Report of the Snowmass Subgroup on Extra Dimensions

G. Pásztor*
CERN
T.G. Rizzo†
Stanford Linear Accelerator Center

In this report we summarize the work performed at Snowmass 2001 on the physics of extra dimensions. We divide these analyses into the following classes: searches for extra dimensional phenomena, identification of specific new physics scenarios, studies of black hole production and non-commutative QED.

1. Searches for extra dimensions

Many models predict the existence of additional dimensions which lead to new and distinct phenomenological signatures that can be searched for at future colliders which have center of mass energies in the TeV range and above. Most of the models in the literature fall into one of the three following broad classes: (*i*) The large extra dimensions scenario of Arkani-Hamed, Dvali and Dimopoulos (ADD) [1] predicts the emission and exchange of large Kaluza-Klein (KK) towers of gravitons that are finely-spaced in mass. The emitted gravitons appear as missing energy while the KK tower exchange leads to contact interaction-like dimension-8 operators. (*ii*) A second possibility includes models where the extra dimensions are of TeV scale in size [2]. In these scenarios there are KK excitations of the Standard Model (SM) gauge (and possibly other SM) fields with masses of order TeV, which can appear as resonances at colliders. (*iii*) A last class of models are those with warped extra dimensions, such as the Randall-Sundrum Model (RS) [3], which predict graviton resonances with both weak scale masses and couplings to matter.

1.1. ADD Signatures in Dijets at Hadron Colliders

The exchange of graviton towers in the ADD model can lead to substantial alterations in a number of $2 \rightarrow 2$ processes that lead to dijet production at hadron colliders. Doncheski [4] explored the relevant distributions for dijet production using a number of distinct kinematic variables to explore the sensitivity to ADD contributions at both the LHC and the Tevatron. He found that the invariant mass, the pseudorapidity (η) and the $\chi = exp(\eta)$ distributions were less sensitive than those associated with p_t or p_t^2 in probing for graviton tower exchange. The 95% CL exclusion reach for the Hewett cutoff scale that he obtained using the p_t distribution was $M_s \simeq 3$ TeV at the Tevatron and $M_s \simeq 20$ TeV at the LHC.

1.2. Supersymmetric ADD

Although the ADD model was designed to circumvent the hierarchy problem without the introduction of supersymmetry (SUSY), it may be natural that SUSY be incorporated into any realistic extension of this scenario. Such a possibility has been considered by Hewett and Sadri [5]. In these extensions not only the graviton but also the gravitino obtain a finely grained KK tower whose interactions can be formulated in terms of a linearized version of supergravity. These authors derived the Feynman rules for this theory and obtained the couplings of the gravitino tower to the SUSY SM matter fields. As in the case of graviton exchange in the usual ADD model,

---

*Gabriella.Pasztor@cern.ch; On leave of absence from KFKI RMKI, Budapest, Hungary.
†rizzo@slac.stanford.edu

the virtual exchange of gravitinos requires a cutoff when summing over the KK tower. In the ADD case this generates a set of effective dimension-8 operators describing the exchange of gravitons characterized by the scale M_s. For the gravitino case, operators of several different dimensions are generated, including those of dimension-6, so we might expect gravitino exchange effects to be large. In performing the sum over KK states one must include the shift in the propagator due to the non-zero mass of the zero mode that arises from SUSY breaking; however, in the leading approximation one finds that this zero mode mass can be neglected.

The effects of gravitino exchange may be most noticeable in the case of selectron pair production at a linear collider. In addition to the MSSM contributions (γ and Z s-channel exchange and neutralino t-channel exchange) there are now contributions arising from s-channel gravitons and t-channel gravitinos. The use of beam polarization and the fact that the left and right selectrons can be separately identified helps to isolate the new gravitino exchange contributions to the amplitude. Figure 1 shows the size of the SUSY ADD contributions to selectron pair production in comparison to the usual results from the MSSM for one particular channel for different values of the Hewett cutoff scale. Here we see that values of the scale M_s significantly larger than the collider center of mass energy can be probed via the reconstructed angular distribution of the selectrons. Using this selectron production channel, these authors showed that values of the scale M_s as large as $\simeq 20 - 25\sqrt{s}$ can be probed at linear colliders.
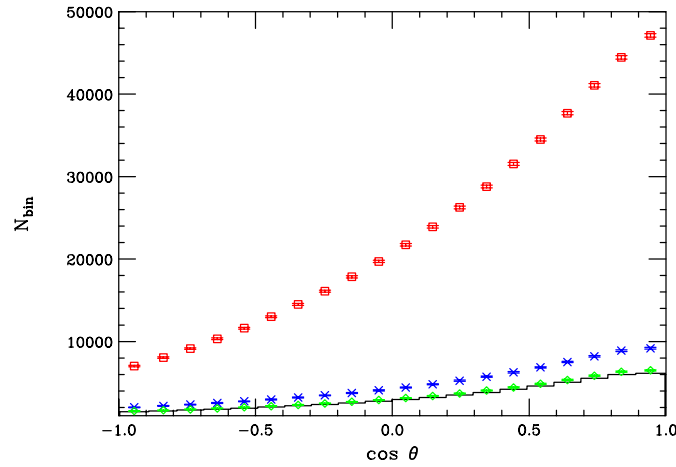

Figure 1: Angular distribution for $e_L^- e^+ \rightarrow \tilde{e}_L \tilde{e}_R$ at a 500 GeV linear collider with 80% beam polarization for the case of six extra dimensions. A luminosity of 500 fb^{-1} has been assumed; the histogram is the MSSM prediction with a bino-like LSP. The red (blue, green) points show the prediction with statistical errors for the SUSY ADD case assuming a cutoff scale of 1.5 (3, 6) TeV.

1.3. Constraints on KK Excitations in TeV-scale extra dimensions

SM gauge bosons propagating in extra dimensions appear from the four dimensional perspective as towers of KK states, which can be produced at colliders if the available energy is higher than the compactification scale M_c. Otherwise if the collision energy is insufficient for direct production, they can still be observed through indirect effects.

Cheung and Landsberg [6] studied the virtual exchange of the KK excitations of the W, Z, γ and g bosons in high energy particle interactions. They performed a combined fit to the dilepton, dijet and top-pair production results at Tevatron, the neutral and charged-current deep-inelastic scattering measurements at HERA and the observables in dijet and dilepton production at LEP2, assuming that the energy scale of these processes are well below M_c and ignoring the mixing among the KK states. In the analysis they take the leading order calculations for both the SM and the new physics processes, and to account for higher order effects they introduce a scale factor, universal for SM and new physics, which is derived either from higher order SM calculations or directly from the data using regions where the contributions from KK states are expected to be vanishing. The best fit for $\pi^2/(3M_c^2)$ gives -0.29 ± 0.09 TeV^{-2} lying in the unphysical region, which is then turned into a lower limit of 6.8 TeV on M_c.

Using the double differential cross-section $\partial^2 \sigma / \partial M_{\ell\ell} \partial \cos \theta$ in the most sensitive Drell-Yan production, they estimated the reach of Tevatron Run II with a luminosity of $\mathcal{L}$=15 fb^{-1} and of LHC with $\mathcal{L}$=100 fb^{-1} to be 4.2 and 13.5 TeV, respectively.

1.4. TeV-Scale Gauge Unification

In models with TeV-scale extra dimensions the evolution of the gauge couplings are modified by power law terms. This effect can be searched for in specific processes which are highly sensitive to one of the gauge couplings and can be well measured at future colliders. C. Balázs and B. Laforge [7] examined a model with TeV-scale extra dimensions with a compactification scale M_c in which SM gauge bosons, especially the gluons, can propagate in the bulk, and gauge and Yukawa unifications happen slightly above the TeV range at M_{GUT}. They demonstrated that by studying dijet production at LHC, which at lowest order is sensitive to α_s^2, one can discover the anomalous running of the strong coupling up to a common compactification scale $M_c = 5 - 10$ TeV depending on how the matching of the evolution below and above the compactification scale is done. To reduce the effect of the KK excitations which compete with the modified running of the gauge coupling, a cut of $p_T < 200$ GeV looks sufficient. The statistical significance defined as the deviation from the SM prediction in units of the statistical uncertainty is shown in Figure 2 for various numbers of extra dimensions as a function of the minimum dijet mass. The Tevatron Run II can observe the effect of the extra dimensions up to a scale of 1 TeV in the most optimistic scenario studied in the paper.
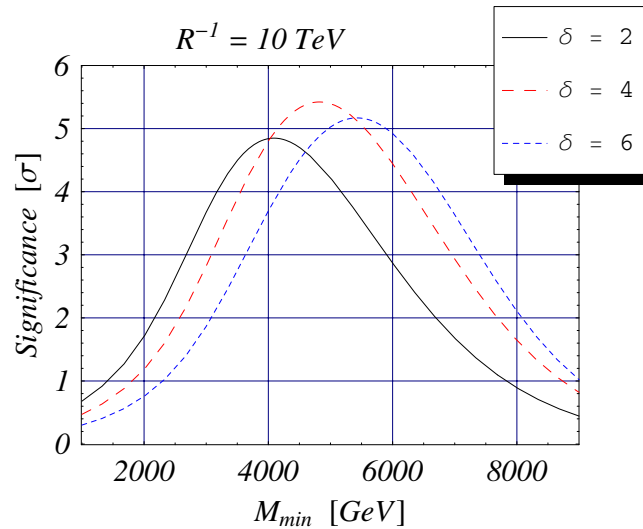

Figure 2: Deviation from the SM in units of the statistical uncertainty in dijet production at LHC in the case of TeV-scale gauge unification for various numbers of extra dimensions for M_c=10 TeV in an optimistic matching scenario.

1.5. Extra Dimensional Signals at Far Future Colliders

Signatures for extra dimensional models at the LHC and TeV-scale linear colliders are well known; many phenomenological studies and several simulation studies already exist. What about machines in the more distant future? Rizzo has begun an examination of the signals for these models at both CLIC ($\sqrt{s} = 3 - 5$ TeV and $L = 1$ ab^{-1}) [8] and VLHC ($\sqrt{s} = 175 - 200$ TeV and $L = 0.2 - 1$ ab^{-1}) [9].

In the case of CLIC, the highest sensitivity to both graviton exchange in the ADD model and gauge KK exchange in TeV-scale theories is to look for deviations in differential cross sections and asymmetries from their SM predictions due to the new particles. $e^+e^- \to f\bar{f}$ provides a set of processes of high sensitivity in either case from which a combined limit can be extracted. For

a luminosity of 1 ab^{-1} the reach was found to be $M_s \simeq 6\sqrt{s}$ for the Hewett cutoff scale in the ADD model and $M_c \simeq 16-20\sqrt{s}$ for the compactification scale in TeV models with one extra dimension compactified on S_1/Z_2. If $\sqrt{s}$ is sufficiently large the details of the KK excitation spectrum can be probed in the TeV case and the number of extra dimensions and the compactification manifold can be determined. Possible separations amongst the fermions in the extra dimensions can also be probed in this case. For the RS model there are two possibilities; if the scale of the graviton spectrum is low enough CLIC can be used to probe the resonance spectrum and the specific decay modes of the graviton including rare decays. If the graviton is somewhat more massive indirect searches can be performed in analogy to the ADD and TeV cases discussed above with the results as shown in Figure 3. In either case CLIC can greatly elucidate the physics of the RS model.
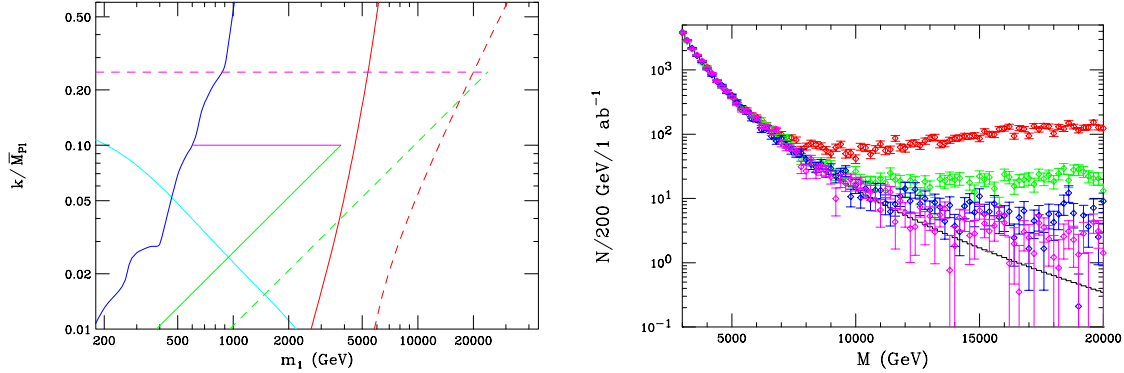

Figure 3: (Left) Allowed regions of the RS model parameter space; m_1 is the lightest KK graviton mass. Current Tevatron (blue) and precision measurements (cyan) forbid regions to the left of their specific curves. The horizontal magenta solid (dashed) lines form the upper bound of the region when $c = k/\overline{M}_* = 0.10$ (0.25) while the solid (dashed) green curve is the corresponding lower bound when $\Lambda_\pi = 10$ (25) TeV. The solid (dashed) red curve is the reach of the LHC (CLIC) with 100 fb^{-1} ($\sqrt{s} = 5$ TeV with 1 ab^{-1}). (Right) Event rate per 200 GeV mass bin for the Drell-Yan process as a function of the lepton pair invariant mass at a $\sqrt{s} = 200$ TeV stage II VLHC. A rapidity cut $\eta_l < 2.5$ on both leptons has been applied. The solid histogram is the SM background in both cases whereas the 'data' points are the predictions of the ADD model. The red, green, blue and magenta points correspond to a Hewett cutoff scale of $M_s = 20, 25, 30$ or 35 TeV, respectively.

The VLHC was shown to have an enormous direct discovery reach for extra dimensional models of all types in the Drell-Yan channel. In the case of ADD the Drell-Yan process was shown to be sensitive to values of M_s as large as $25-40$ TeV through the production of excess events at large invariant masses as demonstrated by Figure 3. In the case of one extra dimension the first KK state of TeV-scale models could be directly observed in Drell-Yan for compactification scales as large as $M_c \simeq 55-60$ TeV. For models with more than one extra dimension, their detailed spectra could be probed in Drell-Yan for compactification scales in excess of $15-20$ TeV thus allowing for a determination of the number of extra dimensions and their compactification manifolds. The production of RS graviton resonances in Drell-Yan when the SM fields remain on the SM brane were shown to be observable for masses as large as $15-40$ TeV depending upon the value of the parameter $c = k/\overline{M}_*$. It was clear that if no sign of the graviton resonances predicted by the RS model show up by the time that CLIC/VLHC probe the relevant mass range then this version of the model would be excluded.

1.6. Radions

In the RS model, the fluctuations of the size of the extra dimension about its stabilized value manifest themselves as a single scalar field, the radion. In the RS model with a bulk scalar field it is expected that the radion is the lightest state beyond the SM fields with a mass probably in the range between $\mathcal{O}(10 \text{ GeV})$ and $\Lambda = e^{-kL}M_{Pl} \sim \mathcal{O}(\text{TeV})$. The couplings of the radion are in the order of $1/\Lambda$ and are very similar to the couplings of the SM Higgs boson, except for one important difference: due to the trace anomaly the radion directly couples to massless gauge bosons at one-loop. Moreover, in the low energy 4D effective theory the radion can mix with the Higgs boson, so

that the physical mass eigenstates are mixtures of the radion and the Higgs boson. This mixing can cause important changes in the contribution to the electroweak observables compared to the no-mixing case as discussed by Kribs [10].

The mixing of the RS radion with the SM Higgs boson can lead to important shifts in the Higgs couplings which become apparent in the Higgs decay widths. This possibility was examined for these proceedings by Hewett and Rizzo [11]. These shifts depend upon the radion and Higgs masses, $m_{r,h}$, the mixing parameter ξ, which is expected to be of order unity, and the ratio of the SM Higgs vacuum expectation value, v, to the RS scale Λ, which is of order one TeV or greater. Their results are shown in Figure 4 for several values of these parameters. Several features can be observed immediately: (*i*) the shifts in the widths to $\bar{f}f/VV$ and $\gamma\gamma$ final states are very similar, while the corresponding shift for the gg final state is quite different. (*ii*) For relatively light radions with a low value of Λ the width into the gg final state can come close to vanishing due to a destructive interference between the contributions to the amplitude for values of ξ near -1. (*iii*) Increasing the value of m_r has less of an effect on the width shifts than does a decrease in the ratio $\frac{v}{\Lambda}$. (*iv*) Since the VV and $\bar{f}f$ final states are dominant for Higgs masses in the region near 125 GeV, we expect that the *branching fractions* for these modes will be little influenced by radion mixing. This implies that it will be imperative to obtain a precise measurement of the Higgs total width in order to probe for mixing with radions.
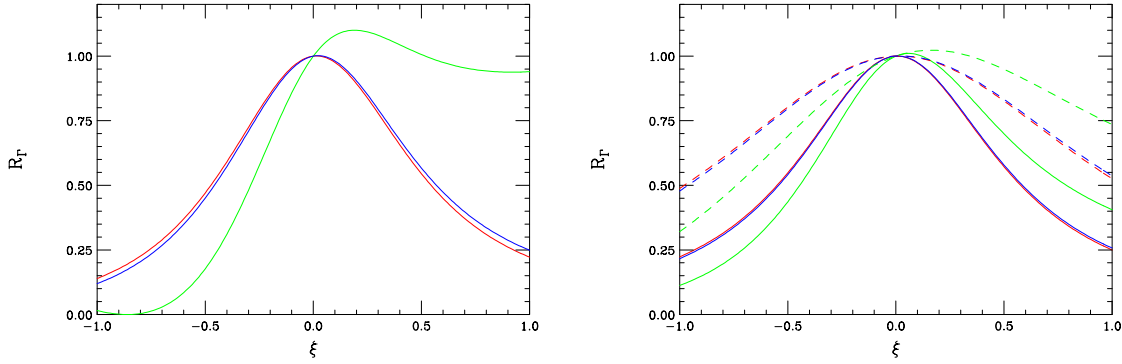

Figure 4: Ratio of Higgs widths to their SM values, R_Γ, as a function of the mixing parameter ξ assuming a Higgs mass of 125 GeV: red for fermion pairs or massive gauge boson pairs, green for gluons and blue for photons. In the left panel we assume a radion mass $m_r = 300$ GeV and $v/\Lambda = 0.2$. In the right panel the solid (dashed) curves are for $m_r = 500$ (300) GeV and $v/\Lambda = 0.2$ (0.1).

2. Identification of New Physics

Once new physics has been discovered the next step is to identify its source. As is well known it is possible that many different new physics scenarios can lead to similar signatures at colliders. This implies that we must have a set of tools available to help us distinguish the various models. For example, an RS graviton and a Z' both yield a resonance structure in the Drell-Yan channel at hadron colliders but they can be distinguished by the angular distributions of their decay products. Several analyses have addressed the issue of new physics identification for the case of models with extra dimensions for these proceedings.

2.1. Extra Dimensions or SUSY in Missing Energy Events?

Missing energy signatures are traditionally used to search for signs of new physics at colliders. The photon plus missing energy final state at a linear collider is particularly useful in discovering two classes of theories. Theories with large extra dimensions predict the associated production of a photon with KK excitations of the graviton ($e^+e^- \rightarrow \gamma G$), while in supersymmetric models with super-light gravitinos the photon can be produced together with a gravitino pair ($e^+e^- \rightarrow \gamma \tilde{G}\tilde{G}$).

Gopalakrishna, Perelstein and Wells [12] have compared the differential cross-sections of the two processes as a function of $x_\gamma = 2E_\gamma/\sqrt{s}$, shown in Figure 5, and $\cos\theta$. They found that a hard cut on x_γ makes it possible to discriminate between supersymmetric gravitino pair-production and KK graviton production if the number of extra dimensions $n < 6$. For the $n = 6$ case the two models give very similar photon energy distributions and are indistinguishable.
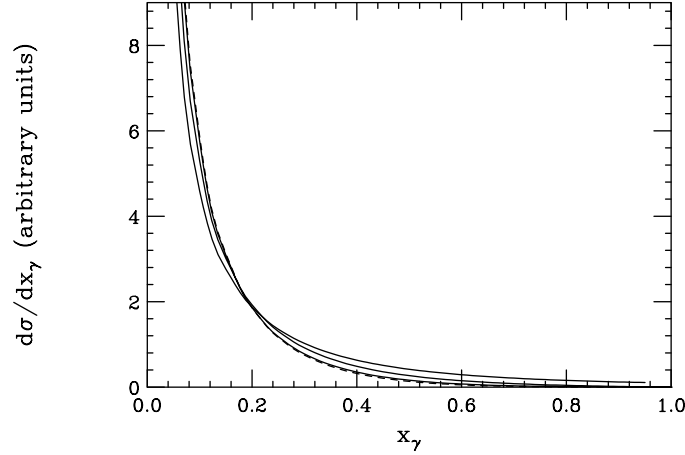

Figure 5: Differential cross-sections of the photon plus missing energy final state as a function of the fractional photon missing energy in large extra dimensional theories with $n = 2, 4, 6$ (solid lines) and in the super-light gravitino model (dashed line) with the scales of the models set so that the total cross-sections are equal.

Another handle to identify the models is to study the cross-section at different center-of-mass energies. Assuming that $\sqrt{s}$ can be varied by 5% close to 500 GeV, the KK graviton models with lower n can again be separated from each other and from the supersymmetric theory, but $n = 6$ gives the same scaling of the cross-section as the super-light gravitino model. Thus one should look for other consequences of the models, like dimension-8 contact interactions in extra dimensional models and the production of superpartners in the super-light gravitino model to identify the physics responsible for the production of missing energy events above the SM expectation.

2.2. KK Resonances or Z' at the LHC?

Precision measurement constraints tell us [2] that the mass of the lightest gauge KK excitation in models with TeV-scale extra dimensions is most likely greater than about 4 TeV when SM matter fields lie at the orbifold fixed points. If this bound is correct then the *second* set of KK excitations will be too massive to be produced at the LHC at an observable rate. An essentially degenerate combination of Z and γ KK excitations has a sufficiently large cross section as to be observable at the LHC for masses as great as $\simeq 6.5 - 7$ TeV in the Drell-Yan channel. If such a gauge KK resonance is indeed observed at the LHC and the second excitation is too massive to be produced, how do we know that it is not a more conventional Z'? One argument that can be made is that in TeV KK models there will also be a first KK excited W state which is degenerate with the first excitation of the γ/Z so that a resonance should also appear in the charged leptons plus missing energy channel. However, there are many extended gauge models in the literature which predict degenerate W'/Z' states so this observation alone will not provide proof in either direction.

Rizzo [13] has compared the Drell-Yan excitation spectrum associated with the production of the first KK γ/Z excitation to that for an arbitrary Z' with generation-independent couplings. (This is a 5 parameter fit assuming that the Z' couples to a gauge group which commutes with weak isospin.) In particular, one makes the assumption that the KK state is actually a Z' and tries to fit to the Z''s quark and lepton couplings. A poor fit then implies that the state is not a Z'. For simplicity this analysis assumed that the KK excitation arose from a S_1/Z_2 compactification and that all the SM fermions were at the same fixed point ($D = 0$) or that quarks and leptons were at opposite fixed points ($D = \pi R_c$, where R_c is the compactification radius). Note that below the

KK resonance there is a strong destructive (constructive) interference between the SM amplitudes and the KK exchanges when $D = 0$ (πR_c).

The results of this analysis were as follows: for $D = \pi R_c$, the KK state could easily mask itself as a Z' for all masses $M_{KK} \geq 4$ TeV. For $D = 0$ and relatively low values of the KK mass, $M_{KK} \simeq 4 - 5$ TeV, the Z' hypothesis failed and the two scenarios were distinguishable. For heavier KK states, the statistics was so poor that model separation was lost. An increase in luminosity or the use of more than one channel may allow these results to be extended to KK states with larger masses.

2.3. Contact Interactions at a Linear Collider

While all present solutions to the hierarchy problem of the SM require the appearance of new physics at energies close to the TeV scale, the predicted new particles can not always be produced directly. However there may be a significant contribution to processes such as fermion or gauge boson pair production due to the effect of virtual states of the new heavy particles.

If a contact interaction type correction to the Standard Model is observed, studying its detailed properties may shed light on the fundamental physics behind it. Pásztor and Perelstein [14] considered several models leading to such indirect effects in lepton pair production at a 500 GeV to 1 TeV linear collider with the aim to determine how well one can discriminate between them. They studied models with large and TeV-scale extra dimensions and lepton compositeness [15]. In the large extra dimension scenario contributions can come from two distinct sources: the virtual exchange of KK excitations of the graviton or of string Regge excitations of the photon and the Z boson [16]. In the case of TeV-scale extra dimensions, accessible for the SM gauge bosons, the contact interactions arise from the exchange of the virtual KK excitations of the photon and the Z boson.

It was shown that for a wide range of model parameters, measuring the lepton pair production cross-section and angular distributions allows the identification of the correct candidate model. The reach in the model scale parameter for the selection of the right model (the exclusion of *all* other models considered) is close, in most cases within $5 - 15\%$, to the sensitivity reach (the exclusion of the SM) of the model.

The result is demonstrated in Figure 6. Here the expected confidence level (CL) at which the various models can be excluded is shown as a function of the scale parameter of the true model assuming that one of them (the true model) is manifested in Nature. The combination of the electron, muon and tau pair final states proved to be a major tool in distinguishing the models.
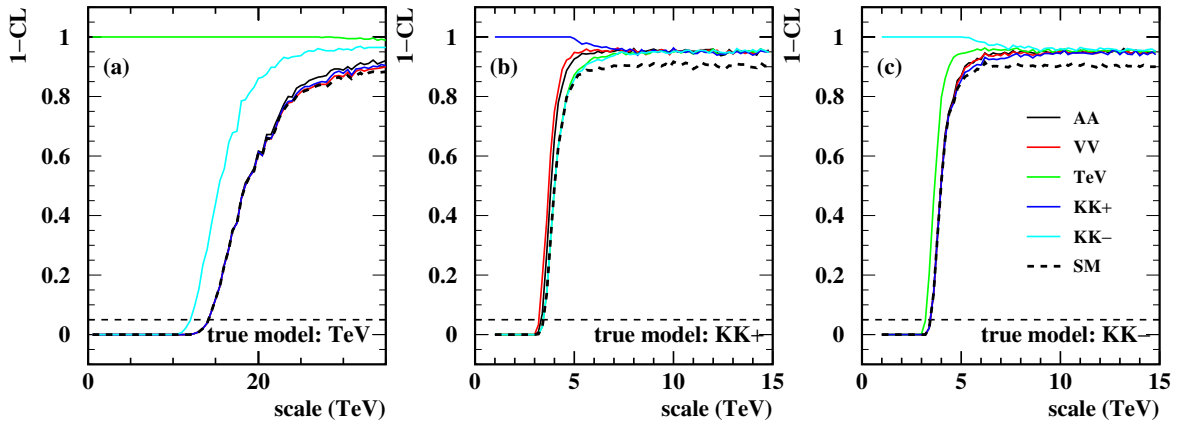

Figure 6: Expected $1-$CL at a 500 GeV linear collider with 100 fb^{-1} luminosity after combining the three lepton pair final states, as a function of the scale parameter of the true model, assumed to be (a) TeV-scale extra dimensions, large extra dimension with dominant contribution from KK excitations of the graviton with (b) $\lambda=+1$ and (c) $\lambda=-1$ using the notation of Hewett, for various tested models. AA and VV stands for composite lepton models with axial-axial and vector-vector couplings. The large extra dimension scenario with dominant contribution from string Regge states is omitted from this combined plot due to the lack of calculations for the muon and tau pair final states.

2.4. Identifying Graviton Resonances at a Linear Collider

The production of a single spin-2 resonance at a collider may be the first signature for the RS model. Could it be something else? For example, some models predict Regge-like excitations of the SM gauge fields which also have spin-2 [16]. However, the branching fractions of these particles and gravitons are quite different; if the various final states are accessible at a given collider, it will allow these two possibilities to be rather easily differentiated. This can be done in a straightforward manner at a linear collider.

The model by Dvali and co-workers [17] predicts the existence of a single graviton-like resonance. In this class of models the propagator of the graviton obtains a rather complicated structure that arises due to novel brane interactions, including a dimension-dependent, as well as energy-dependent, imaginary part and a real part which vanishes at a fixed value of s. Hence one produces an effective resonance which is in some sense a "collective" mode. Since this mode is constructed out of a superposition of the KK states in the graviton tower, it has the same branching fractions as an RS graviton and thus the two models cannot be distinguished using such measurements. The Dvali *et al.* model has three parameters: $d \geq 2$, the number of extra dimensions, M_d, the effective resonance mass and M_*, the d−dimensional Planck scale, which is of the order of a few TeV. Note that in the limit $M_d \to \infty$ we recover the ADD model. For some values of the parameters, in particular when $d = 5$, the effective resonance shape looks very much like that of a relativistic Breit-Wigner (BW) RS graviton.

Rizzo [18] undertook a preliminary study of the line shapes of the resonances appearing in the RS and the Dvali *et al.* models which are imagined to take place after an unfolding of the initial state radiation and beamstrahlung spectra; in particular one tries to fit the non-BW Dvali *et al.* "effective" resonance under the assumption that it is instead a BW RS graviton and perform a fit for the RS model parameter $c = k/\overline{M}_*$. A poor quality fit would thus indicate that the two scenarios are distinguishable. It was shown for a wide range of the other model parameters that the RS and Dvali *et al.* model line shapes could be distinguished provided the ratio $M_*/\sqrt{s} \leq 6$ and the linear collider could scan the region around M_d with adequate luminosity.

3. Black Hole Production at Future Colliders

An exciting possibility in theories with extra dimensions and a low Planck scale is that the production rate of black holes (BH) somewhat more massive than $\overline{M}_*$ can be quite large at future colliders, *e.g.*, of order 100 pb at the LHC [19] and even larger cross sections at the VLHC; the actual production cross section critically depends on the BH mass, the exact value of $\overline{M}_*$ and the number of extra dimensions. Early discussions on this possibility have been extended for these proceedings by several authors [20] so we only review the essential points here.

The basic claim of the original BH papers is as follows: we imagine the collision of 2 high energy partons which are confined to a 3-brane; gravity is free to propagate in δ extra dimensions with the $4 + \delta$ dimensional Planck scale assumed to be $\overline{M}_* \sim 1$ TeV. The curvature of the space is assumed to be small compared to the energy scales involved in the collision so that quantum gravity effects can be neglected. When these partons have a center of mass energy in excess of $\sim \overline{M}_*$ and the impact parameter is less than the Schwarzschild radius, R_S, associated with this center of mass energy, a $4+\delta$-dimensional BH is formed with very high efficiency. The subprocess cross section for BH production is thus essentially geometric, *i.e.*, $\hat{\sigma} = \pi R_S^2$; note that R_S scales as $[M_{BH}/\overline{M}_*^{2+\delta}]^{\frac{1}{1+\delta}}$ apart from an overall δ- and author-dependent numerical factor. (This is due to the different expressions for the Schwarzschild radius used by the two sets of authors.) This numerical factor is relatively important since it leads to a very different δ dependence for the BH production cross section. This cross section expression is claimed to hold when $M_{BH}/\overline{M}_*$ is "large", *i.e.*, when the system can be treated semi-classically and quantum gravitational effects are small; one may debate just what "large" really means, but it most certainly means "at least a few". Voloshin has argued that an additional exponential suppression $\sim exp\ [-C(\delta)(M_{BH}/\overline{M}_*)^{\frac{2+\delta}{1+\delta}}]$, where C is a constant, is also present, which seriously damps the cross section for this process [21] even in the semi-classical case. This possibility remains controversial at this time and strong arguments have been made on either side. For purposes of our discussion we will entertain the criticisms by Voloshin but remind the reader that the jury is still out on this issue. If they are valid one worries that the resulting cross sections for heavy BH will possibly be too small to be

observable at the LHC; as we will see below this need not be the case for modest BH masses. In either case we anxiously await the resolution of this important issue.

As can be seen in Figure 7 the rates for BH production at the LHC are quite large over a wide range of masses and numbers of extra dimensions using either set of authors' cross section expressions. Note that the difference between the two sets of predictions increases as δ increases. Once produced these BHs essentially decay semi-classically via Hawking radiation into a reasonably large number $\simeq 25$ or more final state partons in a highly spherical pattern. Hadrons will dominate over leptons by a factor of order $5-10$ for such final states. Such signatures would not be missed at either hadron or lepton colliders. In addition to the above authors, Borissov and Lykken [22] have performed Monte Carlo studies of the decays of TeV scale BH which shed charge and angular momentum modeled via chemical potentials. These authors obtained plots of decay number densities for final state particles with different spins as δ, $\overline{M}_*$ and M_{BH} were varied. From this they can extract the hadronic and leptonic content of BH decays as well as the contributions due to photons, gravitons and Higgs bosons. We note that an alternative decay scenario has been advocated by Casadio and Harms [23].

Note that Figure 7 also shows the effects of the suppression predicted by Voloshin using the authors' respective expressions for R_S [24]; here we make the important observation that at least for some range of parameters BHs will still be produced at significant rates to be observable at the LHC *even if the Voloshin suppression is active.*
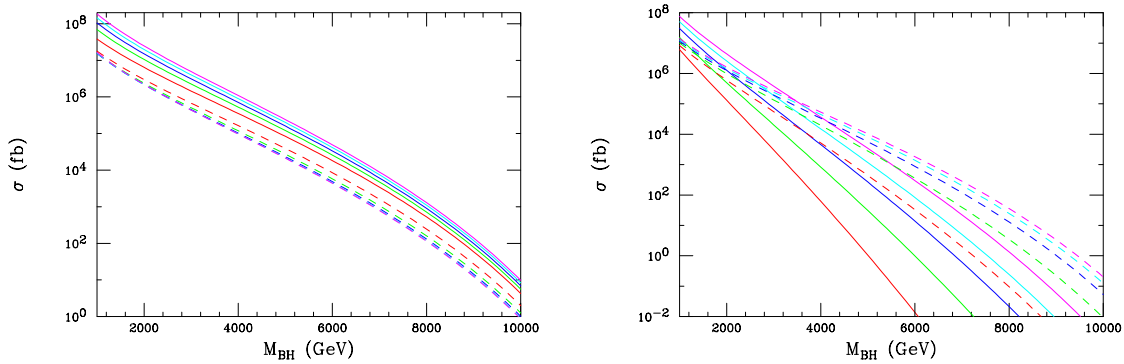

Figure 7: (Left) Cross section for the production of BHs more massive than M_{BH} at the LHC assuming $\overline{M}_* = 1$ TeV for $\delta = 2$ (3, 4, 5, 6) extra dimensions corresponding to the red (green, blue, cyan, magenta) curves. The solid (dashed) curves are the results from the work by Giddings and Thomas (Dimopoulos and Landsberg). (Right) Same as on the left but now including the effects of the Voloshin damping factor. Note that large cross sections are still possible for certain ranges of the parameters.

4. Non-commutative QED

If the Planck scale is indeed of order of a few TeV, other possible stringy effects may be observable at future colliders. One possibility is that space-time is non-commutative (NC), *i.e.*, $[x_\mu, x_\nu] = i\theta_{\mu\nu} = ic_{\mu\nu}/\Lambda_{NC}^2$, where the $c_{\mu\nu}$ is a set of parameters of order unity and $\Lambda_{NC} \sim$ a few TeV. NC effects thus only appear as the TeV scale is approached, generally appearing as dimension-8 operators. NC theories are Lorentz violating (but CPT conserving) since we can decompose the frame-independent quantity $c_{\mu\nu}$ into two 3-vectors (in analogy with the E and B fields) which point in some fixed but *a priori* unknown preferred directions. NC QED provides a potential experimental testing ground for such theories and has interesting collider signatures [25]. NC QED differs from ordinary QED in several ways: (*i*) the $ee\gamma$ vertex picks up a Lorentz violating phase factor which is dependent upon the electron momenta and the components of $\theta_{\mu\nu}$; (*ii*) the NC theory predicts trilinear and quartic couplings between the photons that are, to leading order, linear and quadratic in the parameters $\theta_{\mu\nu}$ and are kinematics dependent; (*iii*) only the charges $Q = 0, \pm 1$ are allowed by gauge invariance in NC QED and thus quarks cannot be treated in the theory as it presently exists and an extension to the full NC SM is required. The hallmark signal at colliders for NC QED is the appearance of an azimuthally-dependent cross section in $2 \rightarrow 2$ processes; the azimuthal dependence arises from the existence of preferred directions.

Godfrey and Doncheski [26] have examined the two processes $\gamma\gamma \to e^+e^-$ and $\gamma e \to \gamma e$ for the effects of NC at linear colliders in the 0.5-8 TeV energy range. Their analysis assumed an integrated luminosity of 500 fb^{-1} and sets a series of lower bounds on the scale Λ_{NC}. (Note that while $\gamma\gamma$ probes space-time (*i.e.*, c_{0i}) NC, γe was shown to probe both space-space (*i.e.*, c_{ij}) and space-time NC.) Figure 8 from [26] shows the azimuthal dependence of the Compton scattering cross section at a 500 GeV collider when Λ_{NC}=500 GeV for a specific orientation of the c_{0i} and c_{ij} vectors relative to the beam axis; we observe that the deviations are quite statistically significant. For the $\gamma\gamma$ process with c_{0i} perpendicular to the beam axis the authors' 95% CL lower bounds on Λ_{NC} are in the range $\simeq 0.42 - 0.52\sqrt{s}$ while bounds as high as $\simeq 1.5\sqrt{s}$ are obtained for the γe process. In the $\gamma\gamma$ case the size of the NC effect was shown to depend only upon the cosine of the angle between the beam axis and the NC direction while in the Compton case the angular dependence was found to be somewhat more complex depending on two orientation angles.
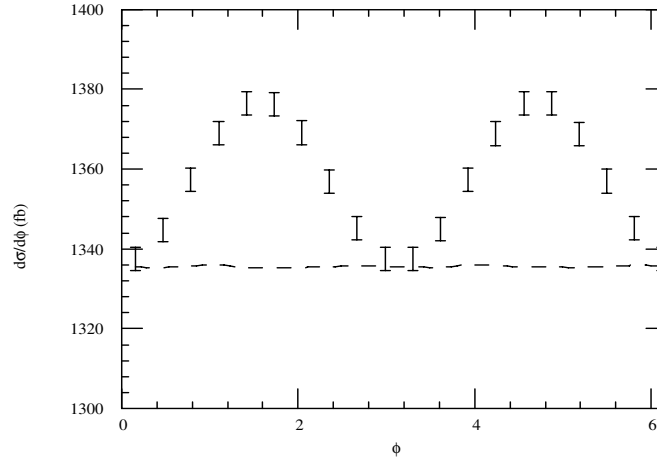

Figure 8: Binned azimuthal cross section dependence for the Compton scattering of unpolarized photons at a 500 GeV linear collider assuming $\Lambda_{NC} = 500$ GeV and an integrated luminosity of 500 fb^{-1}. The dashed line in the SM expectation.

References

[1] N. Arkani-Hamed, S. Dimopoulos, and G. Dvali, Phys. Lett. **B429**, 263 (1998), and Phys. Rev. **D59**, 086004 (1999); I. Antoniadis, N. Arkani-Hamed, S. Dimopoulos, and G. Dvali, Phys. Lett. **B436**, 257 (1998). For an introduction to ADD phenomenology, see G.F. Giudice, R. Rattazzi and J.D. Wells, Nucl. Phys. **B544**, 3 (1999); T. Han, J.D. Lykken and R. Zhang, Phys. Rev. **D59**, 105006 (1999), E.A. Mirabelli, M. Perelstein and M.E. Peskin, Phys. Rev. Lett. **82**, 2236 (1999); J.L. Hewett, Phys. Rev. Lett. **82**, 4765 (1999); T.G. Rizzo, Phys. Rev. **D60**, 115010 (1999).

[2] I. Antoniadis, Phys. Lett. **B246**, 377 (1990); I. Antoniadis, C. Munoz and M. Quiros, Nucl. Phys. **B397**, 515 (1993); I. Antoniadis and K. Benalki, Phys. Lett. **B326**, 69 (1994)and Int. J. Mod. Phys. **A15**, 4237 (2000); I. Antoniadis, K. Benalki and M. Quiros, Phys. Lett. **B331**, 313 (1994). For an overview of phenomenology, see, for example, T.G. Rizzo and J.D. Wells, Phys. Rev. **D61**, 016007 (2000); P. Nath and M. Yamaguchi, Phys. Rev. **D60**, 116006 (1999); M. Masip and A. Pomarol, Phys. Rev. **D60**, 096005 (1999); L. Hall and C. Kolda, Phys. Lett. **B459**, 213 (1999); R. Casalbuoni, S. DeCurtis, D. Dominici and R. Gatto, Phys. Lett. **B462**, 48 (1999); A. Strumia, Phys. Lett. **B466**, 107 (1999); F. Cornet, M. Relano and J. Rico, Phys. Rev. **D61**, 037701 (2000); C.D. Carone, Phys. Rev. **D61**, 015008 (2000); T.G. Rizzo, Phys. Rev. **D61**, 055005 (2000) and Phys. Rev. **D64**, 015003 (2001).

[3] L. Randall and R. Sundrum, Phys. Rev. Lett. **83**, 3370 (1999). For an overview of RS phenomenology, see H. Davoudiasl, J.L. Hewett and T.G. Rizzo, Phys. Rev. Lett. **84**, 2080 (2000); Phys. Lett. **B493**, 135 (2000); and Phys. Rev. **D63**, 075004 (2001).

[4] M.A. Doncheski, these proceedings and hep-ph/0111149.

[5] J.L. Hewett and D. Sadri, these proceedings.

[6] K. Cheung and G. Landsberg, these proceedings and hep-ph/0110346.

[7] C. Balázs and B. LaForge, these proceedings and hep-ph/0110217.

[8] T.G. Rizzo, these proceedings and hep-ph/0108235.

[9] T.G. Rizzo, these proceedings and hep-ph/0108234.

[10] G.D. Kribs, these proceedings and hep-ph/0110242.

[11] J.L. Hewett and T.G. Rizzo, these proceedings.

[12] S. Gopalakrishna, M. Perelstein and J.D. Wells, these proceedings and hep-ph/0110339.

[13] T.G. Rizzo, these proceedings and hep-ph/0109179.

[14] G. Pásztor and M. Perelstein, these proceedings and hep-ph/0111471.

[15] E.Eichten, K.D. Lane and M.E. Peskin, Phys. Rev. Lett. **50**, 811 (1983)

[16] S. Cullen, M. Perelstein and M.E. Peskin, Phys. Rev. **D62**, 055012 (2000).

[17] G. Dvali, G. Gabadadze and M. Porrati, Phys. Lett. **B485**, 208 (2000); G. Dvali and G. Gabadadze, Phys. Rev. **D63**, 065007 (2001).

[18] T.G. Rizzo, these proceedings and hep-ph/0110202.

[19] S.B. Giddings and S. Thomas, hep-ph/0106219; S. Dimopoulos and G. Landsberg, Phys. Rev. Lett. **87**, 16162 (2001).

[20] S.B. Giddings, these proceedings and hep-ph/0110127; G. Landsberg, these proceedings.

[21] M.B. Voloshin, Phys. Lett. **B518**, 137 (2001)and hep-ph/0111099.

[22] L. Borissov and J. Lykken, these proceedings.

[23] R. Casadio and B. Harms, hep-th/0110255.

[24] T.G. Rizzo, these proceedings and hep-ph/0111230.

[25] J.L. Hewett, F.J. Petriello and T.G. Rizzo, Phys. Rev. **D64**, 075012 (2001); P. Mathews, Phys. Rev. **D63**, 075007 (2001); S. Baek *et al.*, Phys. Rev. **D64**, 056001 (2001).

[26] S. Godfrey and M. Doncheski, these proceedings and hep-ph/0108268.