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Equation for Doublets of Particles with Half-Integer Spin and Parasupersymmetry

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In the present paper we consider the tensor-bispinor equations, which describe two particles with arbitrary half-integer spin and opposite parity. We obtain the energy levels of particle with half-integer spin s in constant magnetic field and find the parasupersymmetry which is connected with extra degeneracy of energy levels in constant magnetic field.

1 Introduction

It is well-known that equations for particles with spin more than $\frac{1}{2}$ have a lot of defects such as: acausality [1], incompatibility of the theoretical value of gyromagnetic ratio $g = \frac{1}{s}$ with experimental value $g = 2$ for any spin, complex value of energy of charged particle in constant magnetic field and others. In the papers [2, 3] tensor-bispinor equation was considered, which describes the motion of particle with half-integer spin. It was shown [2, 3], that tensor-bispinor equation remains causal after introducing the interaction with electromagnetic field. Moreover, this equation, in the case of minimal interaction, is equivalent to the equation in Dirac-like form, which was considered in the paper [4].

The aim of this paper is generalization of the result that was obtained in the papers [2,3], for the case of anomalous interaction quadratic in electromagnetic field. We note that there is a kind of anomalous interaction quadratic in electromagnetic field at which the extra degeneration of energy levels of particle appear. This degeneratior is caused by parasupersymmetry of considered equations.

In Section 2 we consider tensor-bispinor equations for free particle with arbitrary half-integer spin. We also show the equivalence of such equations and the Dirac-like equations [4]. In Section 3 the minimal and anomalous interaction with electromagnetic field is discussed. Section 4 is dedicated to the problem of motion a particle in constant magnetic field. The condition is found at which the problem of complex energy levels does not arise. Finally, in Section 5, we retrieve the conditions the constants to which should satisfy of interactions in order that the equation for particle in constant magnetic field be parasupersymmetric (It is worth to note that parasupersymmetry of equations for spin-1 and spin-0 was considered by Beckers, Debergh, Nikitin [5] and Sergeyev [6]). We also obtain corresponding parasupercharges.

2 Tensor bispinor equation for free particles with arbitrary spin

Let us consider in this section the description of motion of free particles with arbitrary half-integer spin in terms of irreducible antisymmetric tensor-bispinor $\Psi_\gamma^{[\mu_1,\nu_1][\mu_2,\nu_2]\cdots[\mu_n,\nu_n]}$ [2, 3] of rank n ($n = s - \frac{1}{2}$).

We suppose that the tensor-bispinor $\Psi_\gamma^{[\mu_1,\nu_1][\mu_2,\nu_2]\cdots[\mu_n,\nu_n]}$ satisfies the following conditions

$$\Psi^{[\mu_1\nu_1][\mu_2\nu_2]\cdots[\mu_k\nu_k]\cdots[\mu_n\nu_n]} = -\Psi^{[\mu_1\nu_1][\mu_2\nu_2]\cdots[\nu_k\mu_k]\cdots[\mu_n\nu_n]},$$
$$\Psi^{[\mu_1\nu_1][\mu_2\nu_2]\cdots[\mu_k\nu_k]\cdots[\mu_l\nu_l]\cdots[\mu_n\nu_n]} = \Psi^{[\mu_1\nu_1][\mu_2\nu_2]\cdots[\mu_l\nu_l]\cdots[\mu_k\nu_k]\cdots[\mu_n\nu_n]}, \qquad (1)$$

and

$$\gamma_\mu\gamma_\nu\Psi^{[\mu\nu][\mu_1\nu_1]\cdots[\mu_{n-1}\nu_{n-1}]} = 0, \tag{2}$$

where γ_μ are the Dirac matrices. Moreover, $\Psi_\gamma^{[\mu_1,\nu_1][\mu_2,\nu_2]\cdots[\mu_n,\nu_n]}$ satisfy the Dirac equation

$$(\gamma_\lambda p^\lambda - m)\Psi^{[\mu_1,\nu_1]\cdots[\mu_n,\nu_n]} = 0, \tag{3}$$

where $p_\mu = i\frac{\partial}{\partial x^\mu}$.

Commuting $\gamma_\mu\gamma_\nu$ and using (2), (3) with $(\gamma_\lambda p^\lambda - m)$ we come to the condition

$$p_\mu\gamma_\nu\Psi^{[\mu\nu][\mu_1\nu_1]\cdots[\mu_{n-1}\nu_{n-1}]} = 0. \tag{4}$$

The wave function $\Psi_\gamma^{[\mu_1,\nu_1][\mu_2,\nu_2]\cdots[\mu_n,\nu_n]}$ belongs to the carrier space of the representation

$$\begin{aligned}&\left[D\left(s-\tfrac{1}{2},0\right)\oplus D\left(0,s-\tfrac{1}{2}\right)\right]\otimes\left[D\left(\tfrac{1}{2},0\right)\oplus D\left(0,\tfrac{1}{2}\right)\right]\\ &\quad = D(s,0)\oplus D(0,s)\oplus D\left(s-\tfrac{1}{2},\tfrac{1}{2}\right)\oplus D\left(\tfrac{1}{2},s-\tfrac{1}{2}\right)\oplus D(s-1,0)\oplus D(0,s-1).\end{aligned}$$

of the Lorentz group. Thus, the wave function $\Psi_\gamma^{[\mu_1,\nu_1][\mu_2,\nu_2]\cdots[\mu_n,\nu_n]}$ has $16s$ components. Constraint (2) removed the states which correspond to the representation $D(s-1,0)\oplus D(0,s-1)$. The secondary constraint (4) nullifies the remaining part of non-physical components and, as a result, we have exactly $4(2s+1)$ independent components, i.e. twice more than it is necessary for describing particles with arbitrary spin. So equation (3) with additional conditions (1)–(2), (4) describes two particles with arbitrary spin s, but these particles have opposite parity.

To introduce the interaction with electromagnetic field in equations (1)–(4) it is preferable to write them as a single equation. For this purpose we present the wave function $\Psi_\gamma^{[\mu_1,\nu_1][\mu_2,\nu_2]\cdots[\mu_n,\nu_n]}$ in the form

$$\begin{aligned}\Psi^{[\mu_1\nu_1]\cdots[\mu_n\nu_n]} &= \Phi^{[\mu_1\nu_1]\cdots[\mu_n\nu_n]} + \frac{1}{2}\Big(\gamma^{[\mu_1}A^{\nu_1][\mu_2\nu_2]\cdots[\mu_n\nu_n]}\\ &\quad + \gamma^{[\mu_2}A^{\nu_2][\mu_1\nu_1][\mu_3\nu_3]\cdots[\mu_n\nu_n]} + \cdots + \gamma^{[\mu_n}A^{\nu_n][\mu_1\nu_1]\cdots[\mu_{n-1}\nu_{n_1}]}\Big),\end{aligned} \tag{5}$$

here $\gamma^{[\mu}A^{\nu]} = \gamma^\mu A^\nu - \gamma^\nu A^\mu$. Tensors $\Phi^{[\mu_1\nu_1]\cdots[\mu_n\nu_n]}$ and $A^{\lambda[\mu_1\nu_1]\cdots[\mu_{n-1}\nu_{n-1}]}$ satisfy the following conditions

$$\begin{aligned}&\gamma_\mu\Phi^{[\mu\nu][\mu_1\nu_1]\cdots[\mu_n\nu_n]} = 0,\\ &\gamma_\lambda A^{\lambda[\mu_1\nu_1]\cdots[\mu_{n-1}\nu_{n-1}]} = \gamma_{\mu_1}A^{\lambda[\mu_1\nu_1][\mu_2\nu_2]\cdots[\mu_n\nu_n]} = 0,\\ &A^{\lambda[\mu_1\nu_1]\cdots[\mu_n\nu_n]} + A^{\mu_1[\nu_1\lambda]\cdots[\mu_n\nu_n]} + A^{\nu_1[\lambda\mu_1]\cdots[\mu_n\nu_n]} = 0,\\ &\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\\ &A^{\lambda[\mu_1\nu_1]\cdots[\mu_n\nu_n]} + A^{\mu_n[\mu_1\nu_1]} + A^{\nu_n[\mu_1\nu_1]\cdots[\lambda\mu_n]} = 0.\end{aligned}$$

Moreover, tensors $\Phi^{[\mu_1\nu_1]\cdots[\mu_n\nu_n]}$ and $A^{\lambda[\mu_1\nu_1]\cdots[\mu_{n-1}\nu_{n-1}]}$ satisfy conditions (1). Then we can write the single equation equivalent to (1)–(4) in the form

$$\begin{aligned}&(\gamma_\nu p^\nu - m)\Psi^{[\mu_1\nu_1][\mu_2\nu_2]\cdots[\mu_n\nu_n]} - \frac{n}{2(8n-11)}\sum_{i=1}^{n} P_1(\mu_i\nu_i)\\ &\quad - \frac{n}{8n-11}\sum_{i,j=1(j>i)}^{n} P_2(\mu_i\nu_i;\mu_j\nu_j) = 0,\end{aligned} \tag{6}$$

where

$$P_1(\mu_i\nu_i) = [\gamma_{\mu_i}, \gamma_{\nu_i}]p_\lambda A^{\lambda[\mu_1\nu_1]\cdots[\mu_{i-1}\nu_{i-1}][\mu_{i+1}\nu_{i+1}]\cdots[\mu_n\nu_n]},$$
$$P_2(\mu_i\nu_i;\mu_j\nu_j) = P_1(\mu_i\nu_j) - P_1(\mu_i\mu_j) + P_1(\nu_i\mu_j) - P_1(\nu_i\nu_j). \tag{7}$$

Contracting (6)–(7) with $(p_{\mu_1}\gamma_{\nu_1} - p_{\nu_1}\gamma_{\mu_1})$ we come to constraint (4).

It was shown [2, 3] that equations (5)–(7) can be written in the Dirac-like form [6]

$$(\Gamma_\mu p^\mu - m)\Psi^\epsilon = 0, \qquad \epsilon = -1, 1,$$
$$(\Gamma_\mu p^\mu + m)(1 + \epsilon i\Gamma_4)[S_{\mu\nu}S^{\mu\nu} - 4s(s-1)]\Psi^\epsilon = 24m\Psi^\epsilon. \tag{8}$$

Here Ψ^ϵ are wave functions, which have $8s$ components; matrices Γ_μ have the following form

$$\Gamma_\mu = \gamma_\mu \otimes I,$$

where γ_μ are Dirac matrices and I is $(2\tau+1)\times(2\tau+1)$ unit matrix ($\tau = s - \frac{1}{2}$). In the general case Ψ^ϵ can be expressed in terms of tensor-bispinor $\Psi_\gamma^{[\mu_1,\nu_1][\mu_2,\nu_2]\cdots[\mu_n,\nu_n]}$. For example, in the case of spin $\frac{3}{2}$ we have

$$\Phi_\alpha^{ab} = \frac{1}{2}\varepsilon_{abc}\big(\Phi_{\alpha c}^{(-1)} + \Phi_{\alpha c}^{(1)}\big), \qquad \Phi_\alpha^{oc} = \frac{i}{2}\big(\Phi_{\alpha c}^{(1)} - \Phi_{\alpha c}^{(-1)}\big), \qquad a, b, c = 1, 2, 3, \quad \alpha = 0, 1, 2, 3.$$

Matrices $S_{\mu\nu}$ can be represented in the form

$$S_{\mu\nu} = j_{\mu\nu} + \tau_{\mu\nu}, \qquad j_{\mu\nu} = \frac{i}{4}[\Gamma_\mu, \Gamma_\nu]$$

and $\tau_{\mu\nu}$ satisfy the commutation relations

$$[\tau_{\mu\nu}, \tau_{\rho\sigma}] = i(g_{\mu\sigma}\tau_{\nu\rho} + g_{\nu\rho}\tau_{\mu\sigma} - g_{\mu\rho}\tau_{\nu\sigma} - g_{\nu\sigma}\tau_{\mu\rho}), \qquad [\tau_{\mu\nu}, \Gamma_\rho] = [\tau_{\mu\nu}, j_{\rho\sigma}] = 0,$$

and belong to the representation $D(\tau, 0)$ of algebra $AO(1,3)$, i.e. satisfy the relations

$$\tau_{ab} = \varepsilon_{abc}\tau_c, \qquad \tau_{0a} = i\tau_a, \qquad a, b, c = 1, 2, 3,$$
$$\tau_a\tau_a = \tau(\tau+1), \qquad [\tau_a, \tau_b] = i\varepsilon_{abc}\tau_c. \tag{9}$$

Taking into account (9) we can write τ_a as

$$\tau_a = 1 \otimes \hat{\tau}_a, \qquad a = 1, 2, 3,$$

where 1 is 4×4 unit matrix and $\hat{\tau}_a$ realize irreducible representation $D(\tau)$ of algebra $AO(3)$.

3 Interaction with electromagnetic field

The minimal interaction with electromagnetic field and anomalous interaction linear in electromagnetic field were considered in [2–4]. Using tensor-bispinor formulation we can introduce as minimal as anomalous interaction with electromagnetic field and then we come over to Dirac-like form [2]

$$\Big(\Gamma_\mu\pi^\mu - m + \frac{e}{4m}(1 - \epsilon i\Gamma_4)\Big(\frac{i}{4}(g-2)[\Gamma_\mu, \Gamma_\nu] + g\tau_{\mu\nu}\Big)F^{\mu\nu}\Big)\Psi^\epsilon = 0,$$
$$(\Gamma_\mu\pi^\mu + m)(1 + i\Gamma_4)[S_{\mu\nu}S^{\mu\nu} - 4s(s-1)]\Psi^\epsilon = 24m\Psi^\epsilon. \tag{10}$$

Equations (10) can be generalized by introducing the anomalous interaction quadratic in electromagnetic field

$$\Big(\Gamma_\mu\pi^\mu - m + \frac{e}{4m}(1-\epsilon i\Gamma_4)\big(gS_{\mu\nu}F^{\mu\nu} + g_1(S_{\mu\nu}F^{\mu\nu})^2 + g_2F^{\mu\nu}F_{\mu\nu} - i\Gamma_\mu\Gamma_\nu F^{\mu\nu}\big)\Big)\Psi^\epsilon = 0. \tag{11}$$

Equations (11) admit the Lagrangian formulation and include coupling constants g, g_1 and g_2. We will see that the constraints g_1 and g_2 can be chosen such way that the known difficulties [2] with complex energy levels will be overcomed.

4 Particle with arbitrary half-integer spin s in constant and homogeneous magnetic field

In this section we consider the problem of interaction of a charged particle with arbitrary half-integer spin with a constant and homogeneous magnetic field. We demonstrate that for $g \neq \frac{1}{s}$ this problem leads to the known difficulties with complex energies and demonstrate that these difficulties are successfully overcomed using the generalized model (11) with bilinear in $F_{\mu\nu}$ anomalous interaction.

Equation (11) can be expressed in the form of second order equation

$$\Big(\pi_\mu\pi^\mu - m^2 + \frac{eg}{2}S_{\mu\nu}F^{\mu\nu} + \frac{eg_1}{2}(S_{\mu\nu}F^{\mu\nu})^2 + \frac{eg_2}{2}F^{\mu\nu}F_{\mu\nu}\Big)\Psi_+^\epsilon = 0, \tag{12}$$
$$(S_{\mu\nu}S^{\mu\nu} - 15)\Psi_+^\epsilon = 0, \qquad \Psi_-^\epsilon = \frac{1}{m}\Gamma_\mu\pi^\mu\Psi_+^\epsilon,$$

where Ψ^ϵ can be obtained, using the expression

$$\Psi^\epsilon = \Psi_+^\epsilon + \Psi_-^\epsilon.$$

For the case of the constant and homogeneous magnetic field the vector-potential A_μ and the field tensor $F_{\mu\nu}$ are

$$A_0 = A_2 = A_3 = 0, \qquad A_1 = Hx_2, \qquad F_{0a} = F_{23} = F_{31} = 0, \qquad a = 1, 2, 3,$$
$$F_{12} = H_3 = H, \qquad H \geq 0,$$

H is the magnetic field strength.

The solution of equation (12) can be represented as [4]

$$\Psi_+^\epsilon = \begin{pmatrix} \Phi_s^\epsilon \\ \hat{0} \\ \frac{1}{m}(\varepsilon + \frac{1}{s}S_a\pi_a)\Phi_s^\epsilon \\ -\frac{1}{sm}K_a^s\pi_a\Phi_s^\epsilon \end{pmatrix}.$$

Here $(K_3^s)_{mm'} = \delta_{mm'}\sqrt{s^2 - m^2}$; $(K_1^s)_{mm'} \pm i(K_2^s)_{mm'} = \pm\delta_{m\pm m'}\sqrt{s(s-1) \mp m(m\mp 1) \pm 2ms}$, $m, m' = -s, -s+\frac{1}{2}, \ldots, s$; $\hat{0}$ is zero matrix $1\times(2s-1)$, s is value of spin of the particle, Φ_s^ϵ is a four-component spinor which satisfy the equation

$$\big[p^2 + e^2H^2x_2^2 - eH(gS_3 + 2g_1S_3^2H + 2g_2H + 2x_2p_1)\big]\Phi_s^\epsilon = (\varepsilon^2 - m^2)\Phi_s^\epsilon. \tag{13}$$

So the problem of describing the motion of particle with spin s reduces to solving equation (13).

Using the eigenvectors Ω_ν^s ($\nu = -s, -s+\frac{1}{2}, \dots, s$) of matrix S_3 we can represent Φ_s in the form

$$\Phi_s^\epsilon = \exp(i(p_1x_1 + p_3x_3)) \sum_{\nu=-s}^{s} f_\nu^s(x_2)\Omega_\nu^s,$$

where $f_\nu^s(x_2)$ are unknown functions, Ω_ν^s are 4 components spinor eigenvectors of S_3, $S_3\Omega_\nu^s = \nu\Omega_\nu^s$. The functions f_ν^s satisfy the equation

$$\left(\frac{d^2}{dy^2} + y^2\right) f_\nu^s(y) = \eta {f^s}_\nu(y),$$

where $\eta = \frac{\varepsilon^2 - m^2 - p_3^2}{eH} + \nu(g + 2g_1\nu H + 2g_2H)$, $x_2 = \frac{1}{eH}(p_1 + \sqrt{eH}y)$.

Requiring that ${f^s}_\nu(y) \to 0$ when $y \to \pm\infty$ we have

$$\eta = 2n + 1, \qquad n = 0, 1, 2, 3, \dots.$$

Then the energy levels are

$$\varepsilon^2 = m^2 + p_3^2 + eH(2n + 1 - \nu(g + 2g_1\nu H + 2g_2H))$$

and eigenfunctions $f_\nu^s(y)$ take the form

$$f_\nu^s(x_2) = \exp\left(-\frac{eHx_2 - p_2}{2eH}\right) h_n\left(\frac{eHx_2 - p_2}{\sqrt{eH}}\right),$$

where $h_n(y) = \frac{H_n(y)}{||H_n(y)||}$, $H_n(y)$ are Hermitian polynomials. If $g_1 = 0$ such values of H exists for which $\varepsilon^2 \leq 0$. In order that $\varepsilon^2 \geq 0$ for all H, g and g_1 must satisfy the following condition

$$|sg_1 + g_2| \geq \frac{(sg-1)^2 e}{8ms^2}.$$

5 Parasupersymmetry

In this section the parasupersymmetry of equations for particle with spin $\frac{3}{2}$ in magnetic field is discussed. Let us start with the supersymmetry of Dirac equation for particle with spin $\frac{1}{2}$ in homogeneous magnetic field. The related equation (13) takes the following form

$$\left(\varepsilon^2 - (p_1 - eHx_2)^2 - p_2^2 - p_3^2 - m^2 + 2e\sigma_3H\right)\Phi_{\frac{1}{2}} = 0. \tag{14}$$

Here σ_3 is the Pauli matrix and we put in (13) $g_1 = g_2 = 0$ and $g = 2$. Substituting

$$\Phi_{\frac{1}{2}} = \exp(i(p_1x_1 + p_3x_3))\phi(x_2)$$

into (14) we come to the equation

$$\varepsilon'^2\phi(x_2) = H^2\phi(x_2),$$

where

$$H^2 = -\frac{d^2}{dx_2^2} + (p_1 - eHx_2)^2 - 2e\sigma_3H$$

and $\varepsilon'^2 = \varepsilon^2 - p_3^2 - m^2$.

Following [7] we can build the supercharges, which are related to extra degeneracy of energy levels

$$Q = A\psi^\dagger, \qquad Q^\dagger = A^\dagger\psi. \tag{15}$$

A and $A^\dagger$ are bosonic creation and annihilation operators, which have the following form

$$A = \frac{1}{\sqrt{eH}}\left(\frac{d}{dx_2} - p_1 + eHx_2\right), \qquad A^\dagger = \frac{1}{\sqrt{eH}}\left(-\frac{d}{dx_2} - p_1 + eHx_2\right), \tag{16}$$

and satisfy the conditions

$$[A, A^\dagger] = 1, \qquad [N, A] = -A, \qquad [N, A^\dagger] = A^\dagger, \qquad N = A^\dagger A.$$

Operators ψ and $\psi^\dagger$ are fermionic creation and annihilation operators, which have the form

$$\psi = \sigma_+ = \frac{1}{2}(\sigma_1 + i\sigma_2), \qquad \psi^\dagger = \sigma_- = \frac{1}{2}(\sigma_1 - i\sigma_2).$$

They obey the anticommutation relations

$$\{\psi, \psi^\dagger\} = 1, \qquad \{\psi^\dagger, \psi^\dagger\} = \{\psi, \psi\} = 0,$$

as well as obeying the commutation relation

$$[\psi, \psi^\dagger] = \sigma_3.$$

The operator which plays the role of SUSY Hamiltonian is

$$H^2 = QQ^\dagger + Q^\dagger Q.$$

Now let us consider the spin-$\frac{3}{2}$. In this case the equation (13) has the form

$$[-p_1^2 - p_2^2 - p_3^2 + e^2H^2x_2^2 - geHS_3 - 2eHx_2p_1]\Phi^{\epsilon}_{\frac{3}{2}} = (\varepsilon^2 - m^2)\Phi^{\epsilon}_{\frac{3}{2}},$$

where S_3 belongs to the irreducible representation $D(\frac{3}{2})$ of algebra $AO(3)$ and we put $g_1 = g_2 = 0$, $g = 2$.

Substituting

$$\Phi^{\epsilon}_{\frac{3}{2}} = \exp(i(p_1x_1 + p_3x_3))\phi_{\frac{3}{2}}(x_2),$$

we come to the equation

$$\varepsilon'^2\phi(x_2) = H^2_{\frac{3}{2}}\phi(x_2),$$

where $\varepsilon'^2 = \varepsilon^2 - p_3^2 - m^2$ and

$$H^2_{\frac{3}{2}} = -\frac{d^2}{dx_2^2} + (p_1 - eHx_2)^2 - 2eS_3H.$$

Here S_3 is the spin matrix belonging to the irreducible representation $D(\frac{3}{2})$ of algebra $AO(3)$.

Let us show that we obtain $N = 2$, $p = 3$ parasuperalgebra with parasupercharges (15), where bosonic creation and annihilation operators A, $A^\dagger$ (16) and parafermionic creation and annihilation operators take the following form

$$\psi = S_+ = \frac{1}{2}(S_1 + iS_2), \qquad \psi^\dagger = S_- = \frac{1}{2}(S_1 - iS_2). \tag{17}$$

Here S_1, S_2 and S_3 are spin matrices for spin $\frac{3}{2}$. It can be verified that operators ψ, $\psi^\dagger$ satisfy the double commutation relation

$$[\psi,[\psi^\dagger,\psi]]=2\psi, \qquad [\psi^\dagger,[\psi,\psi^\dagger]]=2\psi^\dagger$$

as well as satisfying the commutation relations

$$[\psi,\psi^\dagger]=S_3.$$

Operators Q, $Q^\dagger$ and $H^2_{\frac{3}{2}}$ satisfy the double commutation relations typical for $p=3$ parasupersymmetry

$$[H^2_{\frac{3}{2}},Q]=[H^2_{\frac{3}{2}},Q^\dagger]=0, \qquad [Q,[Q^\dagger,Q]]=2QH^2_{\frac{3}{2}}.$$

We notice that the mentioned parasupersymmetry appears only for $g_2\neq 0$.

It is possible to show that in the case of arbitrary half-integer spin $s=n+\frac{1}{2}$ we have the $p=n$ parasupersymmetry, where the parasuperchages have the form (15) with bosonic creation and annihilation operators A and $A^\dagger$ (16) and parafermionic creation and annihilation operators (17). The related matrices S_1, S_2 and S_3 belong to the irreducible representation $D(s)$ of algebra $AO(3)$.

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