Complete Reconstruction of the Neutralino System

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Consecutively to the chargino system, $\tilde{\chi}^{\pm}$, in which the SU(2) gaugino parameter M_2, the higgsino mass parameter μ and $\tan\beta$ can be determined, the remaining fundamental supersymmetry parameter, the U(1) gaugino mass M_1 can be analysed in the neutralino system, $\tilde{\chi}^0$, including its modulus and phase in CP–noninvariant theories. First experimental hints on CP violation can be seen in the threshold behaviour of neutralino production.

I. INTRODUCTION

In the minimal supersymmetric extension of the Standard Model (MSSM), the spin-1/2 partners of the neutral gauge bosons, $\tilde{B}$ and $\tilde{W}^3$, and of the neutral Higgs bosons, $\tilde{H}_1^0$ and $\tilde{H}_2^0$, mix to form neutralino mass eigenstates χ_i^0 (i=1,2,3,4). The neutralino mass matrix in the $(\tilde{B}, \tilde{W}^3, \tilde{H}_1^0, \tilde{H}_2^0)$ basis

$$\mathcal{M} = \begin{pmatrix} M_1 & 0 & -m_Z c_\beta s_W & m_Z s_\beta s_W \\ 0 & M_2 & m_Z c_\beta c_W & -m_Z s_\beta c_W \\ -m_Z c_\beta s_W & m_Z c_\beta c_W & 0 & -\mu \\ m_Z s_\beta s_W & -m_Z s_\beta c_W & -\mu & 0 \end{pmatrix} \tag{1}$$

is built up by the fundamental supersymmetry parameters: the U(1) and SU(2) gaugino masses M_1 and M_2, the higgsino mass parameter μ, and the ratio $\tan\beta = v_2/v_1$ of the vacuum expectation values of the two neutral Higgs fields which break the electroweak symmetry. Here, $s_\beta = \sin\beta$, $c_\beta = \cos\beta$ and s_W, c_W are the sine and cosine of the electroweak mixing angle θ_W. In CP–noninvariant theories, the mass parameters are complex. Without loss of generality M_2 can be taken real and positive so that the two remaining non–trivial phases may be attributed to M_1 and μ. Neutralinos are produced in e^+e^- collisions, either in diagonal or in mixed pairs: $e^+e^- \rightarrow \tilde{\chi}_i^0\, \tilde{\chi}_j^0$ $(i, j = 1, 2, 3, 4)$. If the collider energy is sufficient to produce the four neutralino states in pairs, the underlying fundamental SUSY parameters $\{|M_1|, M_2, |\mu|, \Phi_1, \Phi_\mu; \tan\beta\}$ can be extracted from the masses $m_{\tilde{\chi}_i^0}$ (i=1,2,3,4) and couplings. Partial information from the lowest $m_{\tilde{\chi}_i^0}$ (i=1,2) neutralino states is sufficient to extract $\{|M_1|, \Phi_1\}$ if the other parameters have been pre–determined in the chargino sector [1, 2].

II. MIXING FORMALISM

In the MSSM, the four neutralinos $\tilde{\chi}_{1,2,3,4}^0$ are mixtures of the neutral U(1) and SU(2) gauginos and the SU(2) higgsinos. Since the matrix $\mathcal{M}$ is symmetric, one unitary matrix N is sufficient to rotate the gauge eigenstate basis $(\tilde{B}^0, \tilde{W}^3, \tilde{H}_1^0, \tilde{H}_2^0)$ to the mass eigenstate basis of the Majorana fields $\tilde{\chi}_i^0$: $\mathcal{M}_{diag} = N^* \mathcal{M} N^\dagger$ with $(\tilde{\chi}_1^0, \tilde{\chi}_2^0, \tilde{\chi}_3^0, \tilde{\chi}_4^0) = N(\tilde{B}, \tilde{W}^3, \tilde{H}_1^0, \tilde{H}_2^0)$. The mass eigenvalues m_i in $\mathcal{M}_{diag}$ can be chosen positive by a suitable definition of the unitary matrix N, which can be parametrized by 6 angles and 10 phases. It is convenient to factorize the matrix N into a diagonal Majorana M and a Dirac–type D component in the following way:

$$N = \mathsf{M}\,\mathsf{D}, \quad \text{with} \quad \mathsf{M} = \mathsf{diag}\left\{e^{i\alpha_1}, e^{i\alpha_2}, e^{i\alpha_3}, e^{i\alpha_4}\right\} \tag{2}$$

One overall Majorana phase is unphysical and may be chosen to vanish and the matrix D can be written as a sequence of 6 two-dimensional rotations. Complete analytical results are given in [2].

III. THE NEUTRALINO QUADRANGLES

The unitarity constraints on the elements of the mixing matrix N can be formulated by means of unitarity quadrangles which are built up by the links $N_{ik}N_{jk}^*$ connecting two rows i and j,

$$M_{ij} = N_{i1}N_{j1}^* + N_{i2}N_{j2}^* + N_{i3}N_{j3}^* + N_{i4}N_{j4}^* = 0 \qquad \text{for} \;\; i \neq j \tag{3}$$

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and by the links $N_{ki}N^*_{kj}$ connecting two columns i and j

$$D_{ij} = N_{1i}N^*_{1j} + N_{2i}N^*_{2j} + N_{3i}N^*_{3j} + N_{4i}N^*_{4j} = 0 \qquad \text{for} \quad i \neq j \tag{4}$$

of the mixing matrix. The areas of the six quadrangles M_{ij} and D_{ij} are given by

$$\text{area}[M_{ij}] = \frac{1}{4}(|J^{12}_{ij}| + |J^{23}_{ij}| + |J^{34}_{ij}| + |J^{41}_{ij}|), \quad \text{area}[D_{ij}] = \frac{1}{4}(|J^{ij}_{12}| + |J^{ij}_{23}| + |J^{ij}_{34}| + |J^{ij}_{41}|), \tag{5}$$

where J^{kl}_{ij} are the Jarlskog–type CP–odd "plaquettes": $J^{kl}_{ij} = \Im\mathrm{m} N_{ik}N_{jl}N^*_{jk}N^*_{il}$. The matrix N is CP violating, if either any one of the plaquettes is non–zero, or, if the plaquettes all vanish, at least one of the links is non–parallel to the real or to the imaginary axis. The phases of the neutralino fields are fixed (modulo a common phase) and the orientation of the neutralino quadrangles M_{ij} and D_{ij} in the complex plane is physically non–trivial. Since the neutralino mass matrix involves only two invariant phases Φ_1 and Φ_μ, all the physical phases of N are fully determined by these two phases in the mass matrix. Therefore the experimental reconstruction of the unitarity quadrangles overconstrains the neutralino system and numerous consistency relations can be exploited to scrutinize the validity of the underlying theory [2].

IV. THRESHOLD BEHAVIOR OF NEUTRALINO PRODUCTION

In CP–invariant theories the S-wave excitation giving rise to a steep rise $\sim \lambda^{1/2}$ of the cross section for the nondiagonal pair near threshold signals opposite CP parities of produced neutralinos. Not all nondiagonal pairs of neutralinos can be produced in the S–wave; if $\{ij\}$ and $\{ik\}$ have opposite CP parities, the pair $\{jk\}$ has the same and it will be excited in the P-wave characterized by the slow rise $\sim \lambda^{3/2}$ of the cross section. It is important to realize that CP-violation may change the threshold behaviour, cf. Fig.1a, and in particular may allow S-wave excitations in any nondiagonal pair, e.g. finding the $\{ij\}$, $\{ik\}$ and $\{jk\}$ pairs to be excited in the S–wave would uniquely signal CP violation.

V. EXTRACTING THE FUNDAMENTAL PARAMETERS

The measurements of the chargino–pair production processes $e^+e^- \to \tilde{\chi}^+_i \tilde{\chi}^-_j$ (i,j=1,2) carried out with polarized beams can be used for a complete determination of the basic SUSY parameters $\{M_2, |\mu|, \Phi_\mu\,; \tan\beta\}$ in the chargino sector with high precision [4].

Each of the four characteristic equations for the neutralino mass squared can be cast into the form [2]

$$(\Re\mathrm{e} M_1)^2 + (\Im\mathrm{m} M_1)^2 + u_i \Re\mathrm{e} M_1 + v_i \Im\mathrm{m} M_1 = w_i \qquad (i = 1, 2, 3, 4) \tag{6}$$

The coefficients u_i, v_i and w_i are functions of the parameters $\tan\beta$, M_2, $|\mu|$, Φ_μ pre–determined in the chargino sector, and the mass $m^2_{\tilde{\chi}^0_i}$; the coefficient v_i is necessarily proportional to $\sin\Phi_\mu$ because physical masses are CP–even. Each neutralino mass defines a circle in the $\{\Re\mathrm{e} M_1, \Im\mathrm{m} M_1\}$ plane. Thus the measurement of three neutralino masses leads to an unambiguous determination M_1, cf. Fig. 2a. With only two light neutralino masses, the two–fold ambiguity can be resolved by exploiting the measured cross section $\sigma\{12\}$, as shown in Fig. 2b. However, if the phase $\sin\Phi_\mu$ vanishes, there remains a two–fold discrete sign ambiguity [2].

If all four masses are experimentally accessible the complete reconstruction of the mass and mixing parameters is easy. The four–state mixing of neutralinos in the MSSM is reflected in the sum rules for the neutralino couplings [2]. Therefore evaluating these sum rules experimentally, it can be tested whether the four–neutralino system $\{\tilde{\chi}^0_1, \tilde{\chi}^0_2, \tilde{\chi}^0_3, \tilde{\chi}^0_4\}$ forms a closed system, or whether additional states at high mass scales mix in, signaling the existence of an extended gaugino system.

VI. THE SUPERSYMMETRIC YUKAWA COUPLINGS

A linear collider with polarized beams [3] offers the possibility to verify very accurately the fundamental SUSY assumption that the Yukawa couplings, $g_{\tilde{W}}$ and $g_{\tilde{B}}$ are indentical to the SU(2) and U(1) gauge couplings g and g'. Varying the left–handed and right–handed Yukawa couplings leads to a significant change in the corresponding left–handed and right–handed production cross sections. Combining the measurements of σ_R and σ_L for the process $e^+e^- \to \tilde{\chi}^0_1 \tilde{\chi}^0_2$, the Yukawa couplings $g_{\tilde{W}}$ and $g_{\tilde{B}}$ can be determined to quite a high precision as demonstrated in Fig. 1b. The 1σ statistical errors have been derived for an integrated luminosity of $\int \mathcal{L}\, dt = 100$ and 500 fb^{-1} and for partially polarized beams.

VII. SUMMARY

The measurement of the processes $e^+e^- \to \tilde{\chi}^0_i \tilde{\chi}^0_j$ (i,j=1,2,3,4), carried out with polarized beams and combined with the analysis of the chargino system $e^+e^- \to \tilde{\chi}^+_i \tilde{\chi}^-_j$ (i,j=1,2), can be used to perform a complete and precise

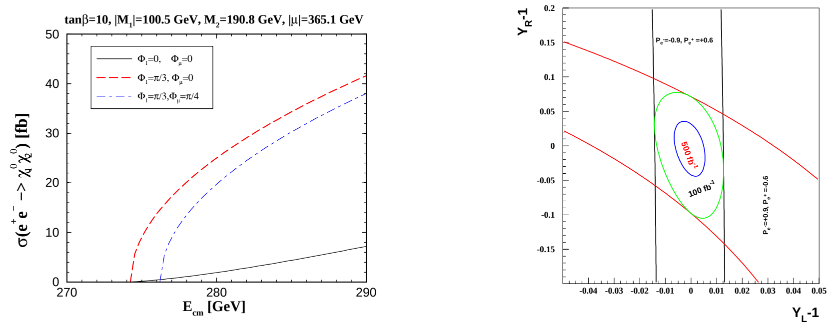

FIG. 1: *a) The threshold behaviour of the neutralino production cross section $\sigma\{12\}$; the shift of the energy threshold is due to the dependence of the neutralino masses on the phases.*
b) Contours of the cross sections $\sigma_L\{12\}$ and $\sigma_R\{12\}$ in the plane of the Yukawa couplings $g_{\tilde{W}}$ and $g_{\tilde{B}}$ normalized to the SU(2) and U(1) gauge couplings g and g' $\{Y_L = g_{\tilde{W}}/g,\, Y_R = g_{\tilde{B}}/g'\}$ for the set RP1 *at the e^+e^- c.m. energy of 500 GeV; the contours correspond to the integrated luminosities 100 and 500 fb^{-1} and the longitudinal polarization of electron and positron beams of 90% and 60%, respectively.*

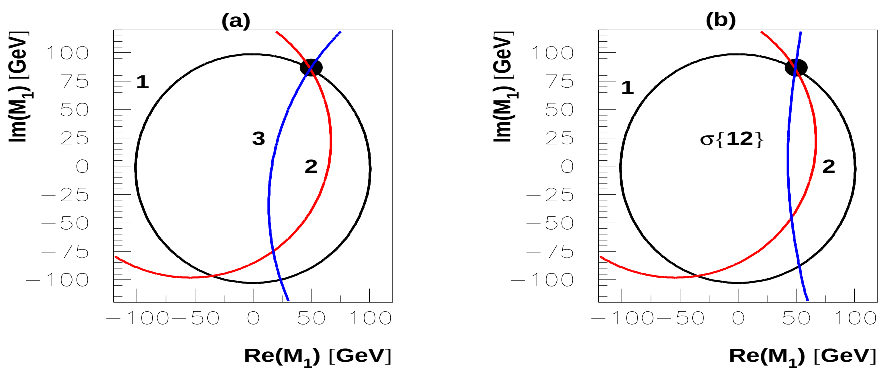

FIG. 2: *The contours of (a) three measured neutralino masses $m_{\tilde{\chi}^0_i}$ $(i = 1, 2, 3)$, and (b) two neutralino masses (1,2) and one neutralino production cross section $\sigma_{tot}\{12\}$ in the $\{\Re\mathrm{e}M_1, \Im\mathrm{m}M_1\}$ plane; the parameter set* RP1$''$ $\{\tan\beta = 10,\ M_2 = 190.8\,\mathrm{GeV},\ |\mu| = 365.1\,\mathrm{GeV},\ \Phi_\mu = \pi/4\}$ *is taken from the chargino sector.*

analysis of the basic SUSY parameters in the gaugino/higgsino sector $\{M_1, M_2, \mu; \tan\beta\}$. The closure of the neutralino system can be verified by exploiting the sum rules for production cross sections.

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