Confinement and domain walls in high density quark matter

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In this talk I will review some recent progress in our understanding of properties of high density quark matter. This talk is based mainly on the papers [1]–[4], done in collaboration with D. Rischke, M. Stephanov, and A. Zhitnitsky.

In recent years, our knowledge of dense quark matter has considerably expanded. We now understand that, quark matter at high densities exhibits the phenomenon of color superconductivity, which determines the symmetry of the ground state and its infrared dynamics. In the simplest case of $N_f = 2$ massless quarks, the ground state of at high baryon densities is the 2SC state [5], characterized by the condensation of diquark Cooper pairs. These pairs are antisymmetric in spin (α, β), flavor (i, j) and color (a, b):

$$\begin{aligned} \langle q^{ia}_{L\alpha} q^{jb}_{L\beta} \rangle^* &= \epsilon_{\alpha\beta} \epsilon^{ij} \epsilon^{abc} X^c \,, \\ \langle q^{ia}_{R\alpha} q^{jb}_{R\beta} \rangle^* &= \epsilon_{\alpha\beta} \epsilon^{ij} \epsilon^{abc} Y^c \,. \end{aligned} \tag{1}$$

X^c and Y^c are complex color 3-vectors. In the ground state, they align along the same direction in the color space and break the color $\mathrm{SU}(3)_c$ group down to $\mathrm{SU}(2)_c$. Thus, five of the original eight gluons acquire "masses" by the Meissner effect, similar to the Higgs mechanism. The remaining three gluons are massless (perturbatively). Because of Cooper pairing, the spectrum of quark excitations carrying SU(2) color charge has a gap Δ.

The physics below the energy scale Δ is governed by the pure gluodynamics in the remaining unbroken SU(2) sector. As shown in Ref. [1], the process of high-density "deconfinement" is quite nontrivial in this case: the quarks are *always* confined, however, the confinement radius grows *exponentially* with increasing density.

Below the scale Δ, heavy (gapped) degrees of freedom decouple and the remaining fields can be described by a local effective Lagrangian. The absence of quarks carrying SU(2) charges below Δ (all are bound into SU(2) singlet Cooper pairs) implies that the medium is transparent to the SU(2) gluons: there is no Debye screening and Meissner effect for these gluons. Mathematically, the gluon polarization tensor $\Pi^{\mu\nu}_{ab}(q)$ vanishes at $q = 0$, which can be checked by a direct calculation of Π at small q [6]. However, although a static SU(2) charge cannot be completely Debye screened by SU(2) neutral Cooper pairs, it can still be *partially* screened if the medium is polarizable, i.e., if it has a *dielectric constant* ϵ different from unity. Analogously, the medium can, in principle, have a magnetic permeability $\lambda \neq 1$.

The requirements of locality, gauge and rotational invariance, and parity fixes the effective action below the scale Δ in the following form

$$S_{\text{eff}} = \frac{1}{g^2} \int d^4x \left(\frac{\epsilon}{2} \mathbf{E}^a \cdot \mathbf{E}^a - \frac{1}{2\lambda} \mathbf{B}^a \cdot \mathbf{B}^a \right), \tag{2}$$

where $E_i^a \equiv F_{0i}^a$ and $B_i^a \equiv \frac{1}{2}\epsilon_{ijk} F_{jk}^a$. Explicit calculations [1] gives

$$\epsilon = 1 + \kappa = 1 + \frac{g^2 \mu^2}{18\pi^2 \Delta^2}, \tag{3}$$

$$\lambda = 1. \tag{4}$$

Since at high densities, the gap Δ is exponentially suppressed compared to the chemical potential μ [7], $\epsilon \gg 1$, i.e., the dielectric constant of the medium is very large. This can be interpreted as a consequence of the fact that the Cooper pairs have large size (of order $1/\Delta$) and so are easy to polarize. The magnetic permeability, in contrast, remains close to 1 due to the absence of mechanisms that would strongly screen the magnetic field.

The effective strong coupling constant α_s' in the theory (2), the equivalence of $\alpha_s = g^2/(4\pi\hbar c)$ in the vacuum, is very small at the matching scale with the microscopic theory (i.e., Δ). In our dielectric medium, the Coulomb potential between two static charges separated by r is $g^2/(\epsilon r)$. Thus, we have to replace g^2 by $g_{\text{eff}}^2 = g^2/\epsilon$. The velocity of light c also needs to be replaced by the velocity of gluons $v = 1/\sqrt{\epsilon}$. This gives

$$\alpha_s' = \frac{g_{\text{eff}}^2}{4\pi\hbar v} = \frac{g^2}{4\pi\sqrt{\epsilon}} = \frac{3}{2\sqrt{2}} \frac{g\Delta}{\mu}. \tag{5}$$

The coupling increases logarithmically as one moves to lower energies, since pure SU(2) Yang-Mills theory is asymptotically free. This coupling becomes large at the confinement scale Λ_{QCD}', which is the mass scale of SU(2) glueballs. Since α_s' is tiny (because $\Delta/\mu \ll 1$), it takes long to grow, and the scale Λ_{QCD}' is thus very small. Using the one-loop beta function, one can estimate

$$\Lambda_{\text{QCD}}' \sim \Delta \exp\left(-\frac{2\pi}{\beta_0 \alpha_s'} \right) \sim \Delta \exp\left(-\frac{2\sqrt{2}\pi}{11} \frac{\mu}{g\Delta} \right), \tag{6}$$

where $\beta_0 = 22/3$ in SU(2) gluodynamics. The possible relevance of the light glueballs for astrophysics was discussed by Ouyed and Sannino [8].

In Ref. [2] we show that at high baryon densities QCD must have domain walls. The simplest case allowing for domain walls is again $N_f = 2$ massless flavors. In perturbation theory, there is a degeneracy of the ground state with respect to the relative U(1) phase between X^a and Y^a in Eq. (1). This is due to the $\mathrm{U(1)_A}$ symmetry of the QCD Lagrangian at the classical level. This fact implies that the $\mathrm{U(1)_A}$ symmetry is spontaneously broken by the color-superconducting condensate. Since this is a global symmetry, its breaking gives rise to a Goldstone boson, which carries the same quantum numbers as the η boson in vacuum.

The field corresponding to η boson can be constructed explicitly. Indeed

$$\Sigma = XY^\dagger \equiv X^a Y^{a*}\,, \tag{7}$$

in contrast to X and Y, is a gauge-invariant order parameter. Furthermore Σ carries a nonzero $\mathrm{U(1)_A}$ charge. Under the $\mathrm{U(1)_A}$ rotations

$$q \to e^{i\gamma_5\alpha/2} q\,, \tag{8}$$

the fields (1) transform as $X \to e^{-i\alpha}X$, $Y \to e^{i\alpha}Y$, and therefore $\Sigma \to e^{-2i\alpha}\Sigma$. Thus, the color-superconducting ground state, in which $\langle\Sigma\rangle \neq 0$, breaks the $\mathrm{U(1)_A}$ symmetry. The Goldstone mode η of this symmetry breaking is described by the phase φ of Σ,

$$\Sigma = |\Sigma| e^{-i\varphi}\,. \tag{9}$$

Under the $\mathrm{U(1)_A}$ rotation (8), φ transforms as

$$\varphi \to \varphi + 2\alpha\,. \tag{10}$$

At low energies, the dynamics of the Goldstone mode φ is described by an effective Lagrangian, which must take the following form,

$$L = f^2[(\partial_0\varphi)^2 - u^2(\partial_i\varphi)^2]\,. \tag{11}$$

Two free parameters of this Lagrangian are the decay constant f of the η boson, and its velocity u. In general, u may be different from 1 since the Lorentz invariance is violated by the dense medium. For large chemical potentials $\mu \gg \Lambda_{\mathrm{QCD}}$, the leading perturbative values for f and u have been determined by Beane *et al.* [9]:

$$f^2 = \frac{\mu^2}{8\pi^2}\,, \qquad u^2 = \frac{1}{3}\,. \tag{12}$$

In particular, the velocity of the η bosons, to this order, is equal to the speed of sound. The fact that $f \sim \mu$ plays an important role in our further discussion.

It is well known that the $\mathrm{U(1)_A}$ symmetry is not a true symmetry of the quantum theory, even when quarks are massless. The violation of the $\mathrm{U(1)_A}$ symmetry is due to non-perturbative effects of instantons. Since at large chemical potentials the instanton density is suppressed (see below), the η boson still exists but acquires a finite mass. In other words, the anomaly adds a potential energy term $V_{\rm inst}(\varphi)$ to the Lagrangian (11),

$$L = f^2[(\partial_0\varphi)^2 - u^2(\partial_i\varphi)^2] - V_{\rm inst}(\varphi)\,. \tag{13}$$

The curvature of $V_{\rm inst}$ around $\varphi = 0$ determines the mass of the η.

A standard symmetry argument determines periodicity of $V_{\rm inst}(\varphi)$. One can formally restore the $\mathrm{U}(1)_{\mathrm{A}}$ symmetry by accompanying (8) by a rotation of the θ-parameter

$$\theta \to \theta + N_f\alpha = \theta + 2\alpha\,. \tag{14}$$

This symmetry must be preserved in the effective Lagrangian, so the latter is invariant under (10) and (14). This means that the potential $V_{\rm inst}$ is a function of the variable $\varphi - \theta$, unchanged under $\mathrm{U}(1)_{\mathrm{A}}$. Since we know that the physics is periodic in θ with period 2π, we can conclude that, at the physical value of the theta angle $\theta = 0$, $V_{\rm inst}$ is a periodic function of φ with period 2π.

Moreover, at large μ, $V_{\rm inst}$ can be found from instanton calculations explicitly. The infrared problem that plagues these calculations in vacuum disappears at large μ: large instantons are suppressed due to Debye screening. As a result, most instantons have small size $\rho \sim \mathcal{O}(\mu^{-1})$ and the dilute instanton gas approximation becomes reliable. One-instanton contribution, proportional to $\cos(\varphi - \theta)$, dominates in $V_{\rm inst}$. Therefore,

$$V_{\rm inst}(\varphi) = -a\mu^2\Delta^2\cos\varphi\,, \tag{15}$$

where Δ is the BCS gap, and a is a dimensionless function of μ estimated in Ref. [2]. Here we only note that a vanishes in the limit $\mu \to \infty$. This is an important fact, since it implies that the mass of the η boson,

$$m = \sqrt{\frac{a}{2}}\frac{\mu}{f}\Delta = 2\pi\sqrt{a}\Delta\,, \tag{16}$$

becomes much smaller than the gap Δ at large μ. In this case the effective theory (13) is reliable, since meson modes other than η have energy of order Δ, i.e., are much heavier than η and decouple from the dynamics of the latter.

The Lagrangian (13) with the potential (15) is just the sine-Gordon model, in which there exist domain-wall solutions to the classical equations of motion. The profile of the wall parallel to the xy plane is

$$\varphi = 4\arctan e^{mz/u}\,, \tag{17}$$

so the wall interpolates between $\varphi = 0$ at $z = -\infty$ and $\varphi = 2\pi$ at $z = \infty$. The tension of the domain wall is

$$\sigma = 8\sqrt{2a}\,uf\mu\Delta\,. \tag{18}$$

A good analog of this domain wall is the $N = 1$ axion domain wall, which also interpolates between the same vacuum.

Another phase with domain walls is that of kaon condensation. The likelihood of this phase was recently emphasized by Schäfer and other authors [10]. The crucial observation is that kaons have small masses [11] in the color-flavor locked phase (CFL) [12] of high-density QCD. A relatively small strangeness chemical potential is thus sufficient to drive kaon condensation. Moreover, it was argued that the mass of the strange quark also works in favor of kaon condensation. In contrast to the conventional charge kaon condensation in nuclear matter, in the CFL phase it is the neutral kaons which are more likely to condense. This is due to the inverse mass ordering [11] of mesons in the CFL phase, which makes the neutral kaons lighter than the charge kaons, at least at very high densities.

In a recent paper [3] we show that the K^0-condensed phase has in its spectrum an extremely light bosonic particle, whose presence implies the existence of non-topological metastable domain walls. Again, this feature can be understood easily from symmetry arguments. The K^0 condensate spontaneously breaks a global U(1) symmetry: the strangeness. The phase of this condensate becomes a Goldstone boson. However, since strangeness is violated by weak processes, the would-be Goldstone boson acquires a very small mass proportional to $G_F^{1/2}$. At very low energies, the effective Lagrangian for φ must have the form of the sine-Gordon theory, which possesses domain wall solutions interpolating between $\varphi = 0$ and $\varphi = 2\pi$. More recently, vortons were discussed by Kaplan and Reddy [13].

It is interesting to note that if baryon number is violated, the superfluid Goldstone mode also acquires a mass. Since the superfluid order parameter is a dibaryon, the mass square of the superfluid Goldstone boson is proportional to the amplitudes of the $\Delta B = 2$ transitions. The experimental bound on $n\overline{n}$ oscillations, $\tau_{n\leftrightarrow\overline{n}} > 10^8$ s, put an upper limit on the mass of the Goldstone boson, $m < 10^{-7}$ eV. The thickness of the corresponding domain wall is larger than about 1 m, and still might be less than the radius of neutron stars. However, unless the neutron star under consideration rotates very slowly, a domain wall that thick is unlikely to exist because of the high density of vortices, which are separated by distances of order 10^{-2} cm.

Finally, domain walls of the type discussed above also exist in two-component Bose-Einstein condensates (BEC) [4]. Recently, two different hyperfine spin states of ^{87}Rb, which were condensed in the same trap by the technique of sympathetic cooling [14]. A similar state has been observed for sodium gas [15]. Binary BEC breaks spontaneously *two* global U(1) symmetries. It is moreover possible to couple two condensates by a driving electromagnetic field tuned to the transition frequency. In this case atoms can be interconverted between the two spin states and the numbers of atoms of each species are not conserved separately anymore; only the total number of atoms is constant. In this case, only one U(1) symmetry remains exact, the other one is explicitly violated. The violated U(1) group corresponds to the relative phase between the two condensate. The Goldstone boson arising from the spontaneous breaking of this U(1) symmetry acquires a gap and gives rise to the domain walls.

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References

[1] D. H. Rischke, D. T. Son, and M. A. Stephanov, Phys. Rev. Lett. **87**, 062001 (2001) [hep-ph/0011379].

[2] D. T. Son, M. A. Stephanov, and A. R. Zhitnitsky, Phys. Rev. Lett. **86**, 3955 (2001) [hep-ph/0012041].

[3] D. T. Son, hep-ph/0108260.

[4] D. T. Son and M. A. Stephanov, cond-mat/0103451.

[5] M. G. Alford, K. Rajagopal, and F. Wilczek, Phys. Lett. B **422**, 247 (1998) [hep-ph/9711395]; R. Rapp, T. Schafer, E. V. Shuryak, and M. Velkovsky, Phys. Rev. Lett. **81**, 53 (1998) [hep-ph/9711396].

[6] D. H. Rischke, Phys. Rev. D **62**, 034007 (2000) [nucl-th/0001040].

[7] D. T. Son, Phys. Rev. D **59**, 094019 (1999) [hep-ph/9812287].

[8] R. Ouyed and F. Sannino, astro-ph/0103022.

[9] S. R. Beane, P. F. Bedaque, and M. J. Savage, Phys. Lett. **B483**, 131 (2000) [hep-ph/0002209].

[10] T. Schäfer, Phys. Rev. Lett. **85**, 5531 (2000) [nucl-th/0007021]; P. F. Bedaque and T. Schäfer, hep-ph/0105150; D. B. Kaplan and S. Reddy, hep-ph/0107265.

[11] D. T. Son and M. A. Stephanov, Phys. Rev. D **61**, 074012 (2000) [hep-ph/9910491]; *ibid.* **62**, 059902(E) (2000) [hep-ph/0004095].

[12] M. G. Alford, K. Rajagopal, and F. Wilczek, Nucl. Phys. B **537**, 443 (1999) [hep-ph/9804403].

[13] D. B. Kaplan and S. Reddy, hep-ph/0109256.

[14] C. J. Myatt, E. A. Burt, R. W. Ghrist, E. A. Cornell, and C. E. Wieman, Phys. Rev. Lett. **78**, 586 (1997).

[15] J. Stenger, S. Inouye, D. M. Stamper-Kurn, H.-J. Miesner, A. P. Chikkatur, and W. Ketterle, Nature **396**, 345 (1998) [cond-mat/9901072]