} = \star h^{(3)} + \text{n. l. t.},
\end{aligned}
\tag{22}
$$

where

$$
\hat{F}^{(4)} = dA^{(3)} + *G_m^{(7)}, \qquad d*G_m^{(7)} = *J_m^{(6)}, \qquad d*G^{(4)} = *J_e^{(3)},
$$

and

$$
h^{(3)} = db^{(2)} - A^{(3)}
$$

is the gauge invariant M5 worldvolume field strength constructing out of the self-dual worldvolume gauge field $b^{(2)}$ and the pullback of the target-space gauge field $A^{(3)}$. The last equation of (22) that generalizes ordinary self-duality condition contains non-linear corrections coming from the Dirac–Born–Infeld-like structure of the M5 kinetic part (see [15–19] for details) and $J_{e,m}$ are the electric and magnetic currents due to the M2 and M5 branes. To distinguish the Hodge operation in target space and on the worldvolume of branes we have denoted the latter by $\star$.

It is easy to verify that the equation of motion of $A^{(3)}$ gauge field leads to the following expression for the field strength dual to $\hat{F}^{(4)}$

$$
\hat{F}^{(7)} = dA^{(6)} + A^{(3)} \wedge dA^{(3)} - 2h^{(3)} \wedge *G_m^{(7)} + *G_e^{(4)},
\tag{23}
$$

if one admits regularization for ill-defined product of two delta-functions $*G_m^{(7)} \wedge *G_m^{(7)}$ entering the $A^{(3)}$ equation of motion with $*G_m^{(7)} \wedge *G_m^{(7)} = 0$ [20, 21]. Such a regularization has been used in [10] and is good enough as far as the presence of classical gravitational anomaly due to the M5-brane is neglected that we will suppose for the sake of simplicity. However, it should be emphasized that the appearance of product of delta-functions is "a symptom of attempting to treat in classical supergravity what really should be treated in quantum M-theory" [22]. This problem has been approached and solved in a rigorous way using a notion of distributions in [13, 14].

The relevant part of the action which leads to the set of equations (22) has the following generating for the doubled field sigma-model action form

$$
\begin{aligned}
\mathcal{L}^{(11)} = {}& \frac{1}{4}\hat{F}^{(4)} * \hat{F}^{(4)} + \frac{1}{4} i_v \hat{\mathcal{F}}^{(4)} * i_v \hat{\mathcal{F}}^{(4)} - \frac{1}{4}\hat{F}^{(7)} * \hat{F}^{(7)} - \frac{1}{4} i_v \hat{\mathcal{F}}^{(7)} * i_v \hat{\mathcal{F}}^{(7)} + \frac{1}{6} F^{(4)} F^{(7)}\\
&+ \left[\mathcal{L}^{(6)}_{\text{M5 kin.}} - \frac{1}{2}\left(A^{(6)} + db^{(2)} A^{(3)}\right)\right] * J_m^{(6)} + \left[\mathcal{L}^{(3)}_{\text{M2 kin.}} - \frac{1}{2} A^{(3)}\right] * J_e^{(3)}\\
&- \frac{1}{2} h^{(3)} \hat{F}^{(4)} * G_m^{(7)}.
\end{aligned}
\tag{24}
$$

To present the latter as the sigma-model-like action one should extend the Cartan form $\mathcal{G}$ with the superalgebra valued element $\mathbf{g} = t_6\big(* G_e^{(4)} - 2h^{(3)} * G_m^{(7)}\big) + t_3 * G_m^{(7)}$, i.e.

$$
\mathcal{G} = d\mathcal{A}\mathcal{A}^{-1} \to \mathcal{G} = d\mathcal{A}\mathcal{A}^{-1} + \mathbf{g}
$$

after that the twisted self-duality condition becomes

$$*\mathcal{G} = \mathcal{S}\mathcal{G}, \tag{25}$$

but the zero-curvature condition is realized now as

$$d\tilde{\mathcal{G}} + \tilde{\mathcal{G}} \wedge \tilde{\mathcal{G}} = 0, \qquad \tilde{\mathcal{G}} = \mathcal{G} - \mathbf{g}. \tag{26}$$

When the twisted self-duality condition holds, i.e. when

$$*\hat{F}^{(4)} = \hat{F}^{(7)}, \qquad *\hat{F}^{(7)} = \hat{F}^{(4)}, \tag{27}$$

equations that follow from the zero-curvature condition (26)

$$dF^{(4)} = 0, \qquad dF^{(7)} = F^{(4)} \wedge F^{(4)}$$

are the same second order equations of motion that follow from the action (24).

Therefore, the sigma-model doubled field action describing the coupling of closed M2 and M5 branes to $D = 11$ supergravity has the following form

$$\begin{aligned} S = S_{EH} &- \mathrm{Tr} \int_{\mathcal{M}^{11}} \left\{ \frac{1}{4} * \mathcal{G} \wedge \mathcal{G} - \frac{1}{12} \tilde{\mathcal{G}} \wedge \mathcal{S} \tilde{\mathcal{G}} - \frac{1}{4} * i_v(\mathcal{S} - *)\mathcal{G} \wedge i_v(\mathcal{S} - *)\mathcal{G} \right\} \\ &+ \int_{\mathcal{M}^3} \left[e^a \wedge \star e^b \eta_{ab} - \frac{1}{2} A^{(3)} \right] + \int_{\mathcal{M}^6} \mathcal{L}^{(6)}_{\mathrm{M5\ kin.}} - \frac{1}{2} \int_{\mathcal{M}^6} \left[A^{(6)} + db^{(2)} \wedge A^{(3)} \right] \\ &- \frac{1}{2} \int_{\mathcal{M}^{11}} h^{(3)} \wedge \hat{F}^{(4)} \wedge *G^{(7)}_m. \end{aligned} \tag{28}$$

6 Discussion

In conclusion let me summarize the obtained results. Following the doubled field approach by Cremmer, Julia, Lü and Pope we have filled the gap in extending this construction to add fermions and have formulated the supersymmeric action in such a way that the twisted self-duality condition which encodes the dynamics of the gauge fields follows from the action as an equation of motion. As the next step towards a fully fledged theory we have extended the construction to include the dynamical sources for antisymmetric gauge fields and have presented the action describing the system of $D = 11$ supergravity interacting with dynamical branes.

The structure of the doubled field action is generic for all maximal supergravities although the relevant superalgebra becomes much more complicated (cf. [4]). The case of $D = 10$ type IIB supergravity, which cannot be recovered by dimensional reduction from $D = 11$ supergravity, also admits the relevant superalgebra structure. This fact motivates understanding the T-duality between type IIA and type IIB theories in framework of the doubled field approach. In our formulation we left the gravity sector out of consideration due to the absence of clear understanding how to present that in a duality-symmetric manner. Having such a recipe it will be interesting to realize the sigma-model construction for the complete sector of fields of $D = 11$ supergravity that possibly could give a new insight on the mysterious M-theory structure.

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[1] Cremmer E., Julia B. and Scherk J., Supergravity theory in 11 dimensions, *Phys. Lett. B*, 1978, V.76, 409–412.

[2] Cremmer E. and Julia B., The SO(8) supergravity, *Nucl. Phys. B*, 1979, V.159, 141–212.

[3] Julia B., Dualities in the classical supergravity limits, hep-th/9805083.

[4] Cremmer E., Julia B., Lü H. and Pope C.N., Dualisation of dualities II: Twisted self-duality of doubled fields and superdualities, *Nucl. Phys. B*, 1998, V.535, 242–292.

[5] Julia B., Superdualities: Below and beyond U-duality, hep-th/0002035.

[6] Cremmer E., Dimensional reduction in field theory and hidden symmetries in extended supergravity, in Supergravity'81, CUP, 1982, 313–369.

[7] Lavrinenko I.V., Lü H., Pope C.N. and Stelle K.S., Superdualities, brane tensions and massive IIA/IIB Duality, *Nucl. Phys. B*, 1999, V.555, 201–227.

[8] Bandos I.A., Nurmagambetov A.J. and Sorokin D.P., Various faces of type IIA supergravity, hep-th/0307153.

[9] Pasti P., Sorokin D.P. and Tonin M., Note on manifest Lorentz and general coordinate invariance in duality symmetric models, *Phys. Lett. B*, 1995, V.352, 59–63; Duality symmetric actions with manifest space-time symmetries, *Phys. Rev. D*, 1995, V.52, 4277–4281; On Lorentz invariant actions for chiral p-forms, *Phys. Rev. D*, 1997, V.55, 6292–6298.

[10] Bandos I., Berkovits N. and Sorokin D., Duality-symmetric eleven-dimensional supergravity and its coupling to M-branes, *Nucl. Phys. B*, 1998, V.522, 214–233.

[11] de Alwis S.P., Coupling of branes and normalization of effective actions in string/M-theory, *Phys. Rev. D*, 1997, V.56, 7963–7977.

[12] Sorokin D., Coupling of M-branes in M-theory, Talk given at the Sixth International Symposium on Particles, Strings and Cosmology (PASCOS 98) (22–29 March, 1998, Boston), hep-th/9806175.

[13] Lechner K., Marchetti P.A. and Tonin M., Anomaly free effective action for the elementary M5-brane, *Phys. Lett. B*, 2002, V.524, 199–207.

[14] Lechner K. and Marchetti P.A., Chern-kernels and anomaly cancellation in M-theory, hep-th/0302108.

[15] Schwarz J.H., Coupling a self-dual tensor to gravity in six dimensions, *Phys. Lett. B*, 1997, V.395, 191–195.

[16] Pasti P., Sorokin D. and Tonin M., Covariant action for a D=11 five-brane with the chiral field, *Phys. Lett. B*, 1997, V.398, 41–46.

[17] Bandos I., Lechner K., Nurmagambetov A., Pasti P., Sorokin D. and Tonin M., Covariant action for the super-five-brane of M-theory, *Phys. Rev. Lett.*, 1997, V.78, 4332–4334.

[18] Aganagic M., Park J., Popescu C. and Schwarz J.H., World-volume action of the M theory five-brane, *Nucl. Phys. B*, 1997, V.496, 191–214.

[19] Bandos I., Lechner K., Nurmagambetov A., Pasti P., Sorokin D. and Tonin M., On the equivalence of different formulations of the M theory five-brane, *Phys. Lett. B*, 1997, V.408, 135-141.

[20] Deser S., Gomberoff A., Henneaux M. and Teitelboim C., p-brane dyons and electric-magnetic duality, *Nucl. Phys. B*, 1998, V.520, 179–204.

[21] Bekaert X. and Gomberoff A., Quantization of the Chern–Simons coupling constant, *JHEP*, 2003, V.0301, 054.

[22] Horava P. and Witten E., Eleven-dimensional supergravity on a manifold with boundary, *Nucl. Phys. B*, 1996, V.475, 94–114.