

# SUSY Parameters from Charginos

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The chargino pair production processes at  $e^+e^-$  collisions are explored to reconstruct the fundamental SUSY parameters: the SU(2) gaugino parameter  $M_2$ , the higgsino mass parameter  $\mu$  and  $\tan\beta$ . Both CP-conserving and CP-violating SUSY sectors are discussed.

## 1. Introduction

The concept of symmetry between bosons and fermions, supersymmetry (SUSY), has so many attractive features that the supersymmetric extension of the Standard Model is still widely considered as a most natural scenario. However, if realized in Nature, supersymmetry must be broken at low energy since no superpartners of ordinary particles have been observed so far. Technically it is achieved by introducing the soft-supersymmetry breaking parameters: gaugino masses  $M_i$ , sfermion masses  $m_{\tilde{f}}$  and trilinear couplings  $A^f$  (gauge group and generation indices are understood). This gives rise to a large number of parameters. Even in the minimal supersymmetric model (MSSM) 105 new parameters are introduced. This number of parameters, reflecting our ignorance of SUSY breaking mechanism, can be reduced by additional physical assumptions.

After discovering supersymmetric particles, however, the priority will be to determine the low-energy Lagrangian parameters. They should be measured independently of any theoretical assumptions. This will allow us to verify the relations among them, if any, in order to distinguish between various SUSY models.

Here we outline how the fundamental SUSY parameters: the SU(2) gaugino parameter  $M_2$ , the higgsino mass parameter  $\mu$  and  $\tan\beta$ , can be determined from the measurements of chargino pair production cross sections with polarized beams at future  $e^+e^-$  linear colliders. The results summarized here have been worked out in a series of papers [1], to which we refer for more detailed discussions and references.

## 2. Chargino sector

After the electroweak symmetry breaking, the mass matrix of the spin-1/2 partners of the  $W^\pm$  gauge bosons and the charged Higgs bosons,  $\tilde{W}^\pm$  and  $\tilde{H}^\pm$ , is non diagonal

$$\mathcal{M}_C = \begin{pmatrix} M_2 & \sqrt{2}m_W \cos\beta \\ \sqrt{2}m_W \sin\beta & \mu \end{pmatrix} \quad (1)$$

The mass eigenstates, the two charginos  $\tilde{\chi}_{1,2}^\pm$ , are mixtures of the charged SU(2) gauginos and higgsinos. Since the chargino mass matrix  $\mathcal{M}_C$  is not symmetric, two different unitary matrices acting on the left- and right-chiral  $(\tilde{W}, \tilde{H})_{L,R}$  two-component states

$$U_{L,R} \begin{pmatrix} \tilde{W}^- \\ \tilde{H}^- \end{pmatrix}_{L,R} = \begin{pmatrix} \tilde{\chi}_1^- \\ \tilde{\chi}_2^- \end{pmatrix}_{L,R} \quad (2)$$

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are needed to diagonalize the matrix eq.(1). In general CP-noninvariant theories the mass parameters are complex. However, by reparametrization of the fields,  $M_2$  can be assumed real and positive without loss of generality so that the only non-trivial reparametrization-invariant phase may be attributed to  $\mu = |\mu| e^{i\Phi_\mu}$  with  $0 \leq \Phi_\mu \leq 2\pi$ . The unitary matrices  $U_L$  and  $U_R$  can be parameterized in the following way:

$$U_L = \begin{pmatrix} \cos \phi_L & e^{-i\beta_L} \sin \phi_L \\ -e^{i\beta_L} \sin \phi_L & \cos \phi_L \end{pmatrix}, \quad U_R = \begin{pmatrix} e^{iy_1} & 0 \\ 0 & e^{iy_2} \end{pmatrix} \begin{pmatrix} \cos \phi_R & e^{-i\beta_R} \sin \phi_R \\ -e^{i\beta_R} \sin \phi_R & \cos \phi_R \end{pmatrix} \quad (3)$$

The mass eigenvalues  $m_{\tilde{\chi}_{1,2}^\pm}^2$  are given by

$$m_{\tilde{\chi}_{1,2}^\pm}^2 = \frac{1}{2} \left[ M_2^2 + |\mu|^2 + 2m_W^2 \mp \Delta_C \right] \quad (4)$$

with  $\Delta_C$  involving the phase  $\Phi_\mu$ :

$$\Delta_C = [(M_2^2 - |\mu|^2)^2 + 4m_W^4 \cos^2 2\beta + 4m_W^2 (M_2^2 + |\mu|^2) + 8m_W^2 M_2 |\mu| \sin 2\beta \cos \Phi_\mu]^{1/2} \quad (5)$$

The four phase angles  $\{\beta_L, \beta_R, y_1, y_2\}$  are not independent: they are functions of the invariant angle  $\Phi_\mu$  and their explicit form can be found in [1]. All four phase angles vanish in CP-invariant theories for which  $\Phi_\mu = 0$  or  $\pi$ . The rotation angles  $\phi_L$  and  $\phi_R$  satisfy the relations:

$$\begin{aligned} c_{2L,R} &\equiv \cos 2\phi_{L,R} = -[M_2^2 - |\mu|^2 \mp 2m_W^2 \cos 2\beta] / \Delta_C \\ s_{2L,R} &\equiv \sin 2\phi_{L,R} = -2m_W [M_2^2 + |\mu|^2 \pm (M_2^2 - |\mu|^2) \cos 2\beta + 2M_2 |\mu| \sin 2\beta \cos \Phi_\mu]^{1/2} / \Delta_C \end{aligned} \quad (6)$$

### 3. Inverting

From the set  $m_{\tilde{\chi}_{1,2}^\pm}$  and  $\cos 2\phi_{L,R}$  the fundamental supersymmetric parameters  $\{M_2, |\mu|, \cos \Phi_\mu, \tan \beta\}$  in CP-(non)invariant theories can be determined unambiguously in the following way:

$$\begin{aligned} M_2 &= m_W [\Sigma - \Delta(c_{2L} + c_{2R})]^{1/2} \\ |\mu| &= m_W [\Sigma + \Delta(c_{2L} + c_{2R})]^{1/2} \\ \cos \Phi_\mu &= [\Delta^2(2 - c_{2L}^2 - c_{2R}^2) - \Sigma] \{ [1 - \Delta^2(c_{2L} - c_{2R})^2] [\Sigma^2 - \Delta^2(c_{2L} + c_{2R})^2] \}^{-1/2} \\ \tan \beta &= \{ [1 - \Delta(c_{2L} - c_{2R})] / [1 + \Delta(c_{2L} - c_{2R})] \}^{1/2} \end{aligned} \quad (7)$$

where we introduced the abbreviations  $\Sigma = (m_{\tilde{\chi}_2^\pm}^2 + m_{\tilde{\chi}_1^\pm}^2 - 2m_W^2) / 2m_W^2$ , and  $\Delta = (m_{\tilde{\chi}_2^\pm}^2 - m_{\tilde{\chi}_1^\pm}^2) / 4m_W^2$ . Therefore to reconstruct the above parameters the chargino masses and  $\cos 2\phi_{L,R}$  have to be measured independently. This can be done from the measurements of the production of chargino pairs at  $e^+e^-$  colliders, where they are produced at tree level via  $s$ -channel  $\gamma$  and  $Z$  exchanges, and  $t$ -channel  $\tilde{\nu}_e$  exchange.

The chargino masses can be measured very precisely from the sharp rise of the cross sections at threshold [2]. The mixing angles  $\phi_{L,R}$  on the other hand can be determined from measured cross sections for the chargino production with polarized beams. For this purpose polarized beams are crucial since the mixing angles  $\phi_{L,R}$  encode the chiral dependence of the chargino couplings to the  $Z$  gauge boson and to the electron-sneutrino current. All the production cross sections  $\sigma_\alpha\{ij\} = \sigma(e^+e^-_\alpha \rightarrow \tilde{\chi}_i^+ \tilde{\chi}_j^-)$  for any beam polarization  $\alpha$  and for any combination of chargino pairs  $\{ij\}$  depend only on  $\cos 2\phi_L$  and  $\cos 2\phi_R$  apart from the chargino masses, the sneutrino mass and the Yukawa  $\tilde{W}e\tilde{\nu}$  coupling.<sup>1</sup> In fact the cross sections are binomials in the  $[\cos 2\phi_L, \cos 2\phi_R]$  plane. Therefore any two contour lines,  $\sigma_L\{11\}$  and  $\sigma_R\{11\}$  for example, will at least cross at one

<sup>1</sup>The explicit dependence on the  $\sin 2\phi_{L,R}$  and on the phase angles  $\beta_L, \beta_R, y_1, y_2$  has to disappear from CP-invariant quantities. The final ambiguity in  $\Phi_\mu \leftrightarrow 2\pi - \Phi_\mu$  in CP-noninvariant theories must be resolved by measuring observables related to the normal  $\tilde{\chi}_1^-$  or/and  $\tilde{\chi}_2^+$  polarization in non-diagonal  $\tilde{\chi}_1^- \tilde{\chi}_2^+$  chargino-pair production.

point in the plane between  $-1 \leq \cos 2\phi_L, \cos 2\phi_R \leq +1$ . However, the contours, being ellipses or hyperbolae, may cross up to four times. Imposing contours for other cross sections  $\sigma_\alpha\{ij\}$  this ambiguity can be resolved and, at the same time, the sneutrino mass and the identity between the  $\tilde{W}e\tilde{\nu}$  Yukawa and the  $W\ell\nu$  gauge couplings can be tested. The sneutrino exchange does not contribute for the right-handed polarized electron beams,  $\alpha = R$ . Therefore, while the curves for  $\sigma_R\{ij\}$  are fixed, the curves for  $\sigma_L\{ij\}$  will move in the  $[\cos 2\phi_L, \cos 2\phi_R]$  plane with changing  $m_{\tilde{\nu}}$  and the Yukawa coupling. All curves will intersect in the same point only if the mixing angles as well as the sneutrino mass and the Yukawa coupling correspond to the correct physical values.

It has been checked that combining the analyses of  $\sigma_R\{ij\}$  and  $\sigma_L\{ij\}$ , the masses, the mixing parameters and the Yukawa coupling can be determined to quite a high precision. For example, for the reference point RR1 introduced in Ref.[2], defined by  $M_2 = 152$  GeV,  $\mu = 316$  GeV and  $\tan\beta = 3$ , one can expect

$$\begin{aligned} m_{\tilde{\chi}_1^\pm} &= 128 \pm 0.04 \text{ GeV} & \cos 2\phi_L &= 0.645 \pm 0.02 & g_{\tilde{W}e\tilde{\nu}}/g_{W\ell\nu} &= 1 \pm 0.001 \\ m_{\tilde{\chi}_2^\pm} &= 346 \pm 0.25 \text{ GeV} & \cos 2\phi_R &= 0.844 \pm 0.005 \end{aligned} \quad (8)$$

where the  $1\sigma$  statistical errors are for an integrated luminosity of  $\int \mathcal{L} = 1 \text{ ab}^{-1}$  collected at  $\sqrt{s} = 800$  GeV.

Using the eqs.(7) the accuracy which can be expected in such an analysis for two CP-invariant reference points, the RR1 defined above and the RR2 defined by  $M_2 = 150$  GeV,  $\mu = 263$  GeV and  $\tan\beta = 30$ , is as follows (errors are  $1\sigma$  statistical only assuming 100% polarized beams)

$M_2$	$152 \pm 1.75 \text{ GeV}$	$150 \pm 1.2 \text{ GeV}$
$\mu$	$316 \pm 0.87 \text{ GeV}$	$263 \pm 0.7 \text{ GeV}$
$\tan\beta$	$3 \pm 0.69$	$> 20.2$

where the first (second) column is for RR1 (RR2). If  $\tan\beta$  is large, this parameter is difficult to extract from the chargino sector. Since the chargino observables depend only on  $\cos 2\beta$ , the dependence on  $\beta$  is flat for  $2\beta \rightarrow \pi$  so that eq.(7) is not very useful to derive the value of  $\tan\beta$  due to error propagation. A significant lower bound can be derived nevertheless in any case.

The errors derived above have been obtained assuming that the sneutrino mass is known from *e.g.* sneutrino pair production. If the sneutrinos, however, are beyond the kinematical reach, their masses can be inferred from the forward-backward asymmetries of the decay leptons [3]. For high precision experimental analyses also radiative corrections should be included [4].

#### 4. Incomplete chargino system

For the above analyses the knowledge of both chargino masses is crucial. However, at an early phase of the  $e^+e^-$  linear collider the energy may only be sufficient to reach the threshold of the light chargino pair  $\tilde{\chi}_1^+\tilde{\chi}_1^-$ . Nearly the entire structure of the chargino system can nevertheless be reconstructed even in this case.

From the  $\sigma_L\{11\}$  and  $\sigma_R\{11\}$  the mixing angles  $\cos 2\phi_L$  and  $\cos 2\phi_R$  can be determined up to at most a four-fold ambiguity assuming that the sneutrino mass and the Yukawa coupling are known. The ambiguity can be resolved within the chargino system by adding the information from measurements with transverse beam polarization or by analyzing the polarization of the charginos in the final state and their spin-spin correlations [1]. The knowledge of  $\cos 2\phi_L$ ,  $\cos 2\phi_R$  and  $m_{\tilde{\chi}_1^\pm}$  is sufficient to derive the fundamental gaugino parameters  $\{M_2, \mu, \tan\beta\}$  in CP-invariant theories up to at most a discrete two-fold ambiguity. This remaining ambiguity can be removed by *e.g.* confronting the ensuing Higgs boson mass  $m_{h^0}$  with the experimental value. Alternatively, the ambiguity can also be resolved by analyzing the light neutralino  $\tilde{\chi}_1^0\tilde{\chi}_2^0$  system for left and right polarized beams. At the same time the U(1) gaugino mass parameter  $M_1$  can also be determined [5].

#### 5. Conclusions

The measured chargino masses  $m_{\tilde{\chi}_{1,2}^\pm}$  and the two mixing angles  $\phi_L$  and  $\phi_R$  are enough to extract the fundamental SUSY parameters  $\{M_2, |\mu|, \cos\Phi_\mu, \tan\beta\}$  unambiguously; a discrete two-fold ambiguity  $\Phi_\mu \leftrightarrow 2\pi - \Phi_\mu$  can be resolved only by measuring the CP-violating observable.

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