Recent progress on the extraction of V_{ud} and V_{us} from meson decays

V. Cirigliano

Departament de Física Teòrica, IFIC, CSIC — Universitat de València
Apt. Correus 22085, E-46071 València, Spain

We discuss radiative corrections to K_{e3} and π_{e3} within chiral perturbation theory with virtual photons and leptons. We then present the extraction of the CKM elements V_{us} and V_{ud} using the presently available experimental input. Finally we discuss the test of CKM unitarity based on the present independent knowledge of V_{ud} and V_{us}, and describe the prospects of improvement with the upcoming new B.R. measurements.

1 Motivation

V_{us} and V_{ud} are, at present, the most precisely known elements of the CKM matrix, with a fractional uncertainty at the level of $\sim 1\%$ and $\sim 0.1\%$ respectively. This high-precision information is extracted from the semileptonic transitions $s \to u$ and $d \to u$, occurring in low-energy hadronic environments. In particular, the best determination of $|V_{us}|$ is obtained from $K \to \pi \ell \nu$ decays ($K_{\ell 3}$), whereas the two most stringent constraints on $|V_{ud}|$ are obtained from superallowed Fermi transitions (SFT), and from the neutron beta decay [1]. The high accuracy of these constraints is due to two key features: the non-renormalization of the vector current at zero momentum transfer in the $SU(N)$ limit (CVC), and the Ademollo Gatto theorem [2]. The latter implies that corrections to the relevant hadronic form factors at zero momentum transfer are of second order in the $SU(N)$-breaking parameters (N=2,3).

The present level of accuracy on $|V_{ud}|$ and $|V_{us}|$ is such that the contribution of $|V_{ub}|$ to the unitarity relation

$$U_{uu} = |V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1 \tag{1}$$

can be safely neglected, and the uncertainty of the first two terms is comparable. In other words, $|V_{ud}|$ and $|V_{us}|$ lead to two independent determinations of the Cabibbo angle both at the 1% level. Plugging in the PDG numbers [3], one finds a discrepancy between the two determinations at the two-sigma level. An opportunity to clarify this long-standing puzzle is offered us by the ongoing experimental efforts which will provide new measurements of π_{e3} [4] and $K_{\ell 3}$ branching ratios [5–7]. Precise data on π_{e3} (B.R. $\sim 10^{-8}$) will allow one to extract V_{ud} from a theoretically very clean environment, while improved $K_{\ell 3}$ B.R.s will allow one to update the extraction of V_{us}. In both cases, however, in order to fully take advantage of new data, an update of the theoretical analysis is in order. In particular, one needs to have good control over the $SU(2)$ and $SU(3)$ breaking effects, as well as on radiative corrections. Chiral Perturbation Theory provides a modern unified framework for studying both problems.

2 Semileptonic decays of light mesons

2.1 Theoretical Framework

Weak decays of hadrons are best treated within an effective theory approach. In the case at hand, one starts by assuming that the Standard Model is the appropriate theory at the scale $\mu \sim 100$ GeV. At scales $\mu \sim M_W$ one integrates out the heavy degrees of freedom and matches the SM to the Fermi Theory of weak interaction, enhanced by QED and QCD, with five active quark flavors. The effective lagrangian governing semileptonic transitions assumes the form:

$$\begin{aligned} \mathcal{L}_{\rm eff} &= -\frac{G_\mu}{\sqrt{2}} \sqrt{S_{\rm ew}(\nu)} \times \mathcal{O}_{JJ}(\nu) \ , \\ \mathcal{O}_{JJ} &= \bar{l}_k \, \gamma_\mu (1-\gamma_5) \, \nu_k \ \times \ \bar{u}_i \, \gamma^\mu (1-\gamma_5) \, \mathbf{V}_{ij} d_j \ . \end{aligned} \tag{2}$$

Here i, j, k are flavor indices (running over active flavors), and $\mathbf{V}$ is the CKM matrix. G_μ is the Fermi constant governing purely leptonic charged current processes (measured in the muon decay). Finally, ν indicates the renormalization scale.

The effective coupling is known up to non-logarithmic terms of order $\alpha \alpha_s$, and is given by [8]:

$$S_{\rm ew}(\mu) = 1 + \frac{2\alpha}{\pi} \left(1 - \frac{\alpha_s}{4\pi}\right) \log \frac{M_Z}{\nu} + O(\frac{\alpha \alpha_s}{\pi^2}) \tag{3}$$

After re-summing the leading logarithms $O[(\alpha \log M_Z)^n, \alpha(\alpha_s \log M_Z)^n]$ via renormalization group, down to the renormalization point $\nu = M_\rho$, one finds (taking into account fermion thresholds):

$$S_{\rm ew}(M_\rho) = 1.0230 \pm 0.0003 \ . \tag{4}$$

The second part of the problem consists in calculating matrix elements of the type $\langle f|\mathcal{O}_{JJ}|i\rangle$. In our case, the relevant matrix element has the form:

$$\langle \pi\ell\nu|\mathcal{O}_{JJ}|P\rangle \sim J_l J_h \left[1 + O\left(\alpha \log \frac{M_P}{m_e}, \alpha c_n\right)\right] \ , \quad (5)$$

where J_l is the leptonic current and J_h is the hadronic current. The radiative corrections will in general involve large long-distance logs as well as non-logarithmic pieces (αc_n). An accurate calculation of semileptonic decays requires control over J_h (QCD dynamics at low energy), and the potentially large long-distance radiative corrections.

The appropriate framework to address both issues is provided by yet another effective theory, valid in the energy range below the first QCD resonance masses: Chiral Perturbation Theory (ChPT) [9]. Active degrees of freedom in ChPT are the eight Goldostone modes (π, K, η) associated with SSB of chiral symmetry in QCD, as well as light leptons and photons. The effective lagrangian of ChPT is constructed according to symmetry principles, taking into account the SSB pattern of chiral symmetry as well as explicit symmetry breaking, due to quark masses. The effective action is organized by chiral power counting as an expansion in the derivatives (momenta) of Goldstone modes (counted as $O(p)$) and the quark masses (counted as $O(p^2)$ in the standard approach). At a given order in p/Λ_χ (with $\Lambda_\chi \sim 4\pi F_\pi$) a finite number of local couplings encodes the short distance physics. Even though a full matching of QCD to ChPT is not available, model-independent phenomenology is possible in several cases of interest. In this work we need the extension of ChPT developed in Ref. [10], appropriate to deal with virtual photons and leptons.

2.2 $P_{\ell 3}$ decays: matrix elements and form factors

Neglecting for the moment radiative corrections, the invariant amplitude for the decay $P(p_P) \longrightarrow \pi(p_\pi)\ \ell^+(p_\ell)\ \nu_\ell(p_\nu)$ factorizes in the product of leptonic and hadronic currents, the latter being decomposed in terms of hadronic form factors. Explicitly one has

$$\mathcal{M} = \frac{G_\mu}{\sqrt{2}} V_{uf}^* \, L^\mu \times$$
$$\times \underbrace{C_P \left[f_+^P(t)\,(p_P + p_\pi)_\mu + f_-^P(t)\,(p_P - p_\pi)_\mu \right]}_{\langle \pi(p_\pi)|\bar{u}\gamma_\mu f|P(p_P)\rangle} , (6)$$

with

$$\begin{array}{lcl} L^\mu & = & \bar{u}(p_\nu)\gamma^\mu(1-\gamma_5)v(p_\ell) \\ t & = & (p_P - p_\pi)^2 \\ C_P & = & (1, 1/\sqrt{2}, \sqrt{2}) \text{ for } (K^0, K^+, \pi^+) \end{array} \quad (7)$$

We recall that the dependence of $|\mathcal{M}|^2$ on f_- is proportional to $(m_\ell/M_P)^2$, thus f_- is completely irrelevant for the electron modes (K_{e3}, π_{e3}). Moreover, it is customary to trade $f_-(t)$ for the scalar form factor $f_0(t)$, defined as:

$$f_0(t) = f_+(t) \ + \ \frac{t}{M_K^2 - M_\pi^2}\, f_-(t) \ . \quad (8)$$

The momentum dependence of the form factors, relevant for the integral over the phase space, is often described in terms of one or at most two parameters (slope and curvature at $t = 0$),

$$f_{+,0}(t) = f_+(0) \left(1 + \lambda_{+,0}\ \frac{t}{M_\pi^2} + \lambda'_{+,0}\frac{t^2}{M_\pi^4} + \ldots\right) . (9)$$

In this approximation the phase space integral depends explicitly only on the slope (and curvature) parameters, and we use for it the generic notation $I_P(\lambda)$. With this notation, the partial width associated to $P \to \pi\ell\nu$ is given by:

$$\begin{array}{lcl} \Gamma_P & = & |V_{uf}|^2 \ \cdot |f_+^P(0)|^2 \cdot \mathcal{N}_P \cdot I_P(\lambda) \ ; \\ \mathcal{N}_P & = & C_P^2 \ \dfrac{G_\mu^2\ M_P^5}{192\pi^3} \ . \end{array} \quad (10)$$

The steps necessary to extract $|V_{uf}| (f = d, s)$ from the measured $P_{\ell 3}$ decay rates are: (1) theoretical evaluation of $f_+(0)$, i.e. the $SU(2)$ and $SU(3)$ corrections; (2) measurement (or, if not available, theoretical evaluation) of the momentum dependence of $f_{+,0}(t)$ (form factor slope and curvature parameters); (3) theoretical treatment of photonic radiative corrections. The short distance contribution is given by the factor $S_{\rm ew}$ and amounts to using a different effective Fermi coupling, while the long-distance corrections will be discussed in the next subsection [note that Eq.10 is not yet general enough to account for these effects, see below].

2.3 Long distance radiative corrections

A recent analysis of this issue has been performed in Ref. [11], to which we refer for details. Long distance radiative corrections involve the effect of (1) loops with virtual photons and (2) emission of real photons:

1. Photon loops modify the very structure of the amplitude, breaking the factorized form. The form factors now depend on another kinematical variable

$$f_\pm(t) \to f_\pm(t, v) \simeq \left[1 + \frac{\alpha}{4\pi}\Gamma_c(v, \lambda_{IR})\right]\ \tilde{f}_\pm(t) \ ,$$

where $v = (p_P - p_l)^2$ in P^+ decays and $v = (p_\pi + p_l)^2$ in P^0 decays. The function $\Gamma_c(v, \lambda_{IR})$

encodes universal long distance corrections, independent of strong dynamics, and is infrared divergent. Since the dependence on the second kinematical variable can be factored out (to a very good approximation), the notion of effective form factor $\tilde{f}_{\pm}(t)$ survives and proves useful in the subsequent analysis. $\tilde{f}_{\pm}(0)$ contains pure QCD effects as well as new local contributions of order α. These, together with the chiral logarithms, are truly structure dependent corrections, which can be described in a model independent way within the ChPT approach.

2. One has to consider how radiation of real photons affects the various observables (e.g. Dalitz Plot density, spectra, branching ratios). For the purpose of extracting $|V_{uf}|$, we need to assess the effect of real photon emission on the partial widths. As is well known, a given experiment measures an inclusive sum of the parent mode and radiative modes:

$$d\Gamma_{\rm obs} = d\Gamma(P_{\ell 3}) \ + \ d\Gamma(P_{\ell 3\gamma}) \ + \ \cdots$$

From the theoretical point of view, only such an inclusive sum is free of infrared singularities. At the precision we aim to work at, a meaningful comparison of theory and experiment can be done only once a clear definition of the inclusive observable is given [12].

In summary, all the universal QED effects, due to both virtual photons $[\Gamma_c(v,\lambda_{IR})]$ and real photons $[d\Gamma(K_{\ell 3\gamma})]$, can be combined to produce a correction to the phase space factor in the expression for the decay width:

$$I_P(\lambda) \longrightarrow I_P(\lambda,\alpha) = I_P(\lambda,0)\left[1+\Delta I_P(\lambda)\right] .$$

This term comes, in principle, with no theoretical uncertainty. The structure dependent electromagnetic corrections, as well as $SU(3)$ breaking corrections are in the form factor $\tilde{f}_{+}(t)$, where all of the theoretical uncertainty concentrates. At zero momentum transfer, this can be conveniently written as

$$\tilde{f}_{+}(0) = f_{+}(0)\,(1+\delta_{SU(2)}+\delta_{EM})$$

Explicit expressions for $\tilde{f}_{\pm}(t)$ at NLO in ChPT, including the SU(2) and EM corrections, can be found in Refs. [11,13].

3 V_{ud} from π_{e3} decay

Theoretically this process is very attractive because it shares the advantages of both Fermi transitions (pure vector transition, no axial-vector contribution) and neutron β-decay (no nuclear structure dependent radiative corrections). The difficulty here lies in the extremely small branching ratio, of order 10^{-8}. With the notations introduced in the previous sections, V_{ud} is related to the partial width by the following relation:

$$|V_{ud}| = \left[\frac{\Gamma_{\pi_{e3[\gamma]}}}{\mathcal{N}_\pi\, I_\pi(\lambda,0)}\right]^{1/2} \left[\frac{1}{S_{\rm ew}(M_\rho)}\right]^{1/2} \times$$
$$\times \frac{1}{1+\delta^{\pi}_{SU(2)}+\delta^{\pi}_{\rm EM}+\frac{1}{2}\Delta I_\pi(\lambda)} \qquad (11)$$

Explicit calculation of the radiative corrections and $SU(2)$ breaking leads to [13]:

$$\begin{array}{rcl} \delta^{\pi}_{SU(2)} & \sim & 10^{-5} \\ \delta^{\pi}_{e^2p^2} & = & (0.46\pm 0.05)\% \\ \Delta I_\pi(\lambda) & = & 0.1\% \ , \end{array} \qquad (12)$$

with a total effect of radiative corrections consistent with previous estimates [14].
The present experimental precision for the branching ratio of the pionic beta decay cannot compete yet with the very small theoretical uncertainties of SFT and neutron beta decay: using the latest PDG value [3] $\mathrm{BR} = (1.025\pm 0.034)\times 10^{-8}$, we find

$$\begin{array}{rcl} |V_{ud}| & = & 0.9675 \pm 0.0160_{\rm exp} \pm 0.0005_{\rm th} \\ & = & 0.9675 \pm 0.0161. \end{array} \qquad (13)$$

However, a substantial improvement of the experimental accuracy is to be expected in the near future. Inserting the present preliminary result obtained by the PIBETA Collaboration [4], $\mathrm{BR} = (1.044 \pm 0.007(\mathrm{stat.}) \pm 0.009(\mathrm{syst.}))\times 10^{-8}$, we find

$$\begin{array}{rcl} |V_{ud}| & = & 0.9765 \pm 0.0056_{\rm exp} \pm 0.0005_{\rm th} \\ & = & 0.9765 \pm 0.0056 \ , \end{array} \qquad (14)$$

where the error should be reduced by about a factor of 3 at the end of the experiment.

4 V_{us} from K_{e3} decays

Each of the four $K_{\ell 3}$ widths is related to V_{us} by a relation similar to Eq.11, modulo an obvious change needed to take into account $SU(3)$ breaking effects. The $K_{\ell 3}$ widths thus allow us to obtain four determinations [15] of $|V_{us}|\cdot f_{+}^{K^0\pi^-}(0)$ which are independent, up to the small correlations of theoretical uncertainties from isospin-breaking corrections $\delta_{\rm EM}$ and $\delta_{SU(2)}$

Table 1. Summary of isospin-breaking factors.

	$\delta_{SU(2)}(\%)$	$\delta_{\rm EM}(\%)$	$\Delta I(\lambda)(\%)$
K^+_{e3}	2.4 ± 0.2	0.32 ± 0.16	-1.27
K^0_{e3}	0	0.46 ± 0.08	-0.32
$K^+_{\mu 3}$	2.4 ± 0.2	0.006 ± 0.16	0 ± 1.0 *
$K^0_{\mu 3}$	0	0.15 ± 0.08	1.7 ± 1.0 *

(almost negligible at present, see Table 1). The master formula for a combined analysis of these modes is:

$$|V_{us}|\cdot f_+^{K^0\pi^-}(0) = \left[\frac{\Gamma_{n[\gamma]}}{\mathcal{N}_n\, I_n(\lambda,0)}\right]^{1/2} \left[\frac{1}{S_{\rm ew}(M_\rho)}\right]^{1/2} \times$$

$$\times \frac{1}{1+\delta^n_{SU(2)}+\delta^n_{\rm EM}+\frac{1}{2}\Delta I_n(\lambda)} \qquad (15)$$

where the index n runs over the four modes ($n = K^+_{e3}, K^0_{e3}, K^+_{\mu 3}, K^0_{\mu 3}$). In writing the above equation, we have chosen to "normalize" the form factors for the various modes to $f_+^{K^0\pi^-}(0)$, evaluated in absence of electromagnetic corrections. Differences between the various modes are due to isospin breaking effects, both of strong ($\delta^i_{SU(2)}$) and electromagnetic ($\delta^i_{\rm EM}$) origin, which have been evaluated at $\mathcal{O}(\epsilon p^2)$ in the chiral expansion [11]:

$$\tilde{f}^n_+(0) \;\; = \;\; f_+^{K^0\pi^-}(0)\;(1+\delta^n_{SU(2)}+\delta^n_{\rm EM})\;. \qquad (16)$$

Let us now discuss the theoretical input needed in Eq.15. In the Table 1 we report estimates of the isospin-breaking parameters [11]. [1] The uncertainty on $\delta^n_{\rm EM}$ and $\delta^n_{SU(2)}$ is due to incomplete knowledge of the relevant NLO ChPT couplings. This is not the dominant uncertainty in the final analysis. The phase space corrections refer to the definition of photon-inclusive width given by Ginsberg [12,11]. Although our calculation of phase space corrections for muonic modes is in progress, in order to include these modes in the phenomenological analysis, we can use the estimates $\Delta I_n, n = K^+_{\mu 3}, K^0_{\mu 3}$ obtained by Ginsberg [12]. Due to potentially large model uncertainty (e.g. dependence on UV cutoff) we assign an error bar of $\pm 1\%$ to these entries.

The expansion of $f_+^{K^0\pi^-}(0)$ in the quark masses has been analyzed up to the next-to-next-to-leading order [17]. At this level of accuracy we write

$$f_+^{K^0\pi^-}(0) \;\; = \;\; 1+f_2+f_4+\mathcal{O}(p^6)\;, \qquad (17)$$

[1] These are based on Ref. [11]. Another calculation [16] for the K^+_{e3} mode leads to consistent results, though within a different framework.

Table 2. $K_{\ell 3}$ branching ratios (BR) and slopes from PDG fits. The lifetimes used as input are: $\tau_{K^\pm} = (1.2384 \pm 0.0024) \times 10^{-8}\, s$ and $\tau_{K_L} = (5.17 \pm 0.04) \times 10^{-8}\, s$.

Mode	BR (%)	$\lambda_+ \times 10^3$	$\lambda_0 \times 10^3$
K^+_{e3}	4.87 ± 0.06	27.8 ± 1.9	
K^0_{e3}	38.79 ± 0.27	29.1 ± 1.8	
$K^+_{\mu 3}$	3.27 ± 0.06	33 ± 10	4 ± 9
$K^0_{\mu 3}$	27.18 ± 0.25	33 ± 5	27 ± 6

where the identity $f_0 = 1$ follows from current conservation in the chiral limit. Because of the Ademollo-Gatto theorem [2], local terms are not allowed to contribute to f_2, and this implies a determination which is practically free of uncertainties:

$$f_2 = -0.023\;\;. \qquad (18)$$

As for f_4 the situation is much less clear, because low energy constants (LECs) of the p^6 Lagrangian can now contribute. Various estimates of the size of f_4 have been given, and for a mini-review we refer to Ref. [1]. In our numerical analysis we use the result of Leutwyler and Roos [17], based on a model-independent parameterization of the asymmetry between kaon and pion wave functions:

$$f_4 = -0.016 \pm 0.008\;\;. \qquad (19)$$

We take this result with the understanding that, e.g., a value of f_4 two sigmas away from the central value in Eq.19 is not strictly forbidden, but rather unlikely.

From the point of view of the pure chiral expansion, the only parameter-free prediction which one can make for f_4 concerns the chiral logs. A first step in this direction was made by Bijnens, Ecker and Colangelo [18], who calculated the double chiral logs contribution to this quantity. The size of this term, however, depends on the renormalization scale μ. By varying the latter within a reasonable range, the numerical estimate $|f_4|_{\rm chiral\ logs} \leq 0.5\%$ was obtained.

Recently two groups [19,20] have performed a full calculation of f_4 to $O(p^6)$, which besides the double chiral logs contains single ones and polynomial contributions. The latter contain LECs of the p^6 Lagrangian, whose value is basically unknown, and make a numerical estimate difficult. However, a key observation [20] is that the relevant local couplings turn out to govern the slope and curvature of the scalar form factor $f_0^{K\pi}(t)$ at $t = 0$. Therefore, at least in principle, such couplings can be related to observables, through a measurement of the the t dependence of $f_0^{K\pi}(t)$ in $K_{\mu 3}$ decays or through dispersive analyses of the scalar

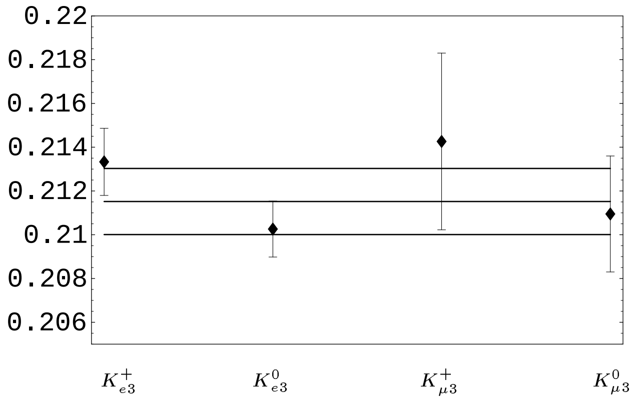

Figure 1. $|V_{us}| \cdot f_+^{K^0\pi^-}(0)$ from the four $K_{\ell 3}$ modes (and average over the electronic modes).

form factor. Finally, let us note that lattice calculations of $f_+(0)$ are certainly called for in order to improve our confidence in the range used for f_4.

Using the experimental input reported in Table 2, an average over the electronic modes (the muonic modes are irrelevant in the average, given the present uncertainties) leads to:

$$|V_{us}| \cdot f_+^{K^0\pi^-}(0) = 0.2115 \pm 0.0015 \ . \tag{20}$$

The error has been multiplied by a scale factor $S = 1.54$, as defined by the PDG [3], in order to take into account the slight inconsistency between K^+_{e3} and K^0_{e3} results. Finally, if we use the Leutwyler-Roos estimate of f_4, or $f_+^{K^0\pi^-}(0) = 0.961 \pm 0.008$, we obtain

$$\begin{aligned} |V_{us}| &= 0.2201 \pm 0.0016_{\rm exp} \pm 0.0018_{{\rm th}(f_4)} \\ &= 0.2201 \pm 0.0024 \ . \end{aligned} \tag{21}$$

5 V_{ud}, V_{us}, and CKM unitarity

Given the tiny size of V_{ub}, the CKM unitarity constraint of Eq.1 provides a relation between V_{ud} and V_{us}. Errors on the direct determinations of V_{ud} and V_{us} are at the level of 0.1% and 1%. Explicitly [1] one has:

$$\begin{aligned} |V_{ud}|_{\rm direct} &= 0.9739 \pm 0.0005 \qquad (n\text{-}\beta \text{ and SFT}) \ ; \\ |V_{us}|_{\rm direct} &= 0.2201 \pm 0.0024 \qquad (K_{e3}) \ . \end{aligned} \tag{22}$$

CKM unitarity implies an independent determination of V_{us} (from V_{ud}) at the 1% level,

$$|V_{us}|_{\rm unit.} = 0.2269 \pm 0.0021 \tag{23}$$

to be compared with the direct determination in Eq.22. The $\sim 2\sigma$ discrepancy between these two determinations could be attributed to an underestimate of i) theoretical uncertainties involved in $|V_{ud}|_{\rm direct}$; ii) theoretical uncertainty in f_4 ; iii) experimental errors in $K_{\ell 3}$ B.R.s. (as probably hinted by the marginal consistency of $f_+(0)|V_{us}|$ from K^+_{e3} and K^0_{e3}). At the moment, in absence of a clear indication of which of the errors is underestimated, it's probably best to treat the two determinations on the same footing, and introduce the PDG scale factor in the final error. Following this procedure one finds

$$|V_{us}|_{{\rm unit.}+K_{\ell 3}} = 0.2240 \pm 0.0034 \ . \tag{24}$$

6 Final remarks

The pion beta decay (π_{e3}) provides in principle a unique test on the existing extractions of V_{ud} from nuclear SFT and neutron beta decay. Theory being extremely clean, one has to wait for the final results from PIBETA [4] to address the impact of this channel on the determination of V_{ud}.

As for $K_{\ell 3}$ decays, our theoretical understanding of radiative corrections is under control (implying an error on V_{us} of about 0.2 %), while improvement is needed in the calculation of $SU(3)$ breaking effects. The experimental situation will soon improve thanks to the high-statistics measurements of $K_{\ell 3}$ widths expected from recently-completed or ongoing experiments, such as BNL-E865 [5], KLOE [6] and NA48 [7]. It should be noted how a single present-day measurement allows us to extract $|V_{us}|$ at the $\sim 1\%$ level [same as in Eq.21]. For example, using the final result from BNL-E865 [5], $\mathrm{BR}(K^+_{e3[\gamma]}) = (5.13 \pm 0.02_{\rm stat.} \pm 0.09_{\rm syst.} \pm 0.04_{\rm norm.})\%$, and the appropriate phase space correction ($\Delta I_{K^+_{e3}} = (-1.27 + 0.5)\%$), one obtains

$$|V_{us}|_{E865} = 0.2272 \pm 0.0030 \ . \tag{25}$$

Although this result is in perfect agreement with what is expected from unitarity, we stress that when combining BNL-E865 with other existing results one finds: 1) internal inconsistencies within K^+_{e3} measurements; 2) the discrepancy with K^0_{e3} is worsened; 3) in a combined analysis, the problem with unitarity persists at the $\sim 2\sigma$ level. In our opinion, a meaningful improvement will be possible once the $K_{\ell 3}$ database is fully updated, including not only BNL-E865 but also the results from KLOE and NA48.

Acknowledgments

I would like to thank the organizers for financial support. This work has been supported in part by

MCYT, Spain (Grant No. FPA-2001-3031), by ERDF funds from the European Commission, and by the EU RTN Network EURIDICE, Grant No. HPRN-CT2002-00311.

References

1. M. Battaglia, A.J. Buras, P. Gambino and A. Stocchi (Eds), "The CKM matrix and the unitarity triangle", arXiv:hep-ph/0304132.
2. M. Ademollo and R. Gatto, Phys. Rev. Lett. **13** (1964) 264.
3. K. Hagiwara *et al.*, Particle Data Group Phys. Rev. D **66** (2002) 010001.
4. D. Počanić (PIBETA Coll.), talk given at the CKM Workshop (Durham, April 5-9 2003).
5. A. Sher et al. (E865 Coll.), arXiv:hep-ex/0305042.
6. B. Sciascia (KLOE Coll.), talk given at the CKM Workshop (Durham, April 5-9 2003).
7. D. Madigozhin (NA48 Coll.), talk given at the CKM Workshop (Durham, April 5-9 2003).
8. J. Erler, arXiv:hep-ph/0211345 ; A. Sirlin, Nucl. Phys. B **196** (1982) 83 ; W.J. Marciano, A. Sirlin, Phys. Rev. Lett. **71** (1993) 3629, and references therein.
9. J. Gasser and H. Leutwyler, Nucl. Phys. **B 250** (1985) 465 ; Ann. Phys. **158** (1984) 142.
10. M. Knecht et al., Eur. Phys. J. C **12** (2000) 469, and references therein.
11. V. Cirigliano et al., Eur. Phys. J. C **23** (2002) 121 [arXiv:hep-ph/0110153]; V. Cirigliano, H. Neufeld, H. Pichl, in progress.
12. E. S. Ginsberg, Phys. Rev. **162** (1967) 1570 [Erratum, ibid. **187** (1969) 2280] ; Phys. Rev. **171** (1968) 1675 [Erratum, ibid. **174** (1968) 2169] ; Phys. Rev. D **1** (1970) 229.
13. V. Cirigliano et al., Eur. Phys. J. C **27** (2003) 255 [arXiv:hep-ph/0209226].
14. W. Jaus, Phys. Rev. D **63** (2001) 053009 ; A. Sirlin, Rev. Mod. Phys. **50** (1978) 579.
15. G. Calderon and G. Lopez Castro, Phys. Rev. D **65** (2002) 073032 [arXiv:hep-ph/0111272].
16. V. Bytev, E. Kuraev, A. Baratt and J. Thompson, arXiv:hep-ph/0210049.
17. H. Leutwyler and M. Roos, Z. Phys. C **25** (1984) 91.
18. J. Bijnens et al., Phys. Lett. B **441** (1998) 437 [arXiv:hep-ph/9808421].
19. P. Post and K. Schilcher, arXiv:hep-ph/0112352.
20. J. Bijnens and P. Talavera, arXiv:hep-ph/0303103.