**CP violation in** $B \rightarrow \pi \pi, \rho \pi$

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In the neutral $B$ meson system, it is possible to measure the CKM angle $\alpha$ using the decay mode $b \rightarrow u \nu d$ in the presence of penguin pollution. Here the recent status of $C$ violation in $B^0 \rightarrow \pi^+\pi^-$ and $B^0 \rightarrow \rho\pi$ decays and the prospects are presented.

1 Introduction

In 1973, Kobayashi and Maskawa (KM) proposed a model where $CP$ violation is accommodated as an irreducible complex phase in the quark mixing matrix [1]. Recent measurements of the $CP$-violating asymmetry parameter $\sin 2\beta$ by the Belle and BaBar Collaborations established $CP$ violation in the neutral $B$ meson [2]. Measurements of other $CP$-violating asymmetry parameters provide important tests of the KM model. Any mode with a contribution from $b \rightarrow u \nu d$ is a possible source of measurement of the Cabibbo-Kobayashi-Maskawa (CKM) angle $\alpha (= \phi_2)$ [3]. Here the status of $CP$ violation in $B^0 \rightarrow \pi^+\pi^-$ and $B^0 \rightarrow \rho\pi$ decays [4] and the prospects are presented.

2 $B^0 \rightarrow \pi^+\pi^-$ decays

The KM model predicts $CP$-violating asymmetries in the time-dependent rates for $B^0$ and $\bar{B}^0$ decays to a common $CP$ eigenstate, $f_{CP}$. In the decay chain $\Upsilon(4S) \rightarrow B^0\bar{B}^0 \rightarrow f_{CP}f_{tag}$, in which one of the $B$ mesons decays at time $t_{CP}$ to $f_{CP}$ and the other decays at time $t_{tag}$ to a final state $f_{tag}$ that distinguishes between $B^0$ and $\bar{B}^0$, the $B^0 \rightarrow \pi^+\pi^-$ decay rate has a time-dependence given by

$$P_{\pi\pi}^q(\Delta t) = \frac{e^{-|\Delta t|/\tau_{B^0}}}{4\tau_{B^0}} \left[ 1 + q \cdot \left\{ S_{\pi\pi} \sin(\Delta m_d\Delta t) - C_{\pi\pi} \cos(\Delta m_d\Delta t) \right\} \right],$$

where $\tau_{B^0}$ is the $B^0$ lifetime, $\Delta m_d$ is the mass difference between the two $B^0$ mass eigenstates, $\Delta t = t_{CP} - t_{tag}$, and the $b$-flavor charge $q = (+1)$ when the tagging $B$ meson is a $B^0$. The $CP$-violating asymmetry parameters $S_{\pi\pi}$ and $C_{\pi\pi}$ defined in Eq. (1) are expressed as $C_{\pi\pi} = (1 - |\lambda_{\pi\pi}|^2)/(1 + |\lambda_{\pi\pi}|^2)$ and $S_{\pi\pi} = 2\text{Im}\lambda_{\pi\pi}/(1 + |\lambda_{\pi\pi}|^2)$, where $\lambda_{\pi\pi}$ is a complex parameter that depends on both $B^0\bar{B}^0$ mixing and the amplitudes for $B^0$ and $\bar{B}^0$ decay to $\pi^+\pi^-$. If the decay proceeded only via a $b \rightarrow u \nu d$ tree amplitude, $S_{\pi\pi} = \sin^2 \alpha$ and $C_{\pi\pi} = 0$. In general, $S_{\pi\pi}$ is given by $\sqrt{1 - C_{\pi\pi}^2} \sin 2\alpha_{eff}$ Here $\alpha_{eff} - \alpha$ depends on the magnitudes and relative...
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weak and strong phases of the tree and penguin amplitudes. With significant contributions from gluonic $b \to d$ penguin amplitudes, $S_{\pi\pi}$ may not be equal to $\sin 2\alpha$ and direct CP violation, $C_{\pi\pi} \neq 0$, may occur.

$B$ candidates are reconstructed using two variables, the energy difference $\Delta E \equiv E_B^{\text{cms}} - E_{\text{beam}}^{\text{cms}}$ and the beam-energy constrained mass $M_{bc} \equiv \sqrt{(E_{\text{beam}}^{\text{cms}})^2 - (p_B^{\text{cms}})^2}$ [6], where $E_B^{\text{cms}}$ is the cms beam energy, and $E_B^{\text{cms}}$ and $p_B^{\text{cms}}$ are the cms energy and momentum of the $B$ candidate. Charged tracks in $B^0 \to h^+h'^-$ candidates are identified as charged pions or kaons. Here $h$ and $h'$ represent a $\pi$ or $K$. The Belle Collaboration uses the likelihood ratio for a particle to be a $K^\pm$ meson, which is the combined information from the Aerogel Cherenkov counter and CDC $dE/dx$. The BaBar Collaboration uses the Cherenkov angle measurement $\theta_c$ from a detector of internally reflected Cherenkov light. The probability density function (PDF) from the difference between measured and expected values of $\theta_c$ is used in the extended likelihood function for the fit to extract yields and CP parameters.

The $q\bar{q}$ continuum ($q = u, d, s, c$) background is suppressed by the event topology. The Belle Collaboration forms signal and background likelihood functions $L_S$ and $L_{BG}$ from a Fisher discriminant using six modified Fox-Wolfram moments [7] and the cms $B$ flight direction. The continuum background is reduced by imposing requirements on the likelihood ratio $LR = L_S/(L_S + L_{BG})$. The BaBar Collaboration uses the angle $\theta_S$ between the sphericity axis of the $B$ candidate and the sphericity axis of the remaining particles in the cms frame, and cut on $|\cos \theta_S|$. The shapes of Fisher discriminant $F$ [8] for signal and background events are included as PDFs in the maximum likelihood fit.

Leptons, kaons, and charged pions that are not associated with the reconstructed $B$ candidate are used to identify the flavor of the accompanying $B$ meson.

The time difference $\Delta t$ is obtained from the measured distance between the $z$ positions along the beam direction of the $B^0_{\pi\pi}$ and $B^0_{\text{tag}}$ decay vertices and the boost factor $\beta \gamma$ of the $e^+e^-$ system.

Fig. 1 and Fig. 2 show $\Delta E$ distributions for events enhanced in signal $\pi^+\pi^-$ and $K^+\pi^\mp$ decays from the Belle Collaboration [9] and the BaBar Collaboration [10], respectively. They obtained the following results based on $85 \times 10^6$ and $88 \times 10^6 B\overline{B}$ pairs, respectively:

$$C_{\pi\pi} = -0.77 \pm 0.27 \pm 0.08, \quad S_{\pi\pi} = -1.23 \pm 0.41 \pm 0.08 \quad \text{(Belle)},$$
$$C_{\pi\pi} = -0.30 \pm 0.25 \pm 0.04, \quad S_{\pi\pi} = -0.02 \pm 0.34 \pm 0.05 \quad \text{(BaBar)}.$$

The first and the second errors are statistical and systematic errors, respectively. The average values of $C_{\pi\pi}$ and $S_{\pi\pi}$ are $C_{\pi\pi} = -0.49 \pm 0.19$ and $S_{\pi\pi} = -0.47 \pm 0.26$ [11]. In Fig. 3 and Fig. 4, the $\Delta t$ distributions are shown from the Belle result [9] and the BaBar result [10], respectively. Fig. 5 shows the two-dimensional confidence regions in the $A_{\pi\pi}$ vs. $S_{\pi\pi}$. The case that CP symmetry is conserved, $A_{\pi\pi} = S_{\pi\pi} = 0$, is ruled out at the 99.93% confidence level (C.L.), and the 95.5% C.L. region of $A_{\pi\pi}$ and $S_{\pi\pi}$ gives $78^\circ \leq \phi_2 \leq 152^\circ$ from the Belle result [9].

Fig. 6 shows the pQCD prediction [12] and other predictions. As described in [12], the pQCD approach predicts large direct CP asymmetry (16~30%), while QCDF approach predicts $-6 \pm 12\%$ [13]. Other interpretations for the current results can be found in ref. [14].

Using isospin relations [15], we constrain $\theta = \alpha_{\text{eff}} - \alpha$. From the central values of the recent world average values of the branching ratios of $B^0 \to \pi^+\pi^-$, $B^+ \to \pi^+\pi^0$ and the 90%
Figure 1: $\Delta E$ distributions for (a) $\pi^+\pi^-$ and (b) $K^+\pi^-$ candidates with $LR > 0.825$ from the Belle Collaboration. The sum of the signal and background functions is shown as a solid curve. The hatched area represents the $\pi^+\pi^-$ component, the dashed curve represents the $K^+\pi^-$ component, the dotted curve represents $q\bar{q}$ background, and the dot-dashed curve represents the charmless three-body $B$ decay background.

Figure 2: $\Delta E$ distributions for events enhanced in signal (a) $\pi^+\pi^-$ and (b) $K^\pm\pi^\mp$ candidates from the BaBar Collaboration. Solid curves represent projections of the maximum likelihood fit, dashed curves represent $q\bar{q}$ and $\pi\pi \leftrightarrow K\pi$ cross-feed background.
Figure 3: The raw, unweighted $\Delta t$ distributions for $\pi^+\pi^-$ candidates with $LR > 0.825$ from the Belle Collaboration: candidates tagged as (a) $B^0$-tag and (b) $\overline{B}^0$-tag; (c) $\pi^+\pi^-$ yields after background subtraction; (d) the $CP$ asymmetry for $\pi^+\pi^-$. In Figs. (a) through (c), the solid curves show the results of the unbinned maximum likelihood fit to the $\Delta t$ distributions of the whole $\pi^+\pi^-$ candidates. In Fig. (d), the dashed (dotted) curve is the contribution from the cosine (sine) term.

Figure 4: Distributions of $\Delta t$ for events enhanced in signal $\pi\pi$ decays from the BaBar Collaboration: candidates tagged as (a) $B^0$- or (b) $\overline{B}^0$-tag, and (c) the time-dependent asymmetry. Solid (dashed) curves represent projections of the maximum likelihood fit (the sum of $q\bar{q}$ and $K\pi$ backgrounds).
Figure 5: Confidence regions for $A_{\pi\pi}$ and $S_{\pi\pi}$ from the Belle and BaBar results.

Figure 6: The left plot is $A_{\pi\pi}$ vs. $S_{\pi\pi}$ for various values of $\phi_2$ in the pQCD method. Dark areas are allowed regions in the pQCD method for different $\phi_2$ values. The right plot is the predictions for $C_{\pi\pi}$ ($= -A_{\pi\pi}$) and $S_{\pi\pi}$ for several analysis steps with experimental and theoretical constraints.
C.L. upper limit on the $B^0 \to \pi^0 \pi^0$ branching ratio [11] together with $C_{\pi\pi}$, the upper limit on $|\theta|$ is 54°.

3 $B^0 \to \rho \pi \to \pi^+ \pi^- \pi^0$ decays

The CKM angle $\alpha$ can be measured in the presence of penguin contributions using a full Dalitz plot analysis of the final state. In order to extract $\alpha$ cleanly, data with large statistics are required. Following a quasi-two-body approach, the analysis is restricted to the two regions of the $\pi^\mp \pi^0 h^\pm$ Dalitz plot ($h = \pi$ or $K$) that are dominated by $\rho^\pm h^\mp$. The decay rate is given by

$$f_q^{\rho^\pm h^\mp}(\Delta t) = (1 \pm A_{\rho h}^{\rho \pi}) \frac{|\Delta A_{\rho h}|}{4\pi m} \times [1 + q\cdot((S_{ph} \pm \Delta S_{ph}) \sin(\Delta m_d \Delta t) - (C_{ph} \pm \Delta C_{ph}) \cos(\Delta m_d \Delta t))], \quad (2)$$

where $\Delta t = t_{\rho h} - t_{tag}$ as the time interval between the decay of $B_{\rho h}$ and that of the other $B^0$ meson. One finds the relations $S_{\rho \pi} \pm \Delta S_{\rho \pi} = \sqrt{1 - (C_{\rho \pi} \pm \Delta C_{\rho \pi})^2 \sin(2\alpha_{\text{eff}} \pm \delta)}$, where $2\alpha_{\text{eff}} = \arg((q/p)(\overline{A}_{\rho \pi} / A_{\rho \pi}))$, $\delta = \arg[A_{\rho \pi}^- / A_{\rho \pi}^+]$, $\arg[q/p]$ is the $B^0 \to B^0$ mixing phase, and $A_{\rho \pi}^0$ ($\overline{A}_{\rho \pi}^0$) and $A_{\rho \pi}^+ / A_{\rho \pi}^-$ are the transition amplitudes of the processes $B^0(\overline{B}^0) \to \rho^+ \pi^-$ and $B^0(\overline{B}^0) \to \rho^- \pi^+$, respectively. The angles $\alpha_{\text{eff}}$ are equal to $\alpha$ if contributions from penguin amplitudes are absent.

The results on direct $CP$ violation can be expressed as

$$A_{+-} = \frac{N(B^0_{\rho \pi} \to \rho^+ \pi^-) - N(B^0_{\rho \pi} \to \rho^- \pi^+)}{N(B^0_{\rho \pi} \to \rho^+ \pi^+) + N(B^0_{\rho \pi} \to \rho^- \pi^+)}; \quad A_{++} = \frac{N(B^0_{\rho \pi} \to \rho^- \pi^+) - N(B^0_{\rho \pi} \to \rho^+ \pi^-)}{N(B^0_{\rho \pi} \to \rho^+ \pi^+) + N(B^0_{\rho \pi} \to \rho^- \pi^+)} \quad (3)$$

With $89 \times 10^6 B\overline{B}$ pairs [16], the BaBar Collaboration measured the asymmetry parameters:

$$A_{\rho \pi}^{\rho \pi} = -0.18 \pm 0.08 \pm 0.03, \quad C_{\rho \pi} = +0.36 \pm 0.18 \pm 0.04, \quad S_{\rho \pi} = +0.19 \pm 0.24 \pm 0.03,$$

$$\Delta C_{\rho \pi} = +0.28 \pm 0.19 \pm 0.04, \quad \Delta S_{\rho \pi} = +0.15 \pm 0.25 \pm 0.03,$$

$$A_{+-} = -0.62^{+0.24}_{-0.25} \pm 0.06, \quad A_{++} = -0.11^{+0.16}_{-0.17} \pm 0.04.$$

The raw time-dependent asymmetry dominated by kaons and leptons is shown in Fig. 7.

4 Prospects

Table 1 shows the expected errors on asymmetry parameters in $B^0 \to \pi^+ \pi^-$ and $\rho^+ \pi^-$ decays. Fig. 8 shows the prospects of $\alpha - \alpha_{\text{eff}}$ in $B \to \pi \pi$ decays [14]. Only a luminosity of around $10^{-1}$ allows to separate the solutions. The detailed interpretation for $B^0 \to \pi^+ \pi^-$ and $\rho \pi$ can be found in [17].

5 Summary

The Belle and BaBar Collaborations obtain the $CP$-violating asymmetries in $B^0 \to \pi^+ \pi^-$ decays:

$$A_{\pi \pi} = -0.77 \pm 0.27 \pm 0.08, \quad S_{\pi \pi} = -1.23 \pm 0.41^{+0.08}_{-0.07} \quad \text{(Belle)},$$

$$A_{\pi \pi} = -0.30 \pm 0.25 \pm 0.04, \quad S_{\pi \pi} = -0.02 \pm 0.34 \pm 0.05 \quad \text{(BaBar)}.$$
Figure 7: Time distributions for events enhanced in the $\rho\pi$ signal tagged as (a) $B^0$-tag and (b) $\overline{B}^0$-tag, and (c) time-dependent asymmetry between $B^0$-tag and $\overline{B}^0$-tag [16]. The solid (dashed) curve is a likelihood projection of the fit result (the sum of $B$- and continuum-background contributions).

Table 1: The errors on asymmetry parameters in $B^0 \rightarrow \pi^+\pi^-$ and $\rho^\pm\pi^\mp$ decays at several luminosities ($L$), assuming statistical and systematic errors are proportional to $1/\sqrt{L}$ and $1/4\sqrt{L}$, respectively.
Figure 8: $\alpha - \alpha_{\text{eff}}$ at several luminosities (87 fb$^{-1}$, 500 fb$^{-1}$, 2 ab$^{-1}$, and 10 ab$^{-1}$).

The asymmetry parameters in $B^0 \to \rho \pi$ decays are obtained by the BaBar Collaboration:

$$A_{\rho \pi}^{CP} = -0.18 \pm 0.08 \pm 0.03, \quad C_{\rho \pi} = +0.36 \pm 0.18 \pm 0.04, \quad S_{\rho \pi} = +0.19 \pm 0.24 \pm 0.03,$$

$$\Delta C_{\rho \pi} = +0.28 \pm 0.19 \pm 0.04, \quad \Delta S_{\rho \pi} = +0.15 \pm 0.25 \pm 0.03.$$

**References**


[3] $\phi_1 (= \beta) \equiv \arg [-V_{cd}V_{cb}^*/V_{td}V_{tb}^*]$ and $\phi_2 (= \alpha) \equiv \arg [-V_{td}V_{tb}^*/V_{ud}V_{ub}^*]$. 

[4] The inclusion of the charge conjugate mode decay is implied unless otherwise stated.

[5] $C_{\pi \pi} = -A_{\pi \pi}$. The BaBar Collaboration uses $C_{\pi \pi}$ and the Belle Collaboration uses $A_{\pi \pi}$.

[6] The BaBar Collaboration uses the beam-energy substituted mass $m_{ES} = \sqrt{(s/2 + p_i \cdot p_B)^2/E_i^2 - p_B^2}$, where $\sqrt{s}$ is the total cms energy, and the $B$ momentum $p_B$ and the four-momentum of the initial state $(E_i, p_i)$ are defined in the laboratory frame.


[8] $F = 0.53 - 0.60 \times \sum_i p_i^* + 1.27 \times \sum_i p_i^* |\cos(\theta_i^*)|^2$, where $p_i^*$ is the momentum of particle $i$ and $\theta_i^*$ is the angle between its momentum and the $B$ thrust axis in the cms frame. The sum is over all particles in the event excluding those of the $B$ candidate.


[17] H. Lacker, these proceedings.