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Topological Charges, Prequarks and Presymmetry: a Topological Approach to Quark Fractional Charges

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A topological approach to quark fractional charges, based on charge constraints unexplained by the Standard Model of particle physics, is discussed. Charge fractionalization is related to a tunneling process occurring in time between pure gauge field configurations at the far past and future associated with integer-charged bare quarks, named prequarks. This transition conforms to a topologically nontrivial configuration of the weak gauge fields in Euclidean space-time. In this context, an electroweak $\mathcal{Z}_2$ symmetry between bare quarks and leptons, named presymmetry, is revealed. It is shown that an effective topological charge equal to the ratio between baryon number and the number of fermion generations may be associated with baryonic matter. The observed conservation of baryon number is then connected with the conservation of this charge on quarks. Similar results are obtained for leptons in the dual scenario with local quark charges.

1 Introduction

At the level of the Standard Model of elementary particle physics, matter consists of two kinds of particles: quarks and leptons, which are divided into three generations. They interact by means of the strong, weak and electromagnetic forces. These three forces are described by theories of the same general kind: gauge theories with local symmetries. The gauge group is $SU(3)_c \times SU(2)_L \times U(1)_Y$, which provides with the force particles: gluons, weak bosons and photons. Table 1 gives a classification of the first generation of quarks and leptons according to the representation they furnish of this gauge symmetry group, where the ν_{eR} is introduced because of the experimental signatures for neutrino masses, and the conventional relation

$$Q = T_3 + \frac{1}{2}\, Y \tag{1}$$

between electric charge and hypercharge is chosen.

Table 1. First generation of quarks and leptons.

Fermions	$SU(3)_c$	$SU(2)_L$	$U(1)_Y$
$\begin{pmatrix} u_L \\ d_L \end{pmatrix}$	3	2	1/3
u_R	3	1	4/3
d_R	3	1	−2/3
$\begin{pmatrix} \nu_{eL} \\ e_L \end{pmatrix}$	1	2	−1
ν_{eR}	1	1	0
e_R	1	1	−2

In the strong interaction sector there are substantial differences between quarks and leptons: quarks come in triplets while leptons do in singlets of the color group $SU(3)_c$. However, in the electroweak sector their properties are so similar that a deep connection between them seems to exist. Their appearance within each generation is symmetrical: left-handed particles occur in doublets while right-handed ones in singlets of $SU(2)_L$. Their hypercharges are related: their sum within each generation vanishes, i.e.,

$$3\left[Y(u_L)+Y(d_L)+Y(u_R)+Y(d_R)\right]+Y(\nu_{eL})+Y(e_L)+Y(\nu_{eR})+Y(e_R)=0, \tag{2}$$

where the factor 3 takes into account the number of quark colors. This relation, magically obeyed, is crucial to cancel the triangle gauge anomalies of the Standard Model. Furthermore, it relates the number of colors to the fractional nature of quark hypercharges.

Another striking pattern is the following one-to-one correspondence between the quark and lepton hypercharges:

$$\begin{aligned} &Y(u_L)=Y(\nu_{eL})+\frac{4}{3}, \qquad Y(u_R)=Y(\nu_{eR})+\frac{4}{3}, \\ &Y(d_L)=Y(e_L)+\frac{4}{3}, \qquad Y(d_R)=Y(e_R)+\frac{4}{3}, \end{aligned} \tag{3}$$

and similarly for the other two generations. Quark and lepton charges are similarly connected:

$$Q(u)=Q(\nu_e)+\frac{2}{3}, \qquad Q(d)=Q(e)+\frac{2}{3}.$$

Equation (3) is a remarkable and unexpected constraint, more fundamental than (2), which shows that the fractional hypercharge of quarks relies just on a global 4/3 value, independent of flavor and handness. And, if the fractional hypercharge is related to the number of colors as noted above, the factor 1/3 must be there because of quark colors. Besides, it clearly signals for a deeper discrete symmetry between quarks and leptons.

The question is whether there is a way to understand these regularities within the Standard Model. The answer is yes, as I show in this talk, just using the nontrivial topological properties of weak gauge field configurations, as recently proposed [1]. In particular, it is seen that the factor 4 in (3) is a topological Pontryagin index fixed by the requirement of self-consistent gauge anomaly cancellation, so that the number 4/3 is a universal value which in a sense gives structure to the fractional hypercharge of quarks and is conserved by the strong and electroweak interactions.

2 Prequarks and presymmetry

In order to understand the pattern in (3) I pursue a conventional procedure in field theory: start with integer bare local charges for quarks and an exact electroweak symmetry between bare quarks and leptons, and end with fractional charges and a broken or hidden symmetry. I refer to quarks with such bare charges as prequarks and denote them by hats over the symbols that represent the corresponding quarks. The universal hypercharge shift 4/3 may be associated with a bare structure that I denote by X. Thus quarks may conveniently be looked upon as "made" of the following bare mixtures:

$$u=\{\hat{u}X\}, \qquad d=\{\hat{d}X\}, \tag{4}$$

and similarly for the other two generations of quarks. Neither new physical fermions nor new binding forces underlying quarks are introduced. I show that the structure X is associated with an Euclidean configuration of the standard gauge fields having topologically nontrivial properties at infinity; in Minkowski space-time it represents a tunneling process occurring in time by which prequarks get fractional charges. Leptons have integer charges with no topological contribution from such a gauge field configuration.

What are the quantum numbers of $\hat{q}$ and X? Each prequark has the spin, isospin, color charge, and flavor of the corresponding quark. Its bare weak hypercharge is the same as its lepton partner specified by (3) and its bare electric charge is defined according to (1). In this scenario all bare charges associated with local fields are integral. Finally, the baryon number of prequarks can be fixed through the generalized Gell-Mann–Nishijima formula

$$Q = I_z + \frac{1}{2}(\mathrm{B} + S + C + B^* + T), \tag{5}$$

where I_z, S, C, B^*, T denote (pre)quark flavors. Tables 2 and 3 list the prequark quantum numbers. Quarks and prequarks are identical in all their properties except hypercharge, electric charge and baryon number assignments.

Table 2. First generation of prequarks.

Fermions	$SU(3)_c$	$SU(2)_L$	$U(1)_Y$
$\begin{pmatrix} \hat{u}_L \\ \hat{d}_L \end{pmatrix}$	3	2	-1
$\hat{u}_R$	3	1	0
$\hat{d}_R$	3	1	-2

Table 3. Prequark additive quantum numbers.

Prequark	$\hat{u}$	$\hat{d}$	$\hat{s}$	$\hat{c}$	$\hat{b}$	$\hat{t}$
B – baryon number	-1	-1	-1	-1	-1	-1
Q – electric charge	0	-1	-1	0	-1	0
I_z – isospin z-component	$\frac{1}{2}$	$-\frac{1}{2}$	0	0	0	0
S – strangeness	0	0	-1	0	0	0
C – charm	0	0	0	1	0	0
B^* – bottomness	0	0	0	0	-1	0
T – topness	0	0	0	0	0	1

Now regarding the bare quantum numbers associated with the X-configuration, one gets, from (3), (1), and (5),

$$Y(X) = \frac{4}{3}, \qquad Q(X) = \frac{1}{2}Y(X) = \frac{2}{3}, \qquad \mathrm{B}(X) = 2Q(X) = \frac{4}{3}.$$

When these fractional charges are added to the prequark bare ones, the fractional quark charges are obtained.

On the other hand, as readily seen from Tables 1 and 2, a discrete $\mathcal{Z}_2$ symmetry between prequarks and leptons is disclosed in the electroweak sector of the Standard Model. Specifically, this symmetry, which I refer to as presymmetry, means invariance of the classical electroweak Lagrangian of prequarks and leptons under the transformation

$$\hat{u}^i_L \leftrightarrow \nu_{eL}, \qquad \hat{u}^i_R \leftrightarrow \nu_{eR}, \qquad \hat{d}^i_L \leftrightarrow e_L, \qquad \hat{d}^i_R \leftrightarrow e_R,$$

and similarly for the other generations, where i denotes the color degree of freedom. Observe that prequarks and leptons have the same $\mathrm{B}-\mathrm{L} = -1$; i.e., $\mathrm{B}-\mathrm{L}$ is the right fermion number to be considered under presymmetry. Besides, it is important to realize that presymmetry is not an ad hoc symmetry; as (3) shows, it underlies the electroweak relationships between quarks and leptons. Also, prequarks do not define a new layer of the structure of matter; as shown in the following section, they are just bare quarks whose charges are normalized by the universal contribution of a topologically nontrivial Chern–Simons configuration of the standard gauge fields.

3 Topological currents and gauge anomaly cancellation

Following the standard work [2], it is found that the $U(1)_Y$ gauge current for prequarks and leptons

$$\hat{J}^{\mu}_{Y} = \overline{\hat{q}}_L \gamma^{\mu} \frac{Y}{2} \hat{q}_L + \overline{\hat{q}}_R \gamma^{\mu} \frac{Y}{2} \hat{q}_R + \overline{\ell}_L \gamma^{\mu} \frac{Y}{2} \ell_L + \overline{\ell}_R \gamma^{\mu} \frac{Y}{2} \ell_R ,$$

with $\hat{q}_L(\ell_L)$ and $\hat{q}_R(\ell_R)$ uniting the left-handed and right-handed prequarks (leptons) in all generations, respectively, exhibits the $U(1)_Y \times [SU(2)_L]^2$ and $[U(1)_Y]^3$ gauge anomalies generated by the integer hypercharge of prequarks:

$$\partial_{\mu} \hat{J}^{\mu}_{Y} = -\frac{g^2}{32\pi^2} \left[\sum_{\hat{q}_L, \ell_L} \frac{Y}{2} \right] \mathrm{tr}\, W_{\mu\nu} \tilde{W}^{\mu\nu} - \frac{g'^2}{48\pi^2} \left[\sum_{\hat{q}_L, \ell_L} \left(\frac{Y}{2} \right)^3 - \sum_{\hat{q}_R, \ell_R} \left(\frac{Y}{2} \right)^3 \right] F_{\mu\nu} \tilde{F}^{\mu\nu} , \qquad (6)$$

where g, g' and $W_{\mu\nu}$, $F_{\mu\nu}$ are the $SU(2)_L$, $U(1)_Y$ couplings and field strengths, respectively. The anomalies are introduced because the sums in (6) do not vanish:

$$\sum_{\hat{q}_L, \ell_L} Y = -8\, N_g , \qquad \sum_{\hat{q}_L, \ell_L} Y^3 - \sum_{\hat{q}_R, \ell_R} Y^3 = 24\, N_g ,$$

where N_g is the number of fermion generations.

Now, as often noted in the literature [2], the terms on the right-hand side of (6) are divergences of gauge-dependent currents:

$$\partial_{\mu} \hat{J}^{\mu}_{Y} = -\frac{1}{2} \left(\sum_{\hat{q}_L, \ell_L} \frac{Y}{2} \right) \partial_{\mu} K^{\mu} - \frac{1}{2} \left[\sum_{\hat{q}_L, \ell_L} \left(\frac{Y}{2} \right)^3 - \sum_{\hat{q}_L, \ell_L} \left(\frac{Y}{2} \right)^3 \right] \partial_{\mu} L^{\mu} , \qquad (7)$$

where

$$K^{\mu} = \frac{g^2}{8\pi^2} \epsilon^{\mu\nu\lambda\rho}\, \mathrm{tr} \left(W_{\nu} \partial_{\lambda} W_{\rho} - \frac{2}{3} i g W_{\nu} W_{\lambda} W \rho \right) , \qquad L^{\mu} = \frac{g'^2}{12\pi^2} \epsilon^{\mu\nu\lambda\rho} A_{\nu} \partial_{\lambda} A_{\rho} , \qquad (8)$$

which are the Chern–Simons classes or topological currents related to the $SU(2)_L$ and $U(1)_Y$ gauge groups, respectively. Equation (7) can be rewritten in the form

$$\partial_{\mu} \hat{J}^{\mu}_{Y} = -N_{\hat{q}}\, \partial_{\mu} J^{\mu}_{X} ,$$

where $N_{\hat{q}} = 12\, N_g$ is the number of prequarks and

$$J^{\mu}_{X} = \frac{1}{4N_{\hat{q}}} K^{\mu} \sum_{\hat{q}_L, \ell_L} Y + \frac{1}{16 N_{\hat{q}}} L^{\mu} \left(\sum_{\hat{q}_L, \ell_L} Y^3 - \sum_{\hat{q}_R, \ell_R} Y^3 \right) = -\frac{1}{6}\, K^{\mu} + \frac{1}{8}\, L^{\mu} \qquad (9)$$

is the current to be associated with the X-configuration of gauge fields introduced in (4). It can be combined with the anomalous fermionic current $\hat{J}^{\mu}_{Y}$ to define a new current

$$J^{\mu}_{Y} = \hat{J}^{\mu}_{Y} + N_{\hat{q}}\, J^{\mu}_{X} , \qquad (10)$$

which is conserved but it is gauge dependent.

The local counterterms to be added to the Lagrangian of prequarks and leptons, needed to obtain the anomaly-free current of (10), are given by

$$\Delta\mathcal{L} = g' N_{\hat{q}}\, J^{\mu}_{X} A_{\mu}, \tag{11}$$

so that, due to the antisymmetry of $\epsilon^{\mu\nu\lambda\rho}$, only the non-Abelian fields will be topologically relevant, as expected. They produce nontrivial effects as described in the following.

The charge corresponding to the current in (10) is

$$Q_Y(t) = \int d^3x\ \hat{J}^o_Y + N_{\hat{q}} \int d^3x\ J^o_X, \tag{12}$$

which is not conserved after all because of the existence of the topological charge associated with gauge fields. To see this, the change in Q_Y between $t = -\infty$ and $t = +\infty$ is calculated. As usual, it is assumed that the region of space-time where the energy density is nonzero is bounded [3]. Therefore this region can be surrounded by a 3-dimensional surface on which the field configuration becomes pure gauge, i.e.,

$$W_\mu = -\frac{i}{g}(\partial_\mu U)U^{-1}.$$

This field can be obtained from $W_\mu = 0$ by a transformation U that takes values in the corresponding gauge group. In this case, using (12), (9) and (8) for the gauge fields, one ends up with

$$Q_Y(t) = \frac{N_{\hat{q}}}{6}\, n_W(t),$$

where

$$n_W(t) = \frac{1}{24\pi^2}\int d^3x\ \epsilon^{ijk}\ \mathrm{tr}\left(\partial_i U U^{-1} \partial_j U U^{-1} \partial_k U U^{-1}\right) \tag{13}$$

is the winding number of the non-Abelian gauge transformation. This number is integer-valued if we consider a fixed time t and assume that $U(t, \boldsymbol{x})$ equals a direction independent constant at spatial infinity, e.g., $U \to 1$ for $|\boldsymbol{x}| \to \infty$. The usual argument to see this property is based on the observation that this U may be viewed as a map from the 3-dimensional space with all points at infinity regarded as the same onto the 3-dimensional sphere of parameters S^3 of the $SU(2)_L$ group manifold. But 3-space with all points at infinity being in fact one point is topologically equivalent to a sphere S^3 in Minkowski space. Therefore U determines a map $S^3 \to S^3$. These maps are characterized by an integer topological index which labels the homotopy class of the map. This integer is analytically given by (13). For the Abelian case, $n_W = 0$.

Thus, for the field configurations joined to prequarks which at the initial $t = -\infty$ and the final $t = +\infty$ are supposed of the above pure gauge form, the change in charge becomes

$$\Delta Q_Y = Q_Y(t = +\infty) - Q_Y(t = -\infty) = \frac{N_{\hat{q}}}{6}\left[n_W(t = +\infty) - n_W(t = -\infty)\right]. \tag{14}$$

The difference between the integral winding numbers of the pure gauge configurations characterizing the gauge fields at the far past and future can be rewritten as the topological charge or Pontryagin index defined in Minkowski space-time by

$$Q_T = \int d^4x\ \partial_\mu K^\mu = \frac{g^2}{16\pi^2}\int d^4x\, \mathrm{tr}\left(W_{\mu\nu}\tilde{W}^{\mu\nu}\right), \tag{15}$$

where it is assumed that K^i decreases rapidly enough at spatial infinity. This topological index is gauge invariant, conserved and, for arbitrary fields, can take any real value. But, as shown above, for a pure gauge configuration it is integer-valued. Thus

$$Q_T = n = n_W(t = +\infty) - n_W(t = -\infty), \tag{16}$$

so one has from (14) that

$$\Delta Q_Y = N_{\hat{q}} \frac{n}{6}. \tag{17}$$

These equations have a special significance when the integral in (15) is analytically continued to Euclidean space-time. The integral topological charge remains the same but it can now be associated with an Euclidean field configuration that at this point we identify with the X in (4). In this case, the meaning of (16) is that the Euclidean X-configuration interpolates in imaginary time between the real time pure-gauge configurations in the far past and future which are topologically inequivalent. The condition is like the one established for instantons. Thus the continuous interpolation is to be considered as a tunneling process, so that a barrier must separate the initial and final gauge field configurations. It should also be noted that if such an analytical continuation to Euclidean space-time is ignored, nonzero topological charge then implies nonvanishing energy density at intermediate real time and so no conservation of energy.

A consequence of (17) is that $N_{\hat{q}}$ prequarks have to change their $Y/2$ in the same amount $n/6$. For each prequark it implies the hypercharge change

$$Y_{\hat{q}} \to Y_{\hat{q}} + \frac{n}{3}. \tag{18}$$

Therefore, the nontrivial topological properties of the X-configuration give an extra contribution to prequark local hypercharge, so that the integer bare values one starts with have to be shifted. Accordingly, the gauge anomalies have to be re-evaluated. It is found that anomalies are cancelled self-consistently for $n = 4$, the number of prequark flavors per generation, since now

$$\sum_{q_L,\ell_L} Y = 0, \qquad \sum_{q_L,\ell_L} Y^3 - \sum_{q_R,\ell_R} Y^3 = 0. \tag{19}$$

The above hypercharge normalization with topological charge $n = 4$ is consistent with (3) and it means restoration of gauge invariance, breaking of the electroweak presymmetry in the Abelian sector, dressing of prequarks into quarks, and the substitution of the bare presymmetric model by the Standard Model. In fact, from (19) and (9) one notes that the gauge-dependent topological current J^μ_X associated with the X-configuration is cancelled. However, the corresponding conserved topological charge is gauge independent and manifests itself as a universal part of the prequark hypercharge according to (3). More precisely, if the topological current introduced in (11) and its induced hypercharge obtained in (17) are considered, an effective prequark current $\hat{J}^\mu_{Y,\text{eff}}$ can be defined by

$$\hat{J}^\mu_{Y,\text{eff}} = \frac{2}{3}\left(\bar{\hat{q}}_L \gamma^\mu \hat{q}_L + \bar{\hat{q}}_R \gamma^\mu \hat{q}_R\right)$$

to absorb the nontrivial effects of such $N_{\hat{q}} J^\mu_X$ current, namely, cancellation of gauge anomalies and inducement of the hypercharge 4/3 on prequarks regardless of flavor and handness. Upon using this, (10) becomes

$$J^\mu_Y = \bar{\hat{q}}_L \gamma^\mu \frac{Y + 4/3}{2} \hat{q}_L + \bar{\hat{q}}_R \gamma^\mu \frac{Y + 4/3}{2} \hat{q}_R + \bar{\ell}_L \gamma^\mu \frac{Y}{2} \ell_L + \bar{\ell}_R \gamma^\mu \frac{Y}{2} \ell_R.$$

At this point, prequarks with fractional hypercharge, which includes the universal 4/3 part, have to be identified with quarks. The replacement of prequarks by quarks in the strong, weak and Yukawa sectors is straightforward as they have the same color, flavor and weak isospin.

All of this is essentially done at the level of the classical Lagrangian where the specific field configuration X, which only mixes with prequarks, is used. Again, this configuration is a pseudoparticle in Euclidean space-time and a tunneling process in Minkowski space-time by which a prequark hypercharge changes from integer to fractional values. It is also interesting to note that as stated by the model self-consistency is the reason for the "magical" cancellation of gauge anomalies in the Standard Model. Moreover, the factor 1/3 in (18) is due to the number of prequark colors (assuming same number of prequark and lepton families) introduced in (9) through $N_{\hat{q}}$, which predicts, as expected, that quarks carry 1/3-integral charge because they have three colors.

As it is shown above, standard bare quarks instead of prequarks are the fermions to start with in the quantum field theory treatment. The novel news is that bare quarks have fractional charges owing to a universal contribution from a classical gauge field configuration with specific topological properties (i.e., $n = 4$). Underlying this charge structure one has presymmetry as reflected in (3); an electroweak $\mathcal{Z}_2$ symmetry between integer-charged bare quarks and leptons which is broken by the vacuum configuration of gauge fields but it accounts for the electroweak similarities between quarks and leptons.

4 Effective topological charge and baryon number

The underlying topologically nontrivial gauge field configuration in bare quarks suggests to associate an effective topological charge with baryonic matter. If it is considered that an effective fractional topological charge equal to $Q_T = n/N_{\hat{q}} = 1/N_c N_g = 1/9$, where N_c is the number of colors, can be associated with each X-configuration, then, within the bare configuration of (4), it is natural to define an effective topological charge for quarks which is conserved and should be related to its fermion number. As a general rule it is found that

$$Q_T = \frac{\mathrm{B}}{N_g}. \tag{20}$$

Thus $\Delta\mathrm{B} = N_g \Delta Q_T$ for any baryon-number violating process, i.e., only topological effects may violate baryon number.

To see the consistency of the definition for the charge in (20), the baryon plus lepton number (B+L) violating processes induced non-perturbatively by electroweak instanton effects may be considered. According to Ref. [4], for three generations, one electroweak instanton characterized by a topological charge $Q_T = 1$ is associated with quark and lepton number violations in three units: $\Delta\mathrm{B} = \Delta\mathrm{L} = -3$, matching three baryons or nine quarks and three antileptons. A decay such as $p + n + n \rightarrow \mu^+ + \bar{\nu}_e + \bar{\nu}_\tau$ is then allowed. Within the bare configuration scheme of (4), this rule implies that one instanton induces a process in which nine X-configurations vanish. The nine X's make precisely the same topological charge of the electroweak instanton. In a sense, the definition in (20) brings back topological charge conservation in quantum flavor dynamics. In Minkowski space-time one electroweak instanton corresponds to a process which has associated the topological charge change $\Delta Q_T = 1$. It induces a process with the effective topological charge change $\Delta Q_T = -1$ associated with the vanishing of nine quarks and baryon number violation $\Delta\mathrm{B} = -3$. It should also be noted that consistency between the topological charge of the instanton and the effective one assigned to quarks corroborates the above value $n = 4$ of the Pontryagin index fixed by gauge anomaly constraints.

Finally, it can be seen from (20) that a baryon number violation $\Delta\mathrm{B}{=}{-1}$ in actual experiments means an effective topological-charge change $\Delta Q_T = -1/N_g$. In particular, proton decay would imply the presence of a background gauge source with topological charge $1/N_g$ [5]. Stability of a free proton is expected anyway because instanton-like events cannot change topological charge by this amount. For three generations, the effective topological charge has a value $Q_T = 1/9$ for quarks and $Q_T = 1/3$ for nucleons.

5 Conclusion

Insights into the classical dynamics of the standard weak gauge fields which give to quarks fractional charge relative to leptons have been given. A self-consistent method to adjust bare local charges with Chern–Simons contributions has been pursued. Presymmetry, a discrete $\mathcal{Z}_2$ symmetry between bare quarks and leptons, has been introduced to understand quark-lepton similarities in the electroweak sector of the Standard Model. I have also presented arguments, based on the topological character of quark fractional charges and conservation of the associated effective topological charge, to explain the observed conservation of baryon number and predict stability of a free proton.

In this talk I have emphasized a picture in which the integral hypercharges of leptons are entirely associated with local fields. An alternative, dual point of view is to consider quark fractional hypercharges as the local ones, so that the integral hypercharge of leptons relies, as seen from (3), just on a universal $-4/3$ value which also has a topological character. In such a case, this number 3 by which the Pontryagin index $n = 4$ is divided cannot correspond to the number of colors because color is not a property of leptons. The number of generations is the only available degree of freedom. This interpretation solves the fermion family problem: presymmetry requires that the number of fermion generations be equal to the number of quark colors. On the other hand, following an analysis similar to the one presented for baryons, it is concluded that leptonic matter has associated an effective topological charge given by $Q_T = -\mathrm{L}/N_g$. Thus, the general result appears to be

$$Q_T = \frac{\mathrm{B} - \mathrm{L}}{N_g}.$$

This charge is absolutely conserved, so that $\Delta(\mathrm{B} - \mathrm{L}) = 0$ in any physical process, as expected.

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