

On the Possibility of Faster-Than-Light Motions in Nonlinear Electrodynamics

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A version of electrodynamics is constructed in which faster-than-light motions of electromagnetic fields and particles with real masses are possible.

1 Introduction

For certainty, by faster-than-light motions we shall understand the motions with velocities $v > 3 \cdot 10^8$ m/sec. The existence of such motions is the question discussed in modern physics.

As already as in 1946–1948 Blokhintsev [1] paid attention to the possibility of formulating the field theory that allows propagation of faster-than-light (superluminal) interactions outside the light cone. Some time later he also noted the possibility of the existence of superluminal solutions in nonlinear equations of electrodynamics [2]. Kirzhnits [3] showed that a particle possessing the tensor of mass $M^i_k = \text{diag}(m_0, m_1, m_1, m_1)$, $i, k = 0, 1, 2, 3$, $g_{ab} = \text{diag}(+, -, -, -)$ can move faster-than-light, if $m_0 > m_1$. Terletsy [4] introduced into theoretical physics the particles with imaging rest masses moving faster-than-light. Feinberg [5] named these particles tachyons and described their main properties.

Research on superluminal tachyon motions opened up additional opportunities which were studied by many authors, for example by Bilaniuk and Sudarshan [6], Recami [7], Mignani (see [7]), Kirzhnits and Sazonov [8], Corben (see [7]), Patty [9], Oleinik [10]. It has led to original scientific direction (several hundred publications). The tachyon movements may formally be described by Special Relativity (SR) expanded to the domain of motions $s^2 < 0$. For comparison, the standard theory describes motions on the light cone $s^2 = 0$ and in the domain $s^2 > 0$ when $v \leq 3 \cdot 10^8$ m/sec.

The publications are also known in which the violation of invariance of the speed of light is considered [11–17]. One can note, for example, Pauli monograph [11] where elements of Ritz and Abraham theory are presented; Logunov's lectures [12] in which SR formulation in affine space was given; Glashow's paper [13] discussing experimental consequences of the violation of Lorentz-invariance in astrophysics; publications [14–17] on the violation of invariance of the speed of light in SR.

A version of the theory permitting faster-than-light motions of electromagnetic fields and charged particles with real masses is proposed below as continuation of such investigations.

2 Formal construction of the theory

Let us introduce the space-time \mathbb{R}^4 , metric properties of which may depend on the velocity of a particle being investigated and take the metric of \mathbb{R}^4 in the form:

$$\begin{aligned} ds^2 &= (c_0^2 + v^2)dt^2 - dx^2 - dy^2 - dz^2 \\ &= (c_0'^2 + v'^2)(dt')^2 - (dx')^2 - (dy')^2 - (dz')^2 - \text{invariant}. \end{aligned} \quad (1)$$

Here x, y, z are the spatial coordinates, t is the time, c_0, c'_0 are the proper values of the speed of light, v is the velocity of a body under study with respect to the reference frame K . Let us connect the co-moving frame K' with this body. Let the proper speed of light be invariant

$$c_0 = c'_0 = 3 \cdot 10^8 \text{ m/sec} - \text{invariant.} \quad (2)$$

As a result, the common time similar to Newton one may be introduced on the trajectory of the frame K' (when $\mathbf{v} = d\mathbf{x}/dt$) and the velocity of light c , corresponding to the velocity \mathbf{v} :

$$dt = dt'_0 \rightarrow t = t'_0, \quad (3)$$

$$c = \pm c_0 \sqrt{1 + \frac{v^2}{c_0^2}}. \quad (4)$$

We shall name the value $c_0 = 3 \cdot 10^8$ m/sec the speed of light, the value c – the velocity of light¹. In accordance with the hypothesis of homogeneity and isotropy of space-time, the velocity v of a free particle does not depend on t and \mathbf{x} . The velocity of light (4) is a constant in this case. Let $dx^0 = cd\tau$ and

$$x^0 = \int_0^t cd\tau = \pm \int_0^t c_0 \sqrt{1 + \frac{v^2}{c_0^2}} d\tau \quad (5)$$

be the “time” x^0 when $v \neq \text{const}$ also. Keeping this in mind, we rewrite the expression (1) in the form

$$ds^2 = (dx^0)^2 - (dx^1)^2 - (dx^2)^2 - (dx^3)^2, \quad (6)$$

where $x^{1,2,3} = (x, y, z)$. As is known, the metric (6) describes the flat homogeneous Minkowski space-time M^4 with $g_{ik} = \text{diag}(+, -, -, -)$, $i, k = 0, 1, 2, 3$. Under the condition (3), infinitesimal space-time transformations, retaining invariance of the form (6), are accompanied by the transformation of the velocity of light [17]:

$$dx'^i = L^i_k dx^k, \quad c' = c(1 - \boldsymbol{\beta} \cdot \mathbf{u}) / \sqrt{1 - \beta^2}, \quad i, k = 0, 1, 2, 3. \quad (7)$$

Here L^i_k is the matrix of Lorentz group with $\boldsymbol{\beta} = \mathbf{V}/c = \text{const}$ [11], $\mathbf{u} = \mathbf{v}/c$. Corresponding homogeneous integral transformations in the case of the one-parametric Lorentz group L_1 are

$$x'^0 = \frac{x^0 - \beta x^1}{\sqrt{1 - \beta^2}}, \quad x'^1 = \frac{x^1 - \beta x^0}{\sqrt{1 - \beta^2}}, \quad x'^2 = x^2, \quad x'^3 = x^3, \quad c' = c \frac{1 - \beta u^1}{\sqrt{1 - \beta^2}}, \quad (8)$$

with $\boldsymbol{\beta} = (V/c, 0, 0)$, $u^1 = dx^1/dx^0 = v_x/c$. For inertial motions in the space-time (1) we have

$$t' = t, \quad x' = \frac{x - Vt}{\sqrt{1 - V^2/c^2}}, \quad y' = y, \quad z' = z, \quad c' = c \frac{1 - Vv_x/c^2}{\sqrt{1 - V^2/c^2}},$$

where we take into account $v_x = x/t$. The transformations (8) are induced by the operator $X = x_1\partial_0 - x_0\partial_1 - u^1 c\partial_c$ which is the sum of Lorentz group L_1 generator $J_{01} = x_1\partial_0 - x_0\partial_1$ and the generator $D = c\partial_c$ of scale transformations group Δ_1 of the velocity of light $c' = \gamma c$. We can say that these generators act in 5-space $M^4 \times V^1$ where V^1 is a subspace of the velocities of light, and that the transformations (8) belong to the group of direct product $L_1 \times \Delta_1$. The generators J_{01} and D and transformations (8) are respectively the symmetry operators and

¹In the form of $c' = c(1 - \beta^2)^{1/2}$ expression (4) was obtained by Abraham in the model of Aether [11].

symmetry transformations for the equation of the zero cone $s^2 = 0$ in the 5-space $V^5 = M^4 \times V^1$ where $|c| < \infty$ includes the subset $c_0 < |c| < \infty$:

$$s^2 = (x^0)^2 - (x^1)^2 - (x^2)^2 - (x^3)^2 = 0, \quad J_{01}(s^2) = 0, \quad D(s^2) = 0, \quad [J_{01}, D] = 0.$$

The relationships between the partial derivatives of the variables (t, x, y, z) and (x^0, x^1, x^2, x^3) are as follows:

$$\begin{aligned} \frac{\partial}{\partial t} &= \frac{\partial x^0}{\partial t} \frac{\partial}{\partial x^0} + \sum_{\alpha} \frac{\partial x^{\alpha}}{\partial t} \frac{\partial}{\partial x^{\alpha}} = c \frac{\partial}{\partial x^0}, \\ \frac{\partial}{\partial x} &= \frac{\partial x^0}{\partial x} \frac{\partial}{\partial x^0} + \sum_{\alpha} \frac{\partial x^{\alpha}}{\partial x} \frac{\partial}{\partial x^{\alpha}} = \left(\int \frac{\partial c}{\partial x} d\tau \right) \frac{\partial}{\partial x^0} + \frac{\partial}{\partial x^1}, \quad \alpha = 1, 2, 3. \end{aligned} \tag{9}$$

The expressions for $\partial/\partial y$ and $\partial/\partial z$ are analogous to the expression $\partial/\partial x$. Further we restrict ourselves by the case of positive values of the velocities of light and by studying a variant of the theory in which the velocity of light in the range of interactions may only depend on the time “ t ”, i.e. $c = c(t) \leftrightarrow c = c(x^0)$. The relationship between x^0 and t may be deduced from the solution of equation (5). Then

$$\nabla c(x^0) = 0, \quad c = c(x^0), \quad u^2 = u^2(x^0) \quad \leftrightarrow \quad \nabla c(t) = 0, \quad c = c(t), \quad v^2 = v^2(t). \tag{10}$$

Let us note some features of motions in this case.

1. As in SR, the parameter $\beta = V/c$ in the present work is in the range $0 \leq \beta < 1$.
2. As in SR, the value dx^0 is the exact differential in view of the condition $\nabla c = 0$.
3. As distinct from SR, the “time” $x^0 = ct$ in the present work is a function of the time t only for the case of a free particle. In the range of interaction the velocity of light may depend on time t , and the value x^0 becomes the functional (5) of the function $c(t)$.
4. The parameter $\beta = V/c$ of the transformations (8) may be constant not only at the constant velocity of light, but also with $c = c(t)$. Indeed we may accept that $0 \leq \beta = V(t)/c_0(1 + v^2(t)/c_0^2)^{1/2} = \text{const} < 1$, which is not in contradiction with $V = V(t)$, $c = c(t)$. This property permits one to use the matrix L^i_k from (7) for constructing Lorentz invariants in the range of interaction where $c = c(t)$.
5. The condition (10) is invariant on the trajectory of a particle because $t' = t$ in this case. Replacing space-time variables in the expression $\nabla c' = 0$ ($c' = \gamma c$) we find that the condition $\nabla c = 0$ follows from $\nabla c' = 0$ if $\nabla \gamma = 0 \rightarrow (\beta - u^1)\partial\beta/\partial x^\alpha + (\beta^3 - \beta)\partial u^1/\partial x^\alpha = 0$, $\alpha = 1, 2, 3$. The system contains the solution $\beta = \text{const}$, $u^1 = u^1(x^0)$ in agreement with (10) and item 4.

Keeping this in mind, let us construct in the space (6) a theory like SR, reflect it on the space-time (1) by means of the formulas (9), (10) and consider the main properties of the theory. Following [18], we may construct the integral of action in the form:

$$\begin{aligned} S &= S_m + S_{mf} + S_f = -mc_0 \int ds - \frac{e}{c_0} \int A_i dx^i - \frac{1}{16\pi c_0} \int F_{ik} F^{ik} d^4x \\ &= \int \left[-mc_0 \sqrt{1 - u^2} + \frac{e}{c_0} (\mathbf{A} \cdot \mathbf{u} - \phi) \right] dx^0 - \frac{1}{8\pi c_0} \int (E^2 - H^2) d^3x dx^0 \\ &= -mc_0 \int ds - \frac{1}{c_0} \int A_i j^i d^4x - \frac{1}{16\pi c_0} \int F_{ik} F^{ik} d^4x. \end{aligned} \tag{11}$$

Here in accordance with [18] $S_m = -mc_0 \int ds = -mc_0 \int (c_0/c) dx^0 = -mc_0 \int (1 - u^2)^{1/2} dx^0$ is the action for a free particle; $S_f = -(1/16\pi c_0) \int F_{ik} F^{ik} d^4x$ is the action for free electromagnetic field, $S_{mf} = -(e/c_0) \int A_i dx^i = -(1/c_0) \int A_i j^i d^4x$ is the action corresponding to the interaction between the charge e of a particle and electromagnetic field; $A^i = (\phi, \mathbf{A})$ is the 4-potential;

$A_i = g_{ik}A^k$; $j^i = (\rho, \rho\mathbf{u})$ is the 4-vector of current density; ρ is the charge density; $F_{ik} = \partial A_k/\partial x^i - \partial A_i/\partial x^k$ is the tensor of electromagnetic field; $i, k = 0, 1, 2, 3$; $\mathbf{E} = -\partial\mathbf{A}/\partial x^0 - \nabla\phi$ is the electrical field; $\mathbf{H} = \nabla \times \mathbf{A}$ is the magnetic field; $F_{ik}F^{ik} = 2(H^2 - E^2)$; $dx^4 = dx^0 dx^1 dx^2 dx^3$ is the element of the invariant 4-volume. The speed of light c_0 , the mass m , the electric charge e are invariant constants of the theory.

In spite of the similarity, the action (11) differs from the action of SR [18]. The current density was taken in the form $j^i = (\rho, \rho\mathbf{u}) = (\rho, \rho\mathbf{v}/c)$ instead of $j^i = (\rho, \rho\mathbf{v})$ [18]. The electromagnetic field was taken in the form $\mathbf{E} = -\partial\mathbf{A}/\partial x^0 - \nabla\phi = -(1/c)\partial\mathbf{A}/\partial t - \nabla\phi$ instead of $\mathbf{E} = -(1/c_0)\partial\mathbf{A}/\partial t - \nabla\phi$ [18]. The current density in (11) is similar to the one from Pauli monograph [11] with the only difference that the 3-current density in (11) is $\rho\mathbf{v}/c$ instead of being $\rho\mathbf{v}/c_0$ [11]. Analogously, the velocity of 4-potential propagation in (11) is c from (4) instead of c_0 in [18].

In addition to Lorentz-invariance [18], the action (11) is also invariant with respect to any transformations of the velocity of light and, consequently, with respect to the transformations $c' = \gamma c$, as the value c is not contained in the expression (11). As a result the action (11) is invariant with respect to the transformations (8) from the group of direct product $L_1 \times \Delta_1 \subset L_6 \times \Delta_1$, containing the Lorentz group L_6 and the scale group Δ_1 ($c' = \gamma c$) as subgroups.

Lagrangian L , generalized 4-momentum \mathbf{P} and generalized energy \mathcal{H} of a particle are:

$$L = -mc_0\sqrt{1-u^2} + \frac{e}{c_0}(\mathbf{A} \cdot \mathbf{u} - \phi), \quad (12)$$

$$\mathbf{P} = \frac{\partial L}{\partial \mathbf{u}} = \frac{mc_0\mathbf{u}}{\sqrt{1-u^2}} + \frac{e}{c_0}\mathbf{A} = \mathbf{p} + \frac{e}{c_0}\mathbf{A}, \quad (13)$$

$$\mathcal{H} = \mathbf{P} \cdot \mathbf{u} - L = \frac{mc_0}{\sqrt{1-u^2}} + \frac{e\phi}{c_0} = \frac{\mathcal{E}}{c_0} + \frac{e\phi}{c_0}. \quad (14)$$

Here $\mathbf{p} = m\mathbf{v}$ is the momentum, $\mathcal{E} = mc_0c$ is the energy, $\mathcal{E}_0 = mc_0^2$ is the rest energy of a particle. As in SR, the values \mathcal{E} and \mathbf{p} may be united into the 4-momentum p^i :

$$p^i = mc_0u^i = \left(\frac{mc_0}{\sqrt{1-u^2}}, \frac{mc_0u^\alpha}{\sqrt{1-u^2}} \right) = \left(\frac{\mathcal{E}}{c_0}, m\mathbf{v} \right), \quad \alpha = 1, 2, 3. \quad (15)$$

The components of p^i are related by the expressions:

$$p_i p^i = \frac{\mathcal{E}^2}{c_0^2} - \mathbf{p}^2 = m^2 c_0^2, \\ \mathbf{p} = \frac{\mathcal{E}}{c_0} \mathbf{v} \quad \text{or} \quad \mathbf{p} = \frac{\mathcal{E}}{c_0} \mathbf{n}, \quad \mathbf{n} = \frac{\mathbf{c}}{c}, \quad \text{if } m = 0, \quad \mathbf{v} = \mathbf{c}. \quad (16)$$

One can see from here that the momentum of a particle with the zero mass $m = 0$ is independent of the particle velocity $v = c$ and only determined by the particle energy. As in SR, for the case of the photon we find: $p^i = (\hbar\omega/c_0, \hbar\omega\mathbf{n}/c_0) = (\mathcal{E}/c_0, \hbar\mathbf{k})$ where $\mathcal{E} = \hbar\omega$, $\mathbf{k} = (\omega/c_0)\mathbf{n}$, ω is the frequency of electromagnetic field, \hbar is the Planck constant.

For constructing the equations of motion for a charged particle and electromagnetic field let us start from the mechanical [18] and the field Lagrange equations [19, 20]

$$\frac{d}{dx^0} \frac{\partial L}{\partial \mathbf{u}} - \frac{\partial L}{\partial \mathbf{x}} = 0, \quad \frac{\partial}{\partial x^k} \frac{\partial \mathcal{L}}{\partial (\partial A_i / \partial x^k)} - \frac{\partial \mathcal{L}}{\partial A_i} = 0. \quad (17)$$

Here L is Lagrangian (12), $\mathcal{L} = -(1/c_0)A_i j^i - (1/16\pi c_0)F_{ik}F^{ik}$. Taking into account the equality $\nabla(\mathbf{a} \cdot \mathbf{b}) = (\mathbf{a} \cdot \nabla)\mathbf{b} + (\mathbf{b} \cdot \nabla)\mathbf{a} + \mathbf{a} \times (\nabla \times \mathbf{b}) + \mathbf{b} \times (\nabla \times \mathbf{a})$, the permutational ratios of the tensor of electromagnetic field, the expression $\partial(F^{ik}F_{ik})/\partial(\partial A_i/\partial x^k) = -4F^{ik}$ [18] we obtain $d(mc_0\mathbf{u}/(1-u^2)^{1/2})/dx^0 = (e/c_0)\mathbf{E} + (e/c_0)\mathbf{u} \times \mathbf{H}$, $\partial F_{ik}/\partial x^l + \partial F_{kl}/\partial x^i + \partial F_{li}/\partial x^k = 0$,

$\partial F^{ik}/\partial x^k + 4\pi j^i = 0$ [18]. With help of relations (9), (10) and expressions $dx^0 = cdt$, $(1 - u^2)^{1/2} = c_0/c$ we may obtain the following equations of motions in the space-time (1) [17]:

$$\begin{aligned} \frac{d\mathbf{p}}{dt} &= m \frac{d\mathbf{v}}{dt} = \frac{c}{c_0} e \mathbf{E} + \frac{e}{c_0} \mathbf{v} \times \mathbf{H}, \\ \frac{d\mathcal{E}}{dt} &= e \mathbf{E} \cdot \mathbf{v} \rightarrow m \frac{dc}{dt} = \frac{e}{c_0} \mathbf{v} \cdot \mathbf{E}; \end{aligned} \quad (18)$$

$$\begin{aligned} \nabla \times \mathbf{E} + \frac{1}{c} \frac{\partial \mathbf{H}}{\partial t} &= 0, & \nabla \cdot \mathbf{H} &= 0, \\ \nabla \times \mathbf{H} - \frac{1}{c} \frac{\partial \mathbf{E}}{\partial t} &= 4\pi \rho \frac{\mathbf{v}}{c}, & \nabla \cdot \mathbf{E} &= 4\pi \rho. \end{aligned} \quad (19)$$

Equations (18) determine dynamics of motion of a charged particle with a mass m in external field. The set (19) consists of Maxwell equations for the velocity of light $c = c_0(1 + v^2/c_0^2)^{1/2}$ with $v = \text{const}$. Considered together, they form the set of nonlinear equations of electrodynamics describing the motion of electrical charge in the field generated by the motion of the this charge.

We may find from here that wave equations for the vector and scalar potentials take the following form under Lorentz gauge $(1/c)\partial\phi/\partial t + \nabla \cdot \mathbf{A} = 0$:

$$\square \mathbf{A} - \frac{\dot{c}}{c^3} \frac{\partial \mathbf{A}}{\partial t} = 4\pi \rho \frac{\mathbf{v}}{c}, \quad \square \phi - \frac{\dot{c}}{c^3} \frac{\partial \phi}{\partial t} = 4\pi \rho,$$

where $\dot{c} = dc/dt = \mathbf{v} \cdot \dot{\mathbf{v}}/c$. For the free field with $\rho = 0$ we have $\dot{c} = 0$, $v^2 = \text{const}$, $\mathbf{v} \cdot \dot{\mathbf{v}} = 0$, $\square \mathbf{A} = 0$, $\square \phi = 0$. When $c = c_0$ they coincide with the equations from SR.

The condition $\nabla c = 0$ imposes certain limits on the possible movement of a particle. Acting by the operator ∇ on the equation $\dot{c} = (e/mc_0)\mathbf{v} \cdot \mathbf{E}$, we find

$$\begin{aligned} \rho \left[(\mathbf{v} \cdot \nabla) \mathbf{E} + (\mathbf{E} \cdot \nabla) \mathbf{v} - \frac{1}{c} \mathbf{v} \times \frac{\partial \mathbf{H}}{\partial t} \right] \\ + \frac{c}{4\pi} \mathbf{E} \times \left(\square \mathbf{H} - \frac{\dot{c}}{c^3} \frac{\partial \mathbf{H}}{\partial t} \right) - \frac{mc_0}{e} \dot{c} \nabla \rho + \mathbf{v} (\mathbf{E} \cdot \nabla \rho) = 0. \end{aligned}$$

For a free particle ($\dot{c} = 0$, $\rho = 0$, $e=0$) we have $\mathbf{E} \times \square \mathbf{H} = 0$, $\dot{\mathbf{v}} = 0$.

As follows from the equations $\nabla \cdot \mathbf{E} = 4\pi \rho$, $c \nabla \times \mathbf{H} - \partial_t \mathbf{E} = 4\pi \rho \mathbf{v}$ from the set (19), in the present theory the law of the electrical charge conservation is valid:

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0.$$

Following [18], we find the well-known relation:

$$\frac{\partial}{\partial t} \left(\frac{E^2 + H^2}{8\pi} \right) = -c \mathbf{j} \cdot \mathbf{E} - \nabla \cdot \mathbf{S}.$$

Here $W = (E^2 + H^2)/8\pi$ is the energy density of electromagnetic field, $\mathbf{S} = (c/4\pi)\mathbf{E} \times \mathbf{H} = (c(0)\mathcal{E}(t)/4\pi\mathcal{E}(0))\mathbf{E} \times \mathbf{H}$ is the Poynting vector.

At last taking into account the expression for the velocity of light

$$c(t) = c_0 \sqrt{1 + \frac{v^2(t)}{c_0^2}} = c(0) \left[1 + \frac{e}{mc_0 c(0)} \int_0^t \mathbf{v} \cdot \mathbf{E} d\tau \right] = c(0) \left[1 + \frac{\mathcal{E}(t) - \mathcal{E}(0)}{\mathcal{E}(0)} \right],$$

where $c(0)$ is the velocity of light at the time $t = 0$, $\mathcal{E}(0) = mc_0c(0)$, we may rewrite the sets (18), (19) in the equivalent form:

$$m\frac{d\mathbf{v}}{dt} = \frac{c(0)}{c_0} \left[1 + \frac{\mathcal{E}(t) - \mathcal{E}(0)}{\mathcal{E}(0)} \right] e\mathbf{E} + \frac{e}{c_0} \mathbf{v} \times \mathbf{H},$$

$$\frac{d\mathcal{E}}{dt} = e\mathbf{v} \cdot \mathbf{E} \rightarrow \mathcal{E}(t) - \mathcal{E}(0) = e \int_0^t \mathbf{v} \cdot \mathbf{E} d\tau; \quad (20)$$

$$\left[1 + \frac{\mathcal{E}(t) - \mathcal{E}(0)}{\mathcal{E}(0)} \right] \nabla \times \mathbf{E} + \frac{1}{c(0)} \frac{\partial \mathbf{H}}{\partial t} = 0, \quad \nabla \cdot \mathbf{E} = 4\pi\rho, \quad (21)$$

$$\left[1 + \frac{\mathcal{E}(t) - \mathcal{E}(0)}{\mathcal{E}(0)} \right] \nabla \times \mathbf{H} - \frac{1}{c(0)} \frac{\partial \mathbf{E}}{\partial t} = 4\pi\rho \frac{\mathbf{v}}{c(0)}, \quad \nabla \cdot \mathbf{H} = 0. \quad (22)$$

The set (20), (22) coincide with the set of equations of Maxwell electrodynamics [18] in the approximation $[\mathcal{E}(t) - \mathcal{E}(0)]/\mathcal{E}(0) \ll 1$ with $c(0) = c_0$.

Further let us pay attention to the expression $v = (\mathcal{E}^2 - m^2c_0^4)^{1/2}/mc_0 > c_0$. It follows from here that in the framework of the present work a free particle will move faster-than-light, if the particle energy satisfies the condition [17]

$$\mathcal{E} > \sqrt{2}mc_0^2 = \sqrt{2}\mathcal{E}_0. \quad (23)$$

The energy $\mathcal{E} = 2^{1/2}\mathcal{E}_0$ is equal ~ 723 keV for the electron ($\mathcal{E}_0 \sim 511$ keV) and ~ 1327 MeV for the proton and neutron ($\mathcal{E}_0 \sim 938$ MeV). We may conclude from here that in the present work the neutron physics of nuclear reactors may be formulated in the approximation $v < c_0$ as in SR. The electrons with the energy $\mathcal{E} > 723$ keV should be faster-than-light particles (for example, the velocity of the 1 GeV electron should be $\sim 2000 c_0$). The particle physics on accelerators with the energy of protons more than 1.33 GeV would be physics of faster-than-light motions, if the present theory be realized in the nature.

3 Application to physics

Let us consider how a set of the well-known experiments may be interpreted in the framework of the present theory. As an example chosen may be: the Michelson experiment, Fizeau experiment, aberration of light, appearance of atmospheric μ -mesons near the surface of the Earth, Doppler effect, known tests to check independence of the speed of light from the velocity of light source, decay of unstable particles, creation of new particles in nuclear reactions, possible faster-than light motion of nuclear reactions products, Compton effect, photo-effect. Below we shall consider some of them.

Michelson experiment. Negative result of the Michelson experiment for an observer with a terrestrial source of light (the reference frame K , the speed of light c_0) may be explained by space isotropy. Since the speed of light c_0 is the same in all directions, a shift of the interference pattern is absent with the interferometer's rotation. Analogously, in the case of an extraterrestrial source (the reference frame K' – the star moving inertially with a velocity v relatively to the Earth), the velocity of light from the star $c = c_0(1 + v^2/c_0^2)^{1/2}$ is the same for an observer on the Earth. As a result, the interference pattern does not change with the interferometer's rotation [17].

Aberration of light. By analogy with SR [11, 18] for one-half of the aberration angle (in arc seconds) we have: $\sin \alpha = V/c$, $\alpha \sim (V/c_0)(c_0/c) \sim 20.5(1 + z_\omega)/(1 + z_\lambda)$ [17]. Here $z_\omega = (c - Vn_x)/c_0 - 1$, $z_\lambda = (c - Vn_x)c/c_0^2 - 1$ are the redshift parameters for frequency and wavelength respectively. It follows from here, for quasar Q1158 + 4635 with $z_\lambda = 4.73$ (Carswell and Hewett, 1990) we obtain at $n_x = -1$, that $z_\omega \sim 2.23$, $c \sim 1.77c_0$, $\alpha \sim 11.6$ instead of being $z_\omega = z_\lambda$, $c = c_0$, $\alpha \sim 20.5$ in SR.

Appearance of atmospheric μ -mesons near the surface of the Earth. Since the time dilatation is absent in the present work, the appearance of air μ -mesons near the surface of the Earth could be explained by faster-than-light motion of the mesons with velocity of the order of $6 \cdot 10^6 / 2.2 \cdot 10^{-6} \sim 3 \cdot 10^{10}$ m/sec, or $100c_0$. This corresponds to the meson energy $\mathcal{E}_\mu = m_\mu c_0 c \sim 10.6$ GeV, where $m_\mu c_0^2 \sim 106$ MeV is the rest energy of μ -meson [17].

Tests to check independence of the speed of light from the velocity of light source. It is, for example, Bonch–Bruevich–Molchanov (1956) experiment, in which the velocities of light radiated by the eastern and western equatorial edges of the solar disk were compared; Sadeh experiment (1963) in which the velocities of γ -quanta, arising as a result of the electron-positron annihilation in flight, were compared; Filippas–Fox experiment (1964), where the effect of the velocity of fast π^0 -meson on the velocities of γ -quanta from decay $\pi^0 \rightarrow \gamma + \gamma$ were investigated. In view of independence of the velocity of light (4) from the direction of emission of light relatively to the vector of velocity \mathbf{v} of a light source (the solar disk, center-of-mass of electron and positron, π^0 -meson) the result in such type of experiments should be negative [17].

Decay of unstable particles. Because of the equality of the rest energy of particles in SR and in the present work, the condition of spontaneous decay of the particles into fragments is the same for the both theories. In particular, for the case of decay of a particle with the mass M into two fragments with the masses m_1 and m_2 , the energy conservation law leads to $Mc_0^2 = \mathcal{E}_1 + \mathcal{E}_2$, where \mathcal{E}_1 and \mathcal{E}_2 are the energies of the particles produced. Since $\mathcal{E}_1 > m_1 c_0^2$, $\mathcal{E}_2 > m_2 c_0^2$, the decay is possible if $M > m_1 + m_2$ (as in SR [18]). From the conservation law of energy-momentum $Mc_0^2 = \mathcal{E}_1 + \mathcal{E}_2$, $\mathbf{p}_1 + \mathbf{p}_2 = 0$, and relations (16) it follows that $\mathcal{E}_1 = (M^2 + m_1^2 - m_2^2)c_0^2/2M$, $\mathcal{E}_2 = (M^2 - m_1^2 + m_2^2)c_0^2/2M$ as in SR [18]. The difference consists in predicting the fragment velocities. Using formula $\mathcal{E} = mc_0 c$, for these velocities in the present work we obtain $v_1 = [((M^2 + m_1^2 - m_2^2)^2/4m_1^2 M^2) - 1]^{1/2} c_0$, $v_2 = [((M^2 - m_1^2 + m_2^2)^2/4m_2^2 M^2) - 1]^{1/2} c_0$. One can see that when $(M^2 + m_1^2 - m_2^2)^2/4m_1^2 M^2 \geq 2$, $(M^2 - m_1^2 + m_2^2)^2/4m_2^2 M^2 \geq 2$, the velocities v_1 and v_2 can exceed or be equal the speed of light c_0 [17].

Creation of new particles. Let us consider the reaction of antiproton creation in the proton-proton collision in a laboratory reference frame K : p^+ (moving) + p^+ (in rest) = p^+ + p^+ + p^+ + p^- , where we denote the energy of the moving proton as \mathcal{E}_1 , the momentum as \mathbf{p}_1 , and the proton mass as m_p . Following [18] and using the relationship between the momentum and energy (16), one can write $(\mathcal{E}_1 + m_p c_0^2)^2 - c_0^2 p_1^2 = 16m_p^2 c_0^4$. In view of the relationship $\mathcal{E}_1^2 - c_0^2 p_1^2 = m_p^2 c_0^4$, one obtains $2\mathcal{E}_1 m_p c_0^2 = 14m_p^2 c_0^4$. From here we find that the threshold energy of the antiproton creation $\mathcal{E}_1 = 7m_p c_0^2 \sim 7$ GeV is the same as in SR [22]. The difference consists in the value of velocities of particles. In particular, according to the formula $p_1 = m_p v_1 = (\mathcal{E}_1^2/c_0^2 - m_p^2 c_0^2)^{1/2} = (49 - 1)^{1/2} m_p c_0$, in the present work the velocity of proton possessing the energy 7 GeV is $v_1 = (48)^{1/2} c_0 = 6.9c_0$ [17] is distinct from SR.

Compton effect and photo-effect. Following [21], we find from the energy-momentum the conservation law (15) $\hbar\omega = \hbar\omega' + m_e c_0^2 [(1 + v^2/c_0^2)^{1/2} - 1]$; $\hbar\omega/c_0 = (\hbar\omega'/c_0) \cos \theta + m_e v \cos \alpha$, $0 = (\hbar\omega'/c_0) \sin \theta - m_e v \sin \alpha$. Here $\hbar\omega$, $\hbar\omega'$ are the energies of incident and scattered γ -quanta, α and θ are the angles of scattering the electron and γ -quantum respectively, m_e is the electron mass. As a result the angular distribution of scattered γ -quanta is $\omega' = \omega/[1 + \hbar\omega(1 - \cos \theta)/m_e c_0^2]$ as in [21]. But the velocity of forward-scattered electron may exceed the speed of light c_0 : $v_e = c_0 (\hbar\omega/m_e c_0^2) [1 - m_e c_0^2/(2\hbar\omega + m_e c_0^2)] > c_0$ if $\hbar\omega > 698$ keV, which differs from SR. The velocity of scattered γ -quantum does not depend on the angle θ and is determined by mechanism of scattering (immediately after interaction of γ -quantum with electron, or in the act of absorption-emission by scattered electron).

Analogously, in the case of photo-effect the velocity of photoelectron is equal $v = c_0 [(\hbar\omega + m_e c_0^2 - U)/m_e c_0^2 - 1]^{1/2}$, where U is the energy of ionization. If the energy of photon $\hbar\omega \geq (2^{1/2} - 1)m_e c_0^2 + U = 211$ keV + U , the velocity of photoelectron is $v \geq c_0$.

4 Conclusion

The $L_6 \times \Delta_1$ invariant theory has been constructed, where L_6 is the Lorentz group, Δ_1 is the scale transformation group of the velocity of light $c' = \gamma c$. In accordance with Blokhintsev papers [1] we may assume that the proposed theory may prove to be useful in the field of particles physics, when the elementary particle is not a point but possesses some dimensions. Indeed, the elementary particles should be points in the L_6 invariant theory (SR) because of the finiteness of the speed of light c_0 . In the $L_6 \times \Delta_1$ theory this requirement is not necessary because of the absence of the limit to the velocity of light c . The postulation $c' = c$ leads to SR.

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