The Nuts and Bolts of Diffraction

Konstantin Goulianos*
The Rockefeller University, New York

Results on soft and hard diffraction are briefly reviewed with emphasis on the interplay among factorization properties, universality of rapidity gap formation and unitarity.

1. Hard diffraction: the question

The signature of a diffractive event in $\bar{p}p$ collisions is a leading proton or antiproton and/or a rapidity gap defined as a region of pseudorapidity, $\eta \equiv -\ln\tan\frac{\theta}{2}$, devoid of particles. Hard diffraction is a term used to refer to a diffractive process containing a hard partonic scattering (Fig. 1). In deep inelastic scattering (DIS), diffraction is identified by a leading proton in the final state adjacent to a rapidity gap (Fig. 2). The rapidity gap is presumed to be due to the exchange of a Pomeron, whose generic QCD definition is a color-singlet combination of quarks and/or gluons carrying the quantum numbers of the vacuum.

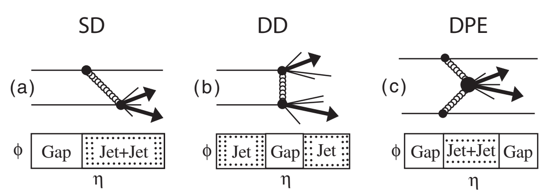

Figure 1: Dijet production in $\bar{p}p$ single (a) and double (b) diffraction, and in double Pomeron exchange (c).

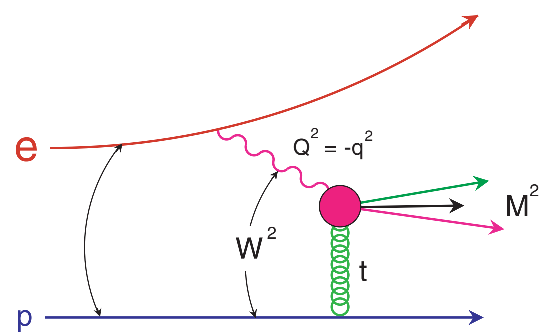

Figure 2: Diagram for diffractive deep inelastic scattering at HERA.

In addition to its dependence on x-Bjorken and Q^2, the *diffractive* structure function (DSF) of the leading nucleon could also depend on the nucleon's fractional momentum loss ξ and its 4-momentum transfer squared t. The central question in diffraction is the validity of QCD factorization, i.e. whether hard diffraction processes can be described in terms of parton level cross sections convoluted with a universal DSF.

2. Data: the answer

The question about QCD factorization in diffraction was addressed "head on" by a comparison [1] between the DSF measured by CDF in dijet production at the Tevatron and the prediction based on parton densities extracted from diffractive DIS at HERA. The DSF at the Tevatron was found to be suppressed relative to the prediction from HERA by a factor of ~ 10. This result confirmed previous CDF results from diffractive W [2], dijet [3] and b-quark [4] production at $\sqrt{s}$=1800 GeV, and was corroborated by more recent CDF results on diffractive J/ψ [5] production at 1800 GeV and dijet production at 630 GeV [6].

* dino@physics.rockefeller.edu

Although factorization breaks down severely between HERA and the Tevatron, it nevertheless holds within the HERA data and within the single-diffractive data at the Tevatron at the same center of mass collision energy. This is demonstrated by the fact that the gluon parton distribution function (PDF) derived from DIS adequately describes diffractive dijet production at HERA [7], while at the Tevatron a consistent gluon PDF is obtained from the measured rates of diffractive W, dijet, b-quark and J/ψ production [5]. However, factorization was found to break down at the Tevatron between the structure functions measured in single-diffraction and in double-Pomeron exchange (DPE) at $\sqrt{s}$ =1800 GeV [8].

3. Soft diffraction: the explanation

The breakdown of QCD factorization observed in hard diffraction is related to the breakdown of Regge factorization responsible for the suppression of soft diffraction cross sections at high energies [9]. This may seem paradoxical, but since the rapidity gap formation is a non-perturbative effect it should not come as a surprise. Thus, "the nuts and bolts of diffraction" are contained in soft diffraction processes.

Soft diffraction has been traditionally treated theoretically in the framework of Regge theory. For large rapidity gaps ($\Delta\eta \geq 3$), the cross sections for single and double (central) diffraction can be written as [10]

$$\begin{aligned}
\frac{d^2\sigma_{SD}}{dt\,d\Delta\eta} &= \left[\frac{\beta^2(t)}{16\pi}e^{2[\alpha(t)-1]\Delta\eta}\right]\left[\kappa\beta^2(0)\left(\frac{s'}{s_\circ}\right)^{\alpha(0)-1}\right] \\
\frac{d^3\sigma_{DD}}{dt\,d\Delta\eta\,d\eta_c} &= \left[\frac{\kappa\beta^2(0)}{16\pi}e^{2[\alpha(t)-1]\Delta\eta}\right]\left[\kappa\beta^2(0)\left(\frac{s'}{s_\circ}\right)^{\alpha(0)-1}\right]
\end{aligned}$$

where $\alpha(t) = 1 + \varepsilon + \alpha' t$ is the Pomeron trajectory, $\beta(t)$ the coupling of the Pomeron to the (anti)proton, and $\kappa \equiv g(t)/\beta(0)$ the ratio of the triple-Pomeron to the Pomeron-proton couplings. The above two equations are remarkably similar. In each case, there are two terms:

- the first term, which is $\sim \{\exp[(\varepsilon + \alpha' t)\Delta\eta]\}^2$ and thus depends on the rapidity gap,
- and the second term, which is $\sim \exp\left[\varepsilon \ln\left(\frac{s'}{s_0}\right)\right]$ and depends on the pseudorapidity interval $\Delta\eta' = \ln\left(\frac{s'}{s_0}\right)$ within which there is particle production.

In the parton model, the second term is interpreted as the sub-energy total cross section, while the first term is the square of the elastic scattering amplitude between the diffractively excited state and the nucleon in SD, or between the two diffractive states in DD. The factor κ in the second term may then be interpreted as being due to the color matching required for a diffractive rapidity gap to occur. Since the sub-energy cross section is properly normalized, the first factor in the equations may be thought of as the rapidity gap probability and *renormalized* to unity. A model based on such a renormalization procedure [9, 11] has yielded predictions in excellent agreement with measured SD and DD cross sections, as seen in Figs. 3 and 4.

The renormalized rapidity gap probability is by definition energy independent and thus represents a scaling behaviour. This procedure has the added advantage of preserving unitarity, which otherwise would be violated. Convoluting the gap probability with partonic level cross sections yields hard diffractive cross sections in general agreement with observations, explaining the factorization properties discussed in the previous section [9].

4. Multiple rapidity gaps in diffraction

The renormalization method used to calculate the SD and DD cross sections can be extended to multi-gap diffractive events. Below, we outline the procedure for calculating the differential cross section for a 4-gap event (we use rapidity, y, and pseudorapidity, η, interchangeably).

The calculation of the differential cross section is based on the parton-model scattering amplitude:

$$\mathrm{Im}\, f(t,\Delta y) \sim e^{(\varepsilon + \alpha' t)\Delta y}$$

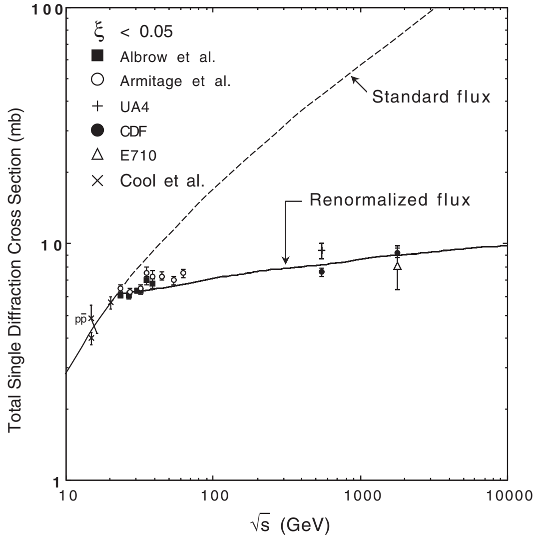

Figure 3: The $\bar{p}p$ total SD cross section exhibits an s-dependence consistent with the renormalization procedure of Ref. [9], contrary to the $s^{2\varepsilon}$ behaviour expected from Regge theory (figure from Ref. [9]).

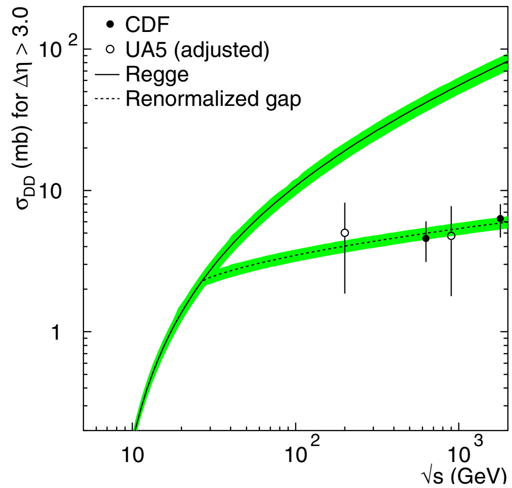

Figure 4: The $\bar{p}p$ total DD (central gap) cross section agrees with the prediction of the *renormalized rapidity gap* model [11], contrary to the $s^{2\varepsilon}$ expectation from Regge theory (figure from Ref. [10]).

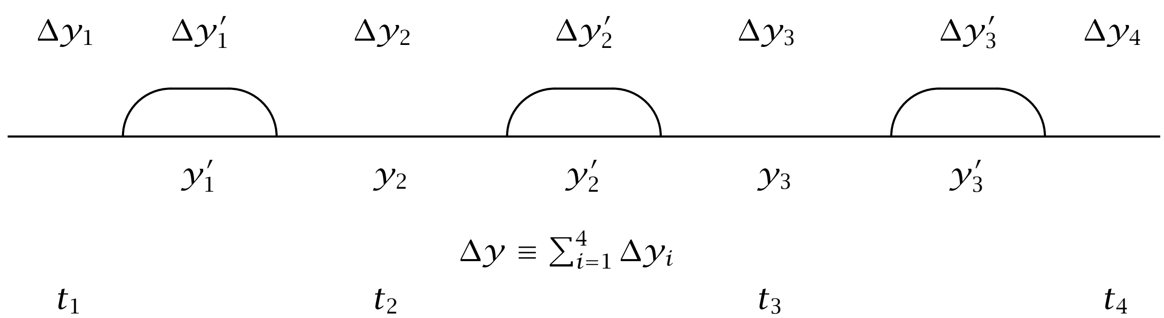

Figure 5: Topology of a 4-gap event in pseudorapidity space.

For the rapidity regions $\Delta y_i'$, where there is particle production, the $t = 0$ parton model amplitude is used and the *sub-energy cross section* is $\sim e^{\varepsilon \Delta y'}$. For rapidity gaps, Δy, which can be considered as resulting from elastic scattering between diffractively excited states, the square of the full parton-model amplitude is used, $e^{2(\varepsilon + \alpha' t_i)\Delta y_i}$, and the form factor $\beta^2(t)$ is included for a surviving nucleon. The *gap probability* (product of all rapidity gap terms) is then normalized to unity, and a *color matching factor* κ is included for each gap.

For the 4-gap example of Fig. 5, which has 10 independent variables, V_i (shown below the figure), we have:

- $\frac{d^{10}\sigma}{\Pi_{i=1}^{10} dV_i} = P_{gap} \times \sigma(\mathrm{sub-energy})$
- $\sigma(\mathrm{sub-energy}) = \kappa^4 \left[\beta^2(0) \cdot e^{\varepsilon \Delta y'}\right]$ $\qquad (\Delta y' = \sum_{i=1}^{3} \Delta y_i')$
- $P_{gap} = N_{gap} \times \Pi_{i=1}^{4} \left[e^{(\varepsilon + \alpha' t_i)\Delta y_i}\right]^2 \times [\beta(t_1)\beta(t_4)]^2 = N_{gap} \cdot e^{2\varepsilon \Delta y} \cdot f(V_i)|_{i=1}^{10}$ $(\Delta y = \sum_{i=1}^{4} \Delta y_i)$
  where N_{gap} is the factor that normalizes P_{gap} over all phase space to unity.

The last equation shows that the renormalization factor of the gap probability depends only on s (since $\Delta y_{max} = \ln s$) and not on the number of diffractive gaps. Thus, the ratio of DPE to SD cross sections is expected to be $\approx \kappa$, with no additional energy dependent suppression. This can be tested at the Tevatron, where one may also study events with a central rapidity gap within single-diffractive clusters. The study of events with more than two gaps will have to await the commissioning of the LHC.

References

[1] T. Affolder *et al.*, CDF Collaboration, Phys. Rev. Lett. **84**, 5083 (2000)
[2] F. Abe *et al.*, CDF Collaboration, Phys. Rev. Lett. **78**, 2698 (1997)
[3] F. Abe *et al.*, CDF Collaboration, Phys. Rev. Lett. **79**, 2636 (1997)
[4] T. Affolder *et al.*, CDF Collaboration, Phys. Rev. Lett. **84**, 232 (2000)
[5] T. Affolder *et al.*, CDF Collaboration, submitted to Phys. Rev. Letters; arXiv:hep-ex/0107071
[6] T. Affolder *et al.*, CDF Collaboration, submitted to Phys. Rev. Letters; arXiv:hep-ex/0109025
[7] C. Adloff *et al.*, H1 Collaboration, arXiv:hep-ex/0012151
[8] T. Affolder *et al.*, CDF Collaboration, Phys. Rev. Lett. **85**, 4217 (2000)
[9] K. Goulianos, Phys. Lett. **B358**, 379 (1995)
[10] T. Affolder *et al.*, CDF Collaboration, Phys. Rev. Lett. **87**, 141802 (2001)
[11] K. Goulianos, arXiv:hep-ph/9806384