$g_\mu - 2$ in Supersymmetry

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The 2.6σ deviation in the muon's anomalous magnetic moment has strong implications for supersymmetry. In the most model-independent analysis to date, we consider gaugino masses with arbitrary magnitude and phase, and sleptons with arbitrary masses and left-right mixings. For $\tan\beta = 50$, we find that 1σ agreement requires at least one charged superpartner with mass below 570 GeV; at 2σ, this upper bound shifts to 850 GeV. The deviation is remarkably consistent with all constraints from colliders, dark matter, and $b \to s\gamma$ in supergravity models, but disfavors the characteristic gaugino mass relations of anomaly-mediation.

The current world average of the muon's anomalous magnetic moment a_μ differs from the standard model prediction by 2.6σ: $a_\mu^{\rm exp} - a_\mu^{\rm SM} = (43 \pm 16) \times 10^{-10}$ [1]. The reported deviation is about three times larger than the standard model's electroweak contribution [2], and so deviations of roughly this order are expected in many models motivated by attempts to understand electroweak symmetry breaking. Its interpretation as supersymmetry is particularly attractive, as supersymmetry naturally provides electroweak scale contributions that are easily enhanced (by large $\tan\beta$) to produce deviations larger than the standard model's electroweak corrections. In addition, a_μ is both flavor- and CP-conserving. Thus, while the impact of supersymmetry on other low energy observables can be highly suppressed by scalar degeneracy or small CP-violating phases in simple models, supersymmetric contributions to a_μ cannot be. In this sense, a_μ is a uniquely robust probe of supersymmetry, and an anomaly in a_μ is a natural place for the effects of supersymmetry to appear.

We begin with the most model-independent analysis possible consistent with slepton flavor conservation. (For more details, see Ref. [3].) In general, the reported deviation may be explained entirely by new physics in the muon's *electric* dipole moment [4]. However, this possibility is not realized in supersymmetry, and we assume that the deviation arises solely from contributions to a_μ. We then consider uncorrelated values of the relevant SUSY parameters, allowing arbitrary gaugino mass parameters and slepton masses and left-right mixing. Despite the generality of this framework, we find stringent upper bounds on charged superpartners, with strong implications for future collider searches. We then consider minimal supergravity, and find remarkable consistency of the a_μ deviation with all present constraints from colliders, dark matter searches, and precision observables, such as $B \to X_s\gamma$. This is not to be taken for granted: as an illustration, we show that the current a_μ result disfavors anomaly-mediated supersymmetry breaking.

Supersymmetric contributions to a_μ have been explored for many years [5]. Following the recent $g_\mu - 2$ result, the implications for supersymmetry have been considered in many studies [6-30] in a variety of frameworks, including minimal supergravity [8, 9, 12, 13, 16, 20, 28], no-scale supergravity [14, 17, 20], models with non-universality [13, 22, 29, 30], inverted hierarchy [20], gauge-mediation [9, 14, 16, 20], anomaly-mediation [14, 20], R-parity violation [15, 26], and CP-violation [11], as well as flavor models [10, 19, 21, 23, 24], and string-inspired frameworks [14, 16, 18, 24].

The anomalous magnetic moment of the muon is the coefficient of the operator $a_\mu \frac{e}{4m_\mu}\, \bar{\mu}\sigma^{mn}\mu\, F_{mn}$, where $\sigma^{mn} = \frac{i}{2}[\gamma^m, \gamma^n]$. The supersymmetric contribution, $a_\mu^{\rm SUSY}$, is dominated by diagrams with neutralino-smuon and chargino-sneutrino loops. In the absence of significant slepton flavor violation, these diagrams are completely determined by only seven supersymmetry parameters: M_1, M_2, μ, $\tan\beta$, $m_{\tilde{\mu}_L}$, $m_{\tilde{\mu}_R}$, and A_μ. In general, M_1, M_2, μ, and A_μ are complex. However, bounds from electric dipole moments generically require their phases to be very small. In addition, $|a_\mu^{\rm SUSY}|$ is typically maximized for real parameters. In deriving model-independent upper bounds on superparticle masses below, we assume real parameters, but consider all possible sign combinations; these results are therefore valid for arbitrary phases.

To determine the possible values of $a_\mu^{\rm SUSY}$ without model-dependent biases, we have calculated $a_\mu^{\rm SUSY}$ in a series of high statistics scans of parameter space, requiring only consistency with collider bounds and a neutral stable lightest supersymmetric particle (LSP). (LSPs that decay visibly in collider detectors are examined in Ref. [3].) We begin by scanning over the parameters M_2, μ, $m_{\tilde{\mu}_L}$, and $m_{\tilde{\mu}_R}$, assuming gaugino mass unification $M_1 = M_2/2$, $A_\mu = 0$, and $\tan\beta = 50$. The free parameters take values up to 2.5 TeV. The resulting values in the $(M_{\rm LOSP}, a_\mu^{\rm SUSY})$ plane are given by the points in Fig. 1. We then consider arbitrary (positive and negative) values of M_2/M_1, leading to possibilities bounded by the solid curve. Finally, we allow any A_μ in the interval $[-100\ {\rm TeV}, 100\ {\rm TeV}]$,

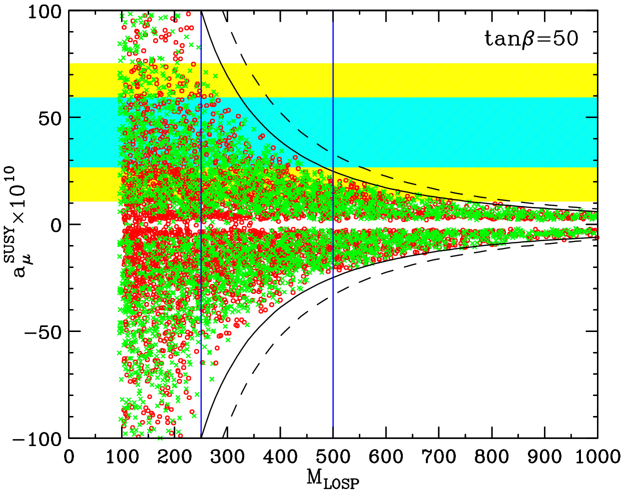

FIG. 1: Allowed values of $M_{\rm LOSP}$, the mass of the lightest observable supersymmetric particle, and $a_\mu^{\rm SUSY}$ from a scan of parameter space with $M_1 = M_2/2$, $A_\mu = 0$, and $\tan\beta = 50$. Green crosses (red circles) have smuons (charginos/neutralinos) as the LOSP. A stable LSP is assumed. Relaxing the relation $M_1 = M_2/2$ leads to the solid envelope curve, and further allowing arbitrary left-right smuon mixing (large A_μ) leads to the dashed curve. The envelope contours scale linearly with $\tan\beta$. The 1σ and 2σ allowed $a_\mu^{\rm SUSY}$ ranges are shown, and the discovery reaches of linear colliders with $\sqrt{s} = 500$ GeV and 1 TeV are given by the vertical blue lines.

which extends the envelope curve to the dashed contour of Fig. 1. As can be seen, allowing large A_μ, *i.e.*, large left-right smuon mixing, significantly enlarges the range of possible a_μ. The envelope contours scale linearly with $\tan\beta$ to excellent approximation.

From Fig. 1 we see that the measured deviation in a_μ is in the range accessible to supersymmetric theories and is easily explained by supersymmetric effects.

The anomaly in a_μ also has strong implications for the superpartner spectrum. Among the most important is that at least two superpartners cannot decouple if supersymmetry is to explain the deviation, and one of these must be charged and so observable at colliders. Non-vanishing $a_\mu^{\rm SUSY}$ thus imply upper bounds on $M_{\rm LOSP}$. The *dashed* contour is parametrized by

$$\begin{array}{l}\text{Arbitrary}\\ \text{LR mixing}\end{array} : \quad \frac{a_\mu^{\rm SUSY}}{43 \times 10^{-10}} = \frac{\tan\beta}{50}\left(\frac{450\ \text{GeV}}{M_{\rm LOSP}^{\rm max}}\right)^2 .$$

If $a_\mu^{\rm SUSY}$ is to be within 1σ (2σ) of the measured deviation, at least one observable superpartner must be lighter than 570 GeV (850 GeV).

These upper bounds have many implications. They improve the prospects for observation of weakly-interacting superpartners at the Tevatron and LHC. They also impact linear colliders: in this model-independent framework, an observable supersymmetry signal is guaranteed at a 1.2 TeV linear collider if one demands 1σ consistency in a_μ; improvements in a_μ measurements may significantly strengthen such

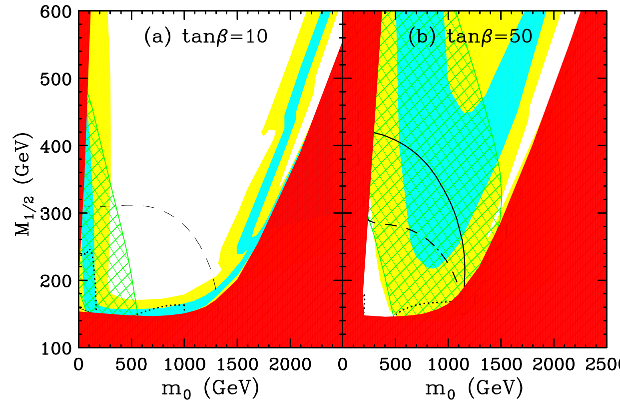

FIG. 2: The 2σ allowed region for $a_\mu^{\rm SUSY}$ (hatched) in minimal supergravity, for $A_0 = 0$, $\mu > 0$, and two representative values of $\tan\beta$. The dark red regions are excluded by the requirement of a neutral LSP and by the chargino mass limit of 103 GeV, and the medium blue (light yellow) region has LSP relic density $0.1 \le \Omega h^2 \le 0.3$ ($0.025 \le \Omega h^2 \le 1$). The area below the solid (dashed) contour is excluded by $B \to X_s\gamma$ (the Higgs boson mass), and the regions probed by the tri-lepton search at Tevatron Run II are below the dotted contours.

conclusions. Finally, these bounds provide fresh impetus for searches for lepton flavor violation, which is also mediated by sleptons and charginos/neutralinos.

We now turn to specific models, where a_μ is correlated to many other observables. We first consider the framework of minimal supergravity, where models are completely specified by the parameters m_0, $M_{1/2}$, A_0, $\tan\beta$, and sign(μ). The first three are the universal scalar, gaugino, and trilinear coupling masses at $M_{\rm GUT} \simeq 2\times 10^{16}$ GeV. We determine the entire weak scale superparticle spectrum through two-loop renormalization group equations [31] with one-loop threshold corrections and superparticle masses [32].

In minimal supergravity, many potential low-energy effects are eliminated by scalar degeneracy. However, $a_\mu^{\rm SUSY}$ is not suppressed in this way and may be large. In this framework, sign$(a_\mu^{\rm SUSY})$ = sign$(\mu M_{1,2})$. As is well-known, however, the sign of μ also enters in the supersymmetric contributions to $B \to X_s\gamma$. Current constraints on $B \to X_s\gamma$ require $\mu M_3 > 0$ if $\tan\beta$ is large. Gaugino mass unification implies $M_{1,2}M_3 > 0$, and so a large discrepancy in a_μ is only possible for $a_\mu^{\rm SUSY} > 0$, in accord with the new measurement.

In Fig. 2, the 2σ allowed region for $a_\mu^{\rm SUSY}$ is plotted for $\mu > 0$. Several important constraints are also included: bounds on the neutralino relic density, the Higgs boson mass limit $m_h > 113.5$ GeV, and the 2σ constraint $2.18\times 10^{-4} < B(B \to X_s\gamma) < 4.10\times 10^{-4}$.

For moderate $\tan\beta$, much of the region preferred by $a_\mu^{\rm SUSY}$ is excluded by m_h. What remains, however, is consistent with all experimental constraints and the requirement of supersymmetric dark matter. For large $\tan\beta$, $a_\mu^{\rm SUSY}$ favors a large allowed area that extends to large $M_{1/2}$ and $m_0 \approx 1.5$ TeV, a region of parameter space again consistent with all constraints and possessing the desired relic density [33]. The cosmologically preferred regions of minimal supergravity are probed by many pre-LHC dark matter experiments [34]. Note, however, that the sign of μ preferred by a_μ implies destructive interference in the leptonic decays of the second lightest neutralino, and so the Tevatron search for trileptons is ineffective for 200 GeV $< m_0 <$ 400 GeV [35].

We close by considering anomaly-mediated supersymmetry breaking. One of the most striking predictions of this framework is that the gaugino masses are proportional to the corresponding beta function coefficients, and so $M_{1,2}M_3 < 0$. Anomaly-mediation therefore most naturally predicts $a_\mu < 0$ [36, 37], in contrast to the observed deviation. A more detailed quantitative analysis [3] in minimal anomaly-mediation finds that, even allowing a 1σ deviation in a_μ, it is barely possible to obtain 2σ consistency with the $B \to X_s\gamma$ constraint. Minimal anomaly mediation is therefore disfavored. The dependence of this argument on the characteristic gaugino mass relations of anomaly mediation suggests that similar conclusions will remain valid beyond the minimal model.

In conclusion, the recently reported deviation in a_μ is easily accommodated in supersymmetric models. Its value provides *model-independent* upper bounds on masses of observable superpartners and already discriminates between well-motivated models.

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[1] H. N. Brown *et al.* [Muon g-2 Collaboration], Phys. Rev. Lett. **86**, 2227 (2001) [hep-ex/0102017].

[2] A. Czarnecki and W. J. Marciano, Phys. Rev. D **64**, 013014 (2001) [hep-ph/0102122].

[3] J. L. Feng and K. T. Matchev, Phys. Rev. Lett. **86**, 3480 (2001) [hep-ph/0102146].

[4] J. L. Feng, K. T. Matchev and Y. Shadmi, Nucl. Phys. B **613**, 366 (2001) [hep-ph/0107182].

[5] P. Fayet, in *Unification of the Fundamental Particle Interactions*, edited by S. Ferrara, J. Ellis, and P. van Nieuwenhuizen (Plenum, New York, 1980), p. 587; J. A. Grifols and A. Mendez, Phys. Rev. **D 26**, 1809 (1982); J. Ellis, J. Hagelin and D. V. Nanopoulos, Phys. Lett. **B116**, 283 (1982); R. Barbieri and L. Maiani, Phys. Lett. **B117**, 203 (1982); D. A. Kosower, L. M. Krauss and N. Sakai, Phys. Lett. **B133**, 305 (1983); T. C. Yuan, R. Arnowitt, A. H. Chamseddine and P. Nath, Z. Phys. **C26**, 407 (1984); T. Moroi, Phys. Rev. **D 53**, 6565 (1996) [hep-ph/9512396]; Erratum, **D 56**, 4424 (1997); M. Carena, G. F. Giudice and C. E. Wagner, Phys. Lett. **B390**, 234 (1997) [hep-ph/9610233]; E. Gabrielli and U. Sarid, Phys. Rev. Lett. **79**, 4752 (1997) [hep-ph/9707546]; K. T. Mahanthappa and S. Oh, Phys. Rev. **D 62**, 015012 (2000) [hep-ph/9908531].

[6] L. L. Everett, G. L. Kane, S. Rigolin, L. T. Wang, Phys. Rev. Lett. **86**, 3484 (2001) [hep-ph/0102145].

[7] E. A. Baltz and P. Gondolo, Phys. Rev. Lett. **86**, 5004 (2001) [hep-ph/0102147].

[8] U. Chattopadhyay and P. Nath, Phys. Rev. Lett. **86**, 5854 (2001) [hep-ph/0102157].

[9] S. Komine, T. Moroi and M. Yamaguchi, Phys. Lett. B **506**, 93 (2001) [hep-ph/0102204].

[10] J. Hisano and K. Tobe, Phys. Lett. B **510**, 197 (2001) [hep-ph/0102315].

[11] T. Ibrahim, U. Chattopadhyay and P. Nath, Phys. Rev. D **64**, 016010 (2001) [hep-ph/0102324].

[12] J. R. Ellis, D. V. Nanopoulos and K. A. Olive, Phys. Lett. B **508**, 65 (2001) [hep-ph/0102331].

[13] R. Arnowitt, B. Dutta, B. Hu and Y. Santoso, Phys. Lett. B **505**, 177 (2001) [hep-ph/0102344].

[14] K. Choi *et al.*, Phys. Rev. D **64**, 055001 (2001) [hep-ph/0103048].

[15] J. E. Kim, B. Kyae and H. M. Lee, hep-ph/0103054.

[16] S. P. Martin and J. D. Wells, Phys. Rev. D **64**, 035003 (2001) [hep-ph/0103067].

[17] S. Komine, T. Moroi and M. Yamaguchi, Phys. Lett. B **507**, 224 (2001) [hep-ph/0103182].

[18] S. Baek, P. Ko and H. S. Lee, hep-ph/0103218.

[19] D. F. Carvalho, J. R. Ellis, M. E. Gomez and S. Lola, Phys. Lett. B **515**, 323 (2001) [hep-ph/0103256].

[20] H. Baer, C. Balazs, J. Ferrandis and X. Tata, Phys. Rev. D **64**, 035004 (2001) [hep-ph/0103280].

[21] S. Baek, T. Goto, Y. Okada and K. i. Okumura, Phys. Rev. D **64**, 095001 (2001) [hep-ph/0104146].

[22] D. G. Cerdeno *et al.*, Phys. Rev. D **64**, 093012 (2001) [hep-ph/0104242].

[23] Z. Chacko and G. D. Kribs, Phys. Rev. D **64**, 075015 (2001) [hep-ph/0104317].

[24] T. Blazek and S. F. King, Phys. Lett. B **518**, 109 (2001) [hep-ph/0105005].

[25] G. C. Cho and K. Hagiwara, Phys. Lett. B **514**, 123 (2001) [hep-ph/0105037].

[26] R. Adhikari and G. Rajasekaran, hep-ph/0107279.

[27] M. Byrne, C. Kolda, J. E. Lennon, hep-ph/0108122.

[28] S. Komine and M. Yamaguchi, hep-ph/0110032.

[29] U. Chattopadhyay and P. Nath, hep-ph/0110341.

[30] M. Endo and T. Moroi, hep-ph/0110383.

[31] I. Jack *et al.*, Phys. Rev. **D50**, 5481 (1994) [hep-ph/9407291].

[32] D. M. Pierce, J. A. Bagger, K. Matchev and R. Zhang, Nucl. Phys. **B491**, 3 (1997) [hep-ph/9606211].

[33] J. L. Feng, K. T. Matchev and T. Moroi, Phys. Rev. Lett. **84**, 2322 (2000) [hep-ph/9908309]; Phys. Rev. D **61**, 075005 (2000) [hep-ph/9909334]; J. L. Feng and K. T. Matchev, Phys. Rev. D **63**, 095003 (2001) [hep-ph/0011356].

[34] J. L. Feng, K. T. Matchev and F. Wilczek, Phys. Lett. B **482**, 388 (2000) [hep-ph/0004043]; Phys. Rev. D **63**, 045024 (2001) [astro-ph/0008115].

[35] K. T. Matchev and D. M. Pierce, Phys. Rev. **D 60**, 075004 (1999) [hep-ph/9904282].

[36] J. L. Feng and T. Moroi, Phys. Rev. **D 61**, 095004 (2000) [hep-ph/9907319].

[37] U. Chattopadhyay, D. K. Ghosh and S. Roy, Phys. Rev. **D 62**, 115001 (2000) [hep-ph/0006049].