String Parton Models in Geant4

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Dual parton or quark gluon string model are the by now almost standard theoretical techniques by which one can arrive at precision description of high energy, soft, inclusive reactions. These reactions make the part of jets at energies that contribute strongly to discovery channels such as H→WWjj, or search for compositeness at the highest transverse momenta. The above modeling approach is available with Geant4 for nucleon induced reactions since the first release. Its object oriented design and parameter set was recently extended to allow for simulation of pion and kaon induced reactions, as well as heavy ion reactions. We will briefly describe the theory and algorithmic approaches that underly the modeling, show the object oriented designs and component structure of the string parton sub-systems of Geant4, present validation/verification results pertaining to these models, as well as results concerning their usage in calorimeter simulation.

1. Overview

The string parton models in Geant4 [1] serve to simulate inelastic reactions of high energy particles with nuclei. The Geant4 string parton models are modular. To simulate the interaction of high energy particle with the nucleus several building parts are used together, and for some of the parts there is more than one choice.

In a first stage the interaction of a high energy particles with at least one nucleon of the nucleus is modeled using a string excitation model. At the moment Geant4 provides two different string excitation models, the diffractive string excitation and the quark gluon string model. In the initial state, a nucleus is built consisting of individual protons and neutrons, the nucleons. The result of an interaction between the primary and the nucleus are one or several excited strings and a nucleus in an excited state. A string consists of two endpoints with defined quark content and carries energy and momentum. The fragmentation of the excited strings into hadrons is handled by a longitudinal string fragmentation model. The interaction of secondaries with the excited nucleus will be handled by a cascade model. Until this implementation will be completed, secondaries are assumed to be produced outside of the nucleus. The de-excitation of the excited nucleus is further simulated by nuclear fragmentation, precompound, and nuclear de-excitation models.

2. Object Oriented Design Overview

The string parton model is part of the Geant4 simulation toolkit. The design of the string parton model was made with a set goals. The use of several models or implementations is made possible through the use of common interfaces, allowing also for easy integration of more models. Common parts between multiple implementations or between various models are shared in common classes. As an example the model of the nucleus is shared between many of the theory driven models for hadronic reactions in Geant4.

An overview of the design of the parton string models of Geant4 using UML notation in shown in figure 1. The interface of the parton string models to the Geant4 toolkit is defined and partly implemented in the abstract *G4VPartonStringModel* class. The classes implementing the diffractive string excitation *G4FTFModel* and the quark gluon string model *G4QGSModel* are concrete implementations of the abstract *G4VPartonStringModel* class. The interface to string fragmentation again is defined in an abstract class, *G4VStringFragmentation*. The model for the longitudinal string fragmentation is implemented by the *G4VLongitudinalStringDecay* class. The latter is abstract, as the fragmentation function is not implemented. This shares the algorithm between concrete implementations and allows specific string models to use specific fragmentation functions. The concrete classes *G4LundStringfragmentation* and *G4QGSMFragmentation* are used, respectively, by the diffractive parton string and by the quark gluon string models. Other string fragmentations schemes are possible, as e.g. indicated by the example of *G4PhythiaFragmentationInterface*. This is foreseen as an interface to the string fragmentation of Phythia7 [2]

3. Modeling the Nucleus

The nucleus is modeled as an ensemble of protons and neutrons. Each nucleon is positioned randomly in configuration and momentum space. The positions are chosen at random following to the nuclear density distribution. For heavy nuclei, ie. nuclei with a mass number above 16, we use a density distribution of the Wood-Saxon form:

$$\rho(r_i) = \frac{\rho_0}{1+\exp(\frac{(r_i-R)}{a})} \qquad (1)$$

where R and a depend on the mass number of the nucleus.

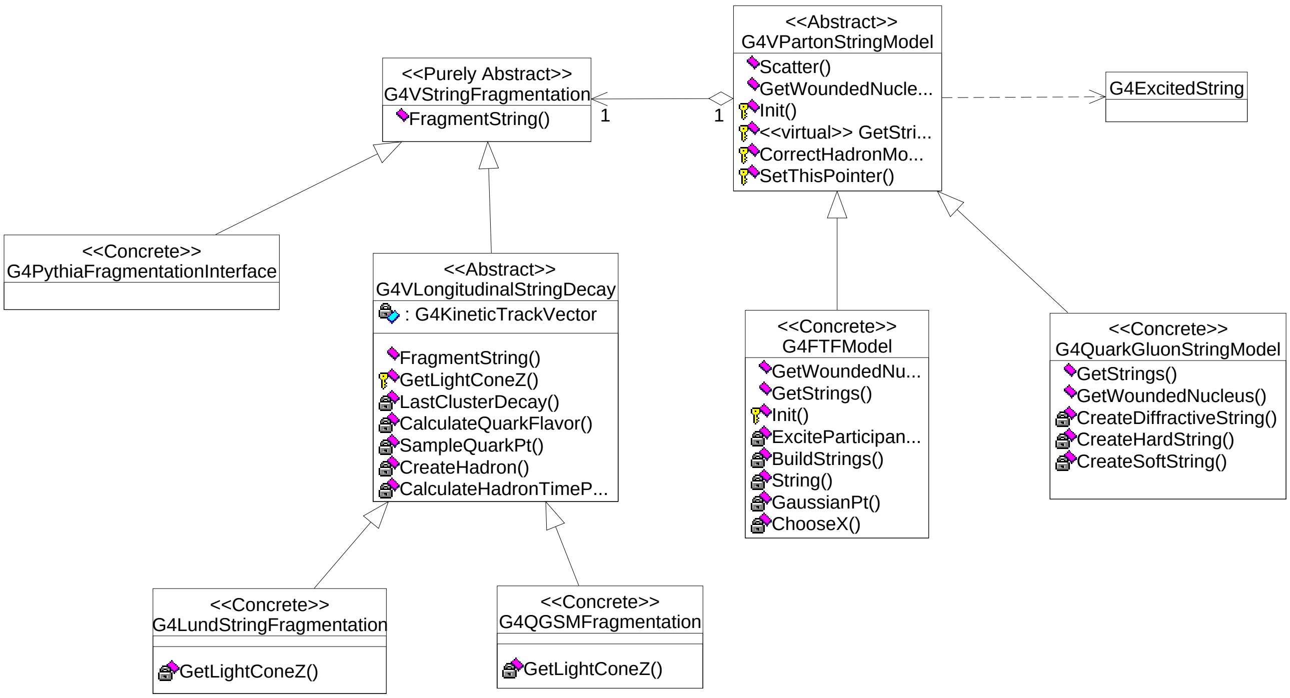

Figure 1: Overview of design for parton string model classes.

For light nuclei we use a density distribution from the harmonic oscillator model:

$$\rho(r_i) = (\pi R'^2)^{-3/2} \exp(-r_i^2/R'^2), \qquad (2)$$

where R' is the effective nuclear radius, and depends on the mass number of the nucleus.

The sampling in configuration space is done such that no two nucleons have a distance from each other less than $0.8fm$.

The momentum of each nucleon is chosen random in momentum space with a maximum momentum p_F^{max}

$$p_F^{max} = \hbar c (3\pi^2 \rho(r_i))^{1/3} \qquad (3)$$

which is a function of the density $\rho(r_i)$ obtained from equation 1 or 2. Momentum balance is achieved by choosing the direction of momentum for the nucleons such that the sum of all nucleon momenta is zero.

For the purpose of further calculations, this nucleus is then collapsed into two dimensions perpendicular to the direction of the primary particle. This way we take into account that at high energies the coherence length of the string fragmentation is large in comparison to the thickness of the (relativistically contracted) nucleus. All scattering is hence assumed to happen independent of any time ordering, and to be correlated only through energy and baryon-number conservation.

4. Diffractive scattering model

The diffractive scattering model simulates the interaction of an high energetic hadron with a nucleus, where the incident particle may interact with one or several nucleons in the nucleons. For each nucleon the impact parameter is calculated, and using the impact parameter and the interaction center of mass energy, the interaction probability is calculated from the inelastic and diffractive cross section respectively using the eiconal model. The interacting nucleons are selected using uniform sampling of the interaction probability.

The diffractive scattering of the primary particles with a nucleon is modeled using an approach similar to the one employed in Fritiof[3]. In this approach the scattering particles only exchange momentum:

$$\begin{aligned} p_1' &= p_1 + q \\ p_2' &= p_2 - q \end{aligned} \qquad (4)$$

where $p_{1,2}$ are the momenta of the incoming, and $p_{1,2}'$ the momenta of the scattered particles, and q is the momentum exchanged. A string is formed for each of the two scattered particles, using the quark content of the original hadron by assigning the quarks of the hadron randomly to the two string ends.

In the center of mass system and using light-cone coordinates, the momenta of the incoming particles are

$$\begin{aligned} p_1 &= (E_1^+, m_1^2/E_1^+, \mathbf{0}) \\ p_2 &= (E_2^-, m_2^2/E_1^-, \mathbf{0}) \end{aligned} \qquad (5)$$

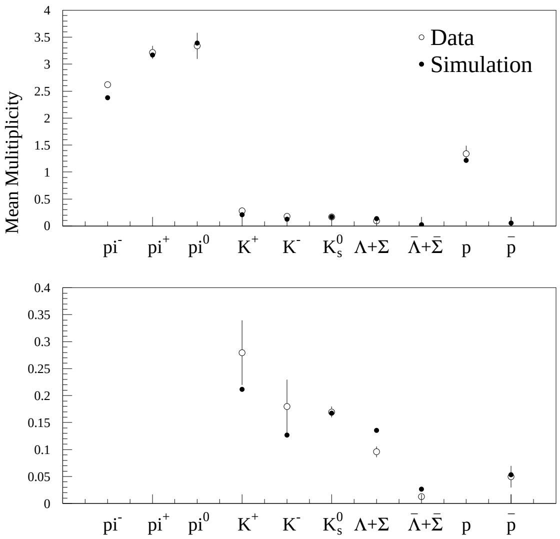

Figure 2: Mean multiplicity for reactions $p\ H \rightarrow\ X$ at $200Gev/c$. Open circles are data, and full circles is Monte Carlo simulation using the diffractive string model.

and the momentum transfer is

$$q = (-q_t^2/x^- E_2^-, q_t^2/x^+ E_2^+, \mathbf{q_t}) \tag{6}$$

The model does not naturally contain transverse momentum, hence the transverse momentum $\mathbf{q_t}$ is sampled from a gaussian distribution with a default width of 0.8 GeV, using a simple multiple small-angle scattering assumption.

The longitudinal components q^+ and q^- of the momentum exchange are obtained sampling x^+ and x^- from the parton distribution:

$$u(x) = x^\alpha (1-x)^\beta \tag{7}$$

where for the diffractive string model the parameters are $\alpha = -1$, and $\beta = 0$.

The masses of the resulting strings must fulfill the kinematic constraint

$$p'^+_{1,2}\, p'^-_{1,2} \ \geq\ p^2_{1,2} + q_t^2 \tag{8}$$

where $p^2_{1,2}$ are the masses of the incident particle and the nucleon.

As an illustration of result obtained from this model, we plot in figure 2 the mean multiplicity for several particles types observed in the final state in reactions $p\ H \rightarrow X$ at a momentum of $200GeV/c$ in laboratory frame for the incoming proton.

5. Quark Gluon String Model

The Quark Gluon string model, too, allows to simulate reactions of high energy hadrons with nuclei and also to simulate high energy electro- and gamma-nuclear reactions. Unlike the diffractive models, in this case, the colour flow is assumed to be between partons from the interaction partners.

The nucleus is modeled as above. The impact parameter for each nucleon b_i is calculated, collapsing the nucleus into a plane orthogonal to the incident primary particle. The hadron nucleon collision probabilities are calculated using the cross-sections of the eiconal model and using gaussian distributions for the wave-functions of both hadrons and nucleons [4]. They are used to determine the number of participating nucleons in the nucleus. In the quark-gluon string model, each hadron-nucleon interaction is assumed to be mediated by the exchange of one or more Pomerons. Hence for each pair of participants the number of Pomerons n is sampled. This is possible as in the Regge Gribov approach the reaction probability can be factorized, and the contribution of any pair of participants can be written as a sum over the number of Pomerons exchanged:

$$P_i(b_i, s) = \frac{1}{c}(1 - \exp\left[-2u(b_i, s)\right]) = \sum_{n=1}^{\infty} P_i^{(n)}(b_i, s) \tag{9}$$

The individual contribution of the N Pomeron graph here reads as

$$P_i^{(n)}(b_i, s) = \frac{1}{c}\exp\left[-2u(b_i^2, s)\right]\frac{(2u(b_i^2, s))^n}{n!} \tag{10}$$

where the Eikonal can be written as

$$u(b_i^2, s) = \frac{z(s)}{2}\exp(\frac{b_i^2}{4\lambda(s)}). \tag{11}$$

Here s is the c.m.s. energy; c, $z(s)$, and $\lambda(s)$ are functions of the eiconal model cross section description, that can be expressed in terms of the Pomeron vertex and trajectory parameters.

In this model a small fraction of interactions is diffractive, its probability is split off using Baker's shower enhancement factor c [5]:

$$P_i^{diff}(b_i, s) = \frac{1-c}{c}(P_i^{tot}(b_i, s) - P_i(b_i, s)) \tag{12}$$

Strings are formed using the parton exchange mechanism by sampling of parton densities and ordering pairs of partons into color coupled entities [6]. Each Pomeron is treated as a pair of colour triplet strings, where the string ends are attached to partons in the interacting hadrons. Strings are then decayed as described later in this paper. The relative contributions from valence and sea quarks are split, so that the fragmentation functions will look like

$$\begin{aligned}\phi_n^h = a^h[&F_v^h(x_+, n)F^h_{anti-v}(x_-, n)\\ &+F_v^h(x_-, n)F^h_{anti-v}(x_+, n)\\ +(n-1)(&F_s^h(x_+, n)F^h_{anti-s}(x_-, n)\\ &+F_s^h(x_-, n)F^h_{anti-s}(x_+, n))]\end{aligned}$$

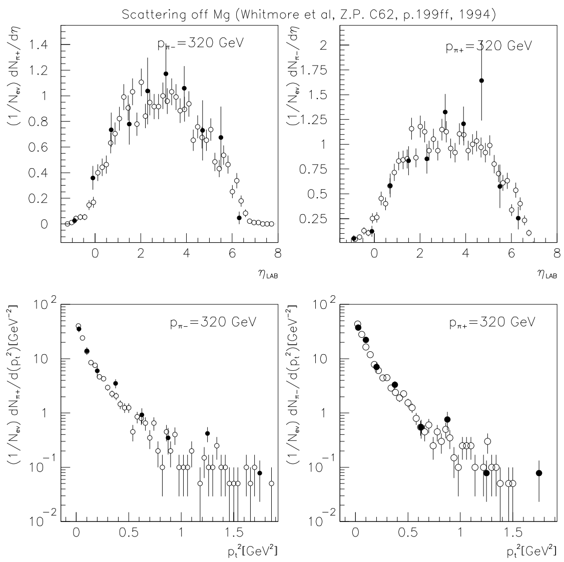

Figure 3: Comparison of data and Monte Carlo prediction for the quark gluon string model. We show rapidity and transverse momentum square distributions of π^+ produced in pion Magnesium reactions at 320 GeV. Open circles are the Monte Carlo predictions, and points are experimental data.

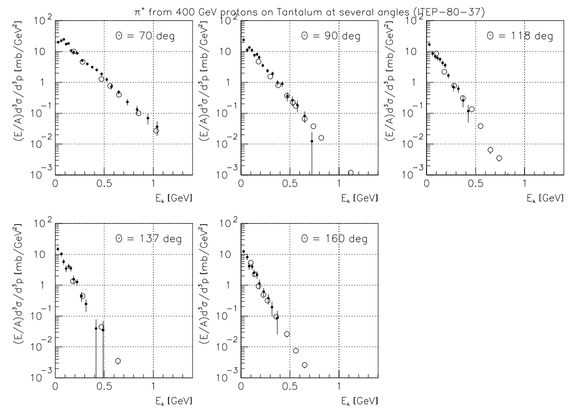

Figure 4: Comparison of data and Monte Carlo prediction for the quark gluon string model. We show invariant cross-sections of π^+ produced in pion Magnesium reactions at 400 GeV as a function of the pion kinetic energy. Each plot describes a different scattering angle. Open circles are experimental data, and points are Monte Carlo prediction.

where v and s stand for valence and see respectively, and the functions F are the parton density functions folded with the fragmentation functions and the transverse momentum function:

$$F^h(x_\pm, n) = \sum_i \int_{x_\pm}^1 f_i(x', n) G_i^h(x_\pm/x') T(p_T, n) dx' \tag{13}$$

Examples of the predictive power of this model are given in Fig.3 and 4.

5.1. Electro- and gamma-nuclear interactions

The quark gluon string model is also used to simulate electro- and gamma nuclear reactions. This is done using a single interaction assumption and vector meson dominance. From there, the quark-gluon string model can be applied as for any other hadron, once the Pomeron vertex and slope parameters are properly adjusted. For electro-nuclear reactions, in addition, it is of course necessary to assume an equivalent photon flux mediating the reactions, and to take into account the virtuality of the photons. The cross section for a high energy gamma ray to interact with a nucleus, and all aspects of the equivalent photon flux hypothesis and momentum transfer calculations is presented in [7].

6. String fragmentation

The string as created by the diffractive or the quark gluon string model is characterized by its four momentum and its constituents, i.e. the quark contents at the two endpoints of the string. The algorithm for hadronisation of the string is common for both string models, except for the fragmentation function used. The string repeatedly is split into a hadron and a new string, until the energy in the strings gets too low for further splitting.

In the current implementation, a constituent can be a up, down, or strange quark or antiquark, or a diquark or anti-diquark of up, down, or strange quarks. The strings must have integer charge, so only the following combinations of constituents plus the charge conjugated combinations are allowed: $q - \overline{q}$, $q - (qq)$, $(qq) - \overline{(qq)}$. In the longitudinal fragmentation model the constituents move in opposite direction increasing the tension on the string. The string then breaks creating a new quark - antiquark $q - \overline{q}$ or diquark - anti-diquark $(qq) - \overline{(qq)}$ pair. The different quark flavours are created with a relative probability of:

$$u \ : \ d \ : \ s \ = \ 1 \ : \ 1 \ : \ 0.27 \tag{14}$$

Diquark - anti-diquark pairs are produced in 10% of all cases.

Half of the newly created pair forms a hadron with one of the constituents, and the other half of the newly created pair together with the remaining constituent forms a new string.

The quark content gives the charge of the hadron and its type. For mesons we create scalar and vector mesons taking into account the mixings of neutral mesons. For barions we construct barions from the lowest SU(3) octet (spin 1/2 barions) and from the lowest SU(3) decuplet (spin 3/2 barions).

The quark or diquark get transverse momentum sampled from a gaussian distribution using a width of $\sigma = 0.5GeV$. The antiquark or anti-diquark gets the opposite transverse momentum to conserve total transverse momentum.

The longitudinal momentum is split off the string longitudinal momentum using a fraction z sampled from a fragmentation function. For the diffractive string model we use the Lund fragmentation function

$$f(z, m_H, q_t) = \frac{1-z}{z}\exp(\frac{-b(m_H^2 + q_t^2)}{z}) \qquad (15)$$

where m_H is the mass of the created hadron, and q_t is the transverse momentum of the hadron. For the quark gluon string model, we use [8]

$$f^h(z, q_t) = [1 + \alpha^h(< q_t >)](1-z)^{\alpha^h(<q_t>)} \qquad (16)$$

where the parameter α depends on the type of newly created quark or diquark, and the average transverse momentum. Finally, the transverse momentum of the hadron is the sum of transverse momenta of the string constituent and of the newly created quark.

This process is iterated until the energy of the string gets too low to form further hadrons.

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